Tuesday, November 16, 2021 3:30 PM

Applications:..

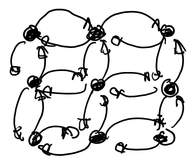
1) Computer-vision
2) Management
3) sports

Recall

1) Given a flow network G with integer capacities, then one can determine a maximum-flow between s & t in $O(nmf)$ time. Furthermore, the max-flow is integral.

2) max-flow from s to t = min. cut separating s & t

Image-segmentation



Given:- 1) Directed graph $G = ([n], E)$

2) Priors
$a_1, a_2, \ldots, a_n$
$b_1, b_2, \ldots, b_n$

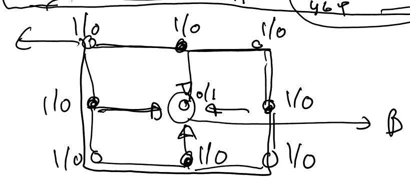
high value $a_i \rightarrow$ vertex/pixel i belongs foreground
high value $b_i \rightarrow$ - - - - - - background

2) $P(x, y)$ = capacities

Goal:- Find partition $[n]$ into X & Y, s.t.

$$\max \boxed{\sum_{v \in X} a_v + \sum_{v \in Y} b_v} - \left(\sum_{\substack{x \in X \\ y \in Y}} P(x,y)\right)$$

f



$\frac{9-4}{=5}$

$8 > 5$

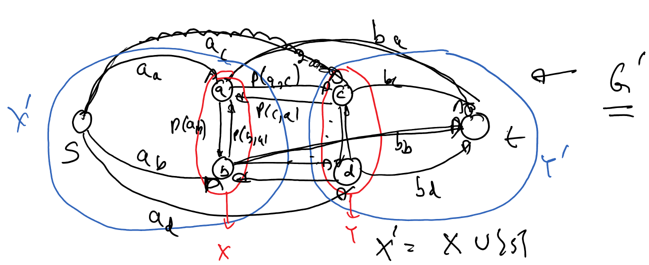
OBJECTIVE

$$\max \left\{ \sum_{v \in X} a_v + \sum_{v \in Y} b_v - \sum_{\substack{x \in X \\ y \in Y}} P(x,y) \right\}$$

$$= \min \left\{ \sum_{\substack{x \in X \\ y \in Y}} P(x,y) - \sum_{v \in X} a_v - \sum_{v \in Y} b_v \right\}$$

Add $\sum_{v \in [n]} a_v + \sum_{v \in [n]} b_v$

$$= \min \left\{ \sum_{\substack{x \in X \\ y \in Y}} P(x,y) + \sum_{v \in Y} a_v + \sum_{v \in X} b_v \right\}$$



$X' = X \cup \{s\}$
$Y' = Y \cup \{t\}$

$$c(X', Y') = \begin{cases} \sum_{v \in Y} a_v & \text{from } s \text{ to vertices in } Y' \\ \sum_{v \in X} b_v & \text{from } X' \text{ to } t \\ \sum_{\substack{x \in X \\ y \in Y}} p(x,y) & \text{from } X \text{ to } Y \end{cases}$$

$$\min \underline{c(X', Y')} = \min \left(\sum_{v \in Y} a_v + \sum_{v \in X} b_v + \sum_{\substack{x \in X \\ y \in Y}} p(x,y) \right)$$

Project selection

Given :- a set of projects $[n]$
profits for the projects $p_1, p_2, \ldots, p_n$.
dependency Graph $G = ([n], E)$

$i \to j$ = Project i can be completed only if project j is completed

Find :- a feasible set of projects X, that maximizes $\sum_{j \in X} p_j$

$$\max \sum_{j \in X} p_j$$

$$\max \sum_{\substack{j \in X \\ p_j > 0}} p_j - \left(\sum_{\substack{j \in X \\ p_j < 0}} - p_j \right)$$

