# HW 2

Due on **Wednesday, September 6, 2023 at 9pm**

**CS 473: Algorithms, Fall 2023** Version: **1.0**

Submissions instructions: As in previous homework.

**4** (100 PTS.) Smallest square

Let $P$ be a set of $n$ points in the plane, and let $k$ be a parameter. Describe a divide and conquer algorithm that computes the smallest (axis-parallel) square containing $k$ points of $P$. Your algorithm should have running time $O(n \log n)$ if $k = O(1)$. What is the running time of your algorithm? Prove that your algorithm indeed outputs the smallest such square.

(Hint: Extend the divide-and-conquer algorithm for closest pair.)

**5** (100 PTS.) Evaluate this.

You are given a sequence $P = \{p_1 < p_2 < \cdots < p_n\}$ of $n$ distinct real numbers. You are also given a set $L = \{f_i(x) = \alpha_i x + \beta_i \mid i = 1, \ldots, n\}$ of $n$ linear functions. Consider the max function $f(x) = \max_i f_i(x)$. Describe a direct (i.e., without computing other structures first) divide-and-conquer algorithm, as fast as possible, that computes the values $y_i = f(p_i)$, for $i = 1, \ldots, n$.

[Hint: (I) Draw an example of how the functions $f_i$ and $f$ look like in the $xy$ plane. (II) Compute the function $f_i$ realizing $f(p_{n/2})$.]

**6** (100 PTS.) Sorting network for $k$-nice inputs.

A sequence of $n$ numbers $x_1, \ldots, x_n$ is ***$k$-nice*** if there is a permutation $\pi : [\![n]\!] \to [\![n]\!]$ such that $x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)}$ is sorted (say in increasing order), and $|\pi(i) - i| \leq k$, for all $i \in [\![n]\!] = \{1, 2, \ldots, n\}$.

Describe a sorting network, using the construction seen in class, that sorts such $k$-nice inputs, where $k$ and $n$ are provided in advance (you can assume both are powers of two). How many gates does your network use? What is the depth of your sorting network?