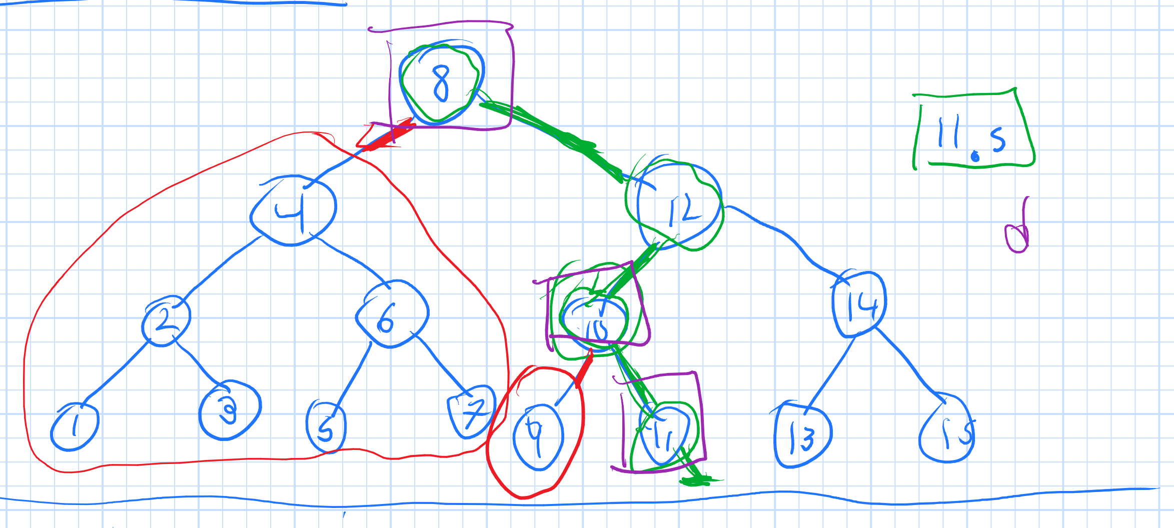
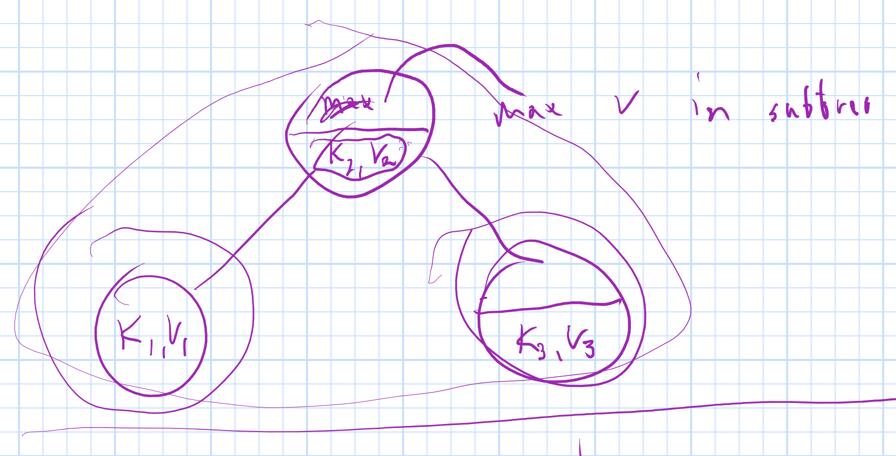
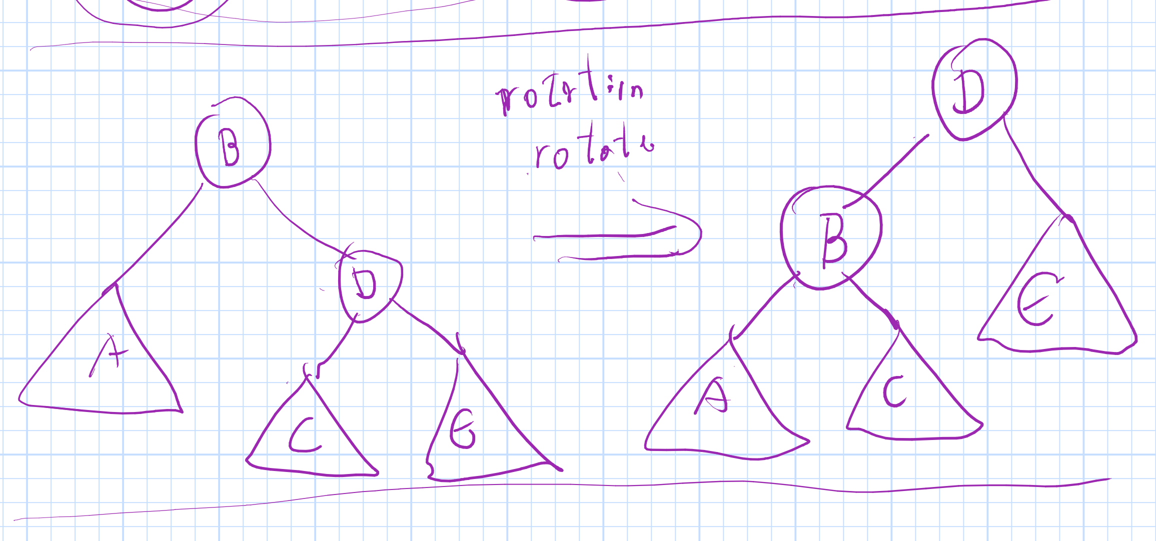
L8: DP [Last]



