cs473: Algorithms	Assigned: *Fri., Oct. 16, 2020*

Problem Set #6

Forbes/Chekuri	Due: *Thu., Oct. 29, 2020 (5:00pm)*

All problems are of equal value.

1. Suppose f is an (s,t)-flow in a network $G = (V, E)$ and let f' be another (s,t)-flow. Prove that there is an (s,t)-flow of value $|f'| - |f|$ in the residual network G_f, where $|f|, |f'|$ are the values of the flows f, f' respectively. Note that $|f'| - |f|$ may be *negative*.

   Using the above briefly argue that if f is an (s,t)-flow then f is a maximum-flow iff there is no s-t path in G_f. Note that this gives an alternate proof of correctness of the correctness of the augmenting path algorithms.

2. Suppose $G = (V, E)$ is a flow-network with integer capacities and f is an integer valued s-t flow in G. Let $G' = (V, E')$ be the support of f; more formally $E' = \{e \in E \mid f(e) > 0\}$.

   - Given G and f describe a linear time algorithm to check whether f is a maximum flow in G.
   - Suppose e is contained in a directed cycle in G'. Show that there is another flow f' such that $f'(e) \le f(e) - 1$ and $|f'| = |f|$.
   - Assuming f is a maximum flow in G describe a linear-time algorithm to find a new maximum flow where the capacity of a specific edge e is reduced by one unit.

3. Let $G = (V, E)$ be a flow network with integer edge capacities. We have seen algorithms that compute *a* minimum s-t cut via maximum flow. For the problem below assume that you only have black box access to an algorithm that given G and nodes s, t outputs a minimum cut between s and t.

   - Describe a simple example of a flow-network G and two nodes s, t such that there are two distinct s-t minimum cuts with the same capacity but different number of edges in the cuts.
   - Given G and s, t and an integer k describe an algorithm that checks whether G has a minimum cut with at most k edges.

   No proof of correctness necessary but we recommend a brief justifaction. And make sure you have a clear and understandable algorithm.