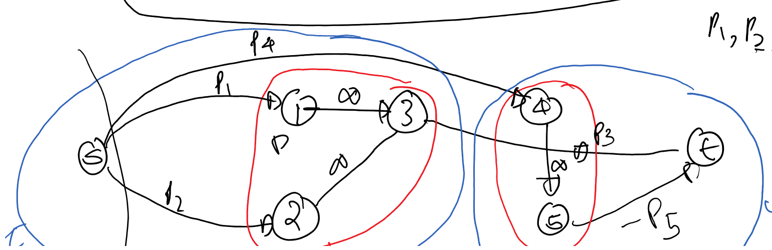
$$\Rightarrow \min \sum_{\substack{j \in X \\ p_j < 0}} (-p_j) - \sum_{\substack{j \in X \\ p_j > 0}} p_j$$

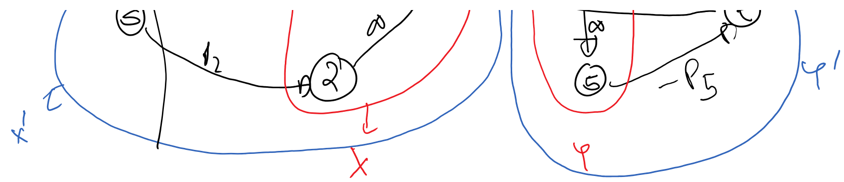
Add $\sum_{p_j > 0} p_j$

$$\boxed{\min \sum_{\substack{j \in X \\ p_j < 0}} (-p_j) + \sum_{\substack{j \notin X \\ p_j > 0}} p_j}$$

$p_1, p_2, p_4 > 0$

$p_3, p_5 < 0$





$X' = X \cup s$

$Y' = Y \cup t$

$$c(X', Y') = \begin{cases} \sum_{j \notin X, \, p_j > 0} p_j & \text{for all edges from } s \text{ to } Y' \\ \sum_{j \in X, \, p_j < 0} (-p_j) & \text{for edges from } X \text{ to } t \\ \infty & \text{for edges from } X \text{ to } Y \end{cases}$$

$$\underline{\min c(X', Y')} = \underline{\min_X \left\{ \sum_{j \notin X, \, p_j > 0} p_j + \sum_{j \in X, \, p_j < 0} (-p_j) \right\}}$$

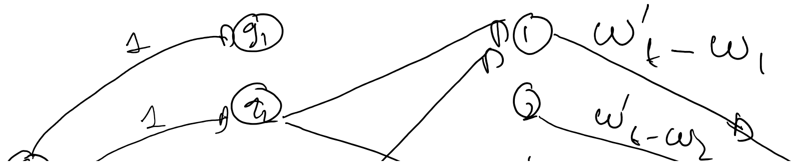
Base-ball elimination

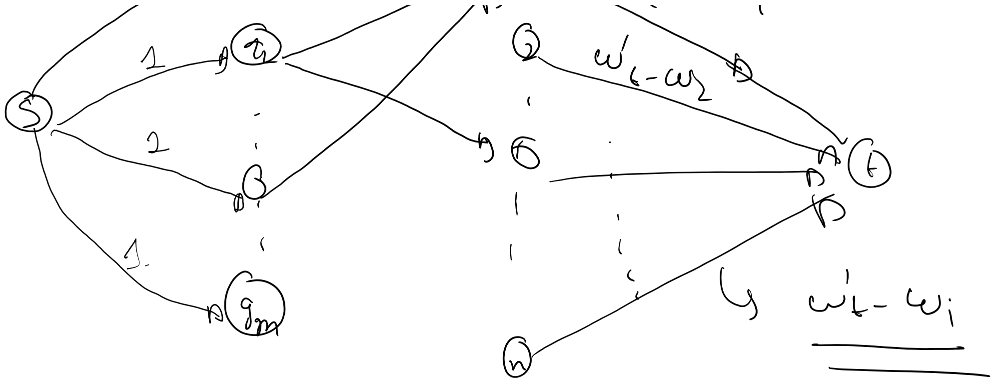
NY 92 Baltimore 91 Toronto 91 Boston 90

5 more games to be played b/w every pair except NY & Boston.

Given: - a set games $g_1, g_2, \ldots, g_m$
- a set of teams (n)
- winning points for every team $w_1, w_2, \ldots, w_n$, w_i = points gathered by team i upto this point
- team $\underline{t}$.

Find: Can team t earn at least as many points as any other team.





"soccer"

3 → 3 0

3 → 1 1

NP-hard!

$w'_t = w_t + $ |total number of games in which team t is involved|

Question: Is it possible to push a flow of m units from s to t?