- Longest increasing subsequence
- Longest common subsequence
- k-Steiner tree.





Query $(k_1,v_1),\dots,(k_n,v_n)$ items currently stored.

Query: $\bar{k}$ - key value

return: $\max_{i: k_i < k} v_i$

Time ins/del/query $O(\log n)$.

Longest increasing subsequence (LIS)

$A[1\dots n]$ real numbers

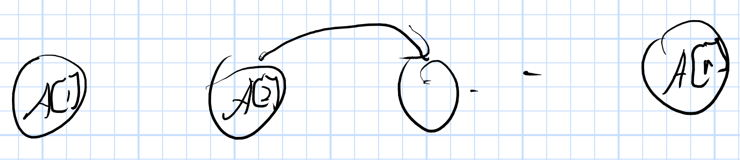
$i_1 < i_2 < i_3 < \dots < i_k$

$A[i_1] < A[i_2] < A[i_3] < \dots < A[i_k]$

7, -5, 13, 473, 3.141592, 1914, 1646

1 n

$i<j$ insert edge $i \to j$ if $A[i]<A[j]$



~~DAG~~ $O(n^2)$

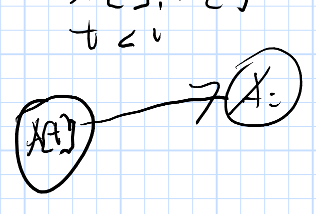
$\ell(i)$: max len of a subseq ending at $A[i]$ and increasing

$$\ell(i) = \max_{\substack{t<i \\ A[t]<A[i]}} (\ell(t)+1)$$

n possible values

$O(n)$

$A[t]<A[i]$, $t<i$



$O(n^2)$

D ← data structure

(K, V)

$(\underbrace{A[i]}_{K}, \underbrace{\ell(i)}_{V})$

Computed for $i=1,\dots,t$.

$$\ell(t+1) = \left(\max_{\substack{(k,v)\in D \\ k<A[t+1]}} v\right) + 1 \Rightarrow (A[t+1], \ell(t+1))$$

insert to D.

query of the data structure!
$O(\log n)$

Overall $O(n \log n)$.

LCS: Longest common subsequence

$A[1\dots n]$

$B[1\dots m]$ two sequences

6, 7, 45, 9, 13, 17, 99, 44

13, 7, 9.9, 99, 6, 44

a_{i-1} | a_i b_j only if equal

$O(mn)$ time [edit distance]

$K = \{(i,j) \mid A[i]=B[j]\}$

Set of all pairs that collide.

