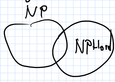
Lecture 3: Some NP complete problems 8/31/21

CSAT circuit satisfiability

Cook-Levin CSAT is NP complete

CSAT
Input: Boolean circuit.
Q: Is there a satisfying assignment?



SATISFIABILITY (SAT)

CNF formula

$x_1, x_2, \ldots, x_n \in \{0,1\}$

$C_1 \cap C_2 \cap C_3 \ldots C_m$

CNF = conjunctive normal form

$C_i = (x_i \vee \bar{x}_j \vee x_k \vee \ldots \vee x_t)$

$\bar{x}_j$: negation

SAT
Given a CNF formula is it satisfiable?

Lemma
SAT is NPC.

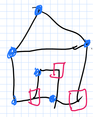
$CSAT \leq_p SAT$.

3SAT
SAT for 3CNF.
3CNF every clause has exactly three literal

$(x_1 \vee \bar{x}_2 \vee x_3)$

2SAT is polynomial!

Vertex Cover
G undirected, parameter k.
Q: Is there a set of k vertices that is a VC.



Coloring, 3-Coloring
Input: G, k
Q: Can the graph be colored by k colors.
k=3 3-coloring is NPC.

Hamiltonian Cycle
G: dir/undirected
Q: Is there a Hamiltonian cycle that visits all the vertices.

Hamiltonian cycle: visit every vertex exactly once
Eulerian cycle: visit every edge exactly once.

TSP: Traveling Salesperson Problem
Hamiltonian cycle of minimum cost.

Subset Sum
S: set of positive integers. T: target number
Q: Is there $X \subseteq S$ s.t.

$\sum_{x \in X} x = t.$ NPC

$U = \max_{s \in S}$

Solve subset sum in time $O((n+U)^{O(1)})$

3DM: Three dimensional matching
X, Y, Z
$n = |X| = |Y| = |Z|$

$T \subseteq X \times Y \times Z$

Q: Is there a set $S \subseteq T$ s.t. every element of $X \cup Y \cup Z$ is covered exactly once?

3DM is NPC.

Partition
S: set of n numbers positive integers.
Q: Is there $X \subseteq S$ s.t.

$\sum_{x \in X} x = \sum_{s \in S} s / 2$?

Partition is NPC

Set Cover
$U = \{1, 2, \ldots n\}$
F: Family of subsets of U
$F = \{\{1,2,4\}, \{7, 9, 12\}, \ldots\}$
k: parameter.
Q: Are there k sets in F that their union is U?

Set cover is NPC.

$3SAT \leq_p$ Independent Set

$F = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\quad)$