CS 473 8/29/2023

L3: Divide and Conquer revisited

$(\alpha x+\beta)(\gamma x+\delta) = \alpha\gamma x^2 + (\alpha\delta+\beta\gamma)x + \beta\delta$

$(\alpha+\beta)(\gamma+\delta) = \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta$

$M_1 = \alpha\gamma$
$M_2 = \beta\delta$
$M_3 = (\alpha+\beta)(\gamma+\delta)$

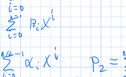
$(\alpha x+\beta)(\gamma x+\delta) = M_1 x^2 + (M_3 - M_1 - M_2)x + M_3$

$p(x) = \sum_{i=0}^{n-1} \alpha_i x^i$ $\qquad n = 2^i$

$q(x) = \sum_{i=0}^{n-1} p_i x^i$

$p_1 = \sum_{i=0}^{n/2-1} \alpha_i x^i \qquad p_2 = \sum_{i=0}^{n/2-1} \alpha_{i+n/2} x^i$

$p(x) = p_1(x) + p_2(x) x^{n/2}$



q_1, q_2

$p(x)q(x) = (p_1 + p_2 x^{n/2})(q_1 + q_2 x^{n/2})$ $\qquad x^{n/2}$

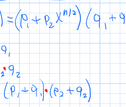
$M_1 = p_1 q_1$
$M_2 = p_2 q_2$
$M_3 = (p_1 + q_1)(p_2 + q_2)$

$p(x)q(x) = M_1 x^n + (M_3 - M_1 - M_2)x^{n/2} + M_2$

$p(x)q(x) = (p_1 + p_2 x^{n/2})(q_1 + q_2 x^{n/2})$
$= p_1 q_1 + (p_1 q_2 + p_2 q_1)x^{n/2} + p_2 q_2 x^n$

$T(n) = O(n) + 4T(\frac{n}{2}) = O(n^2)$

$T(n) = O(n) + 3T(\frac{n}{2})$



ith level $\quad n/2^i$

3^i

$W_i = O(3^i \cdot \frac{n}{2^i}) = (\frac{3}{2})^i n$

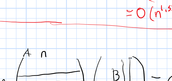
$\sum_{i=0}^{\log_2 n} W_i = \sum_{i=0}^{\log_2 n} (\frac{3}{2})^i n = O(\sum_{i=0}^{\log_2 n} (\frac{3}{2})^i)$

$= O(n \frac{3^{\log_2 n}}{2^{\log_2 n}}) = O(3^{\log_2 n})$

$= 2^{\log_2 n \cdot \log_2 3}$

$= O(n^{\log_2 3})$

$= O(n^{1.585}) << O(n^2)$



$C_{ij} = A_{i:} B_{:j}$ $\quad O(n)$

n^2 $\quad O(n^3)$

A_{11}	A_{12}
A_{21}	A_{22}

B_{11}	B_{12}
B_{21}	B_{22}

$A_{11}B_{11} + A_{12}B_{21}$

$T(n) = O(n^2) + 8T(\frac{n}{2})$
$= O(n^3)$

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7 multiplications

$O(n^{2.31\ldots})$

$O(n^2)$?

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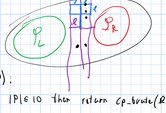
closest pair

$P \subset \mathbb{R}^2 \quad |P| = n$

$cp(P) =$ min pair $p, q \in P$ $\quad \|p-q\|$



$cp(P)$:
if $|P| \le 10$ then return cp_brute(P).
$x_{mid} \leftarrow$ median $X(P)$
$P_L, P_R \leftarrow P$ split by x_{mid}
$d_L \leftarrow cp(P_L)$
$d_R \leftarrow cp(P_R)$
$d = \min(d_L, d_R)$

$T(n) = O(n) + 2T(\frac{n}{2}) = O(n \log n)$

Convolution

$A[1 \ldots n] \quad B[1 \ldots n]$

$A \cdot B = \sum_{i=1}^{n} A[i] B[i]$ $\quad$ dot product

$\langle A, B \rangle$

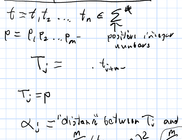
$A_i = [\underbrace{0 \ldots 0}_{i-1 \text{ times}}, A[1], A[2] \ldots A[n-i]]$

$A_{:i} = [A[i+1], \ldots, A[n], 0 \ldots 0]$

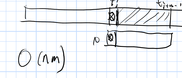
$i = -n+1, \ldots 0, \ldots n-1$

$A_{:} \cdot B$

FFT



$\left[\left(\sum \alpha_i x^i\right)\left(\sum \beta_i x^i\right)\right] = \sum \alpha_i \beta_{-i}$ $\quad$ coefficients of x^k



Text pattern matching

$t = t_1 t_2 \ldots t_n \in \Sigma_n^*$

$p = p_1 p_2 \ldots p_m$ $\quad$ positive integer numbers

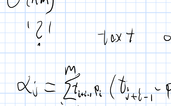
$T_j = \quad . t_{j+m-1}$

$T_j = p$

$\alpha_j =$ "distance" between T_j and p

$= \sum_{i=1}^{m} (t_{j+i-1} - p_i)^2 = \sum_{i=1}^{m} t_{j+i-1}^2 - 2\sum_{i=1}^{m} t_{j+i-1} p_i + \sum_{i=1}^{m} p_i^2$

$O(1)$



$O(nm)$

'?' $\quad$ text $\quad$ or $\quad$ pattern

$\alpha_j = \sum_{i=1}^{m} t_{j+i-1} (t_{j+i-1} - p_i)^2$