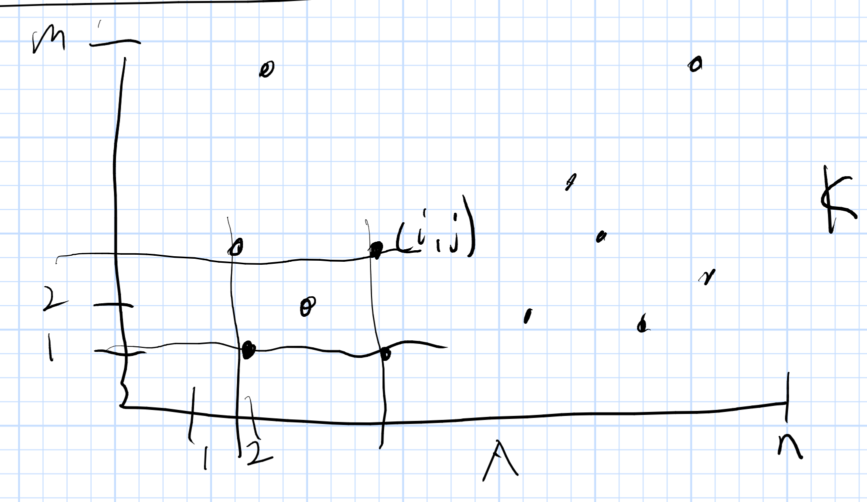
$0 \le |K| \le mn$

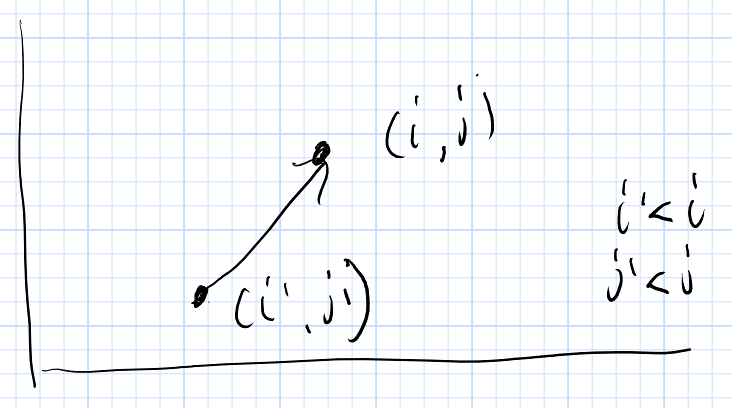
$k=|K|$ this is "small"

sorted $A = [3, 7, 7, 14, 14 \dots]$

sorted $B = [2, 3, 7, 7, 7, 7, 12, 14 \dots]$

$O(n\log n + m\log m + n + m + k)$





Sort the points of K by increasing value in x

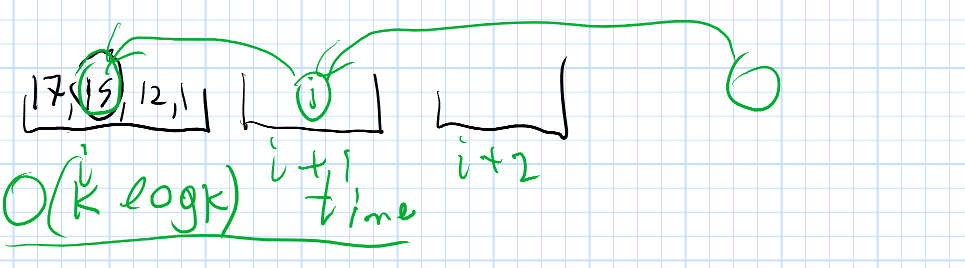
Sort point with same x coord with decreasing y coordinate.

$P_1, P_2, P_3, \dots, P_k$

$y(P_1)$

$j(P_1), j(P_2), j(P_3), \dots, j(P_k)$ sequence

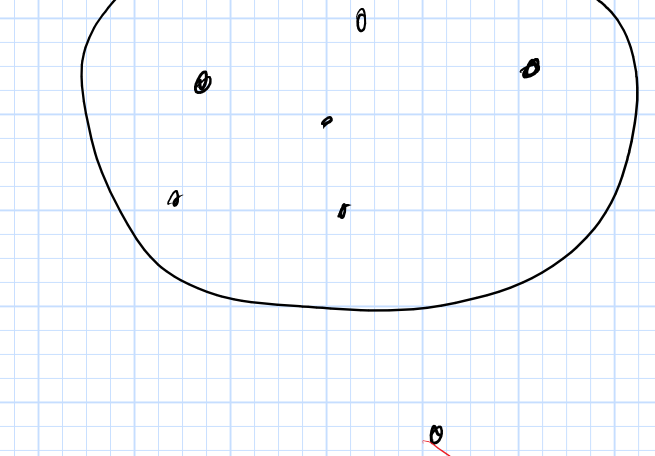
Compute LIS in this sequence



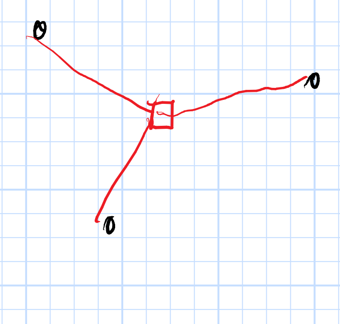
$O(k \log k)$ time

$O(n\log + m\log m + k\log k)$

$= O((n+m+k)\log(nmk))$

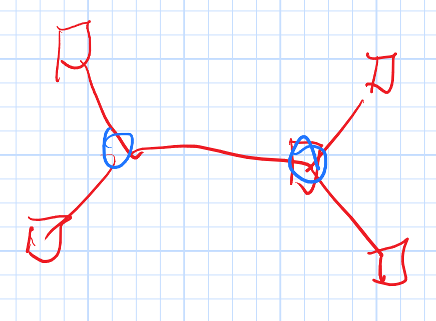


k-Steiner problem
positive weights on edges
k terminals
Connect terminals into one happy connected subgraph.



NP-hard

FPT - fixed parameter tractable



$n^{O(k)}$

$f(k)\, n^{O(1)}$

exponential

$\log 1 + \log 2 \dots + \log n$

$\frac{n}{2} \log \frac{n}{2}$ $\log \frac{n}{2}$ $\log \frac{n}{2} + 1$

$\frac{n}{2} \log \frac{n}{2} \le \log n! \le n \log n$