Single-view 3D Reconstruction

Computational Photography

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Some slides from Alyosha Efros, Steve Seitz

Take-home question

Suppose you have estimated finite three vanishing points corresponding to orthogonal directions:

1) How to solve for intrinsic matrix? (assume K has three parameters)
   - The transpose of the rotation matrix is its inverse
   - Use the fact that the 3D directions are orthogonal
2) How to recover the rotation matrix that is aligned with the 3D axes defined by these points?
   - In homogeneous coordinates, 3d point at infinity is (X, Y, Z, 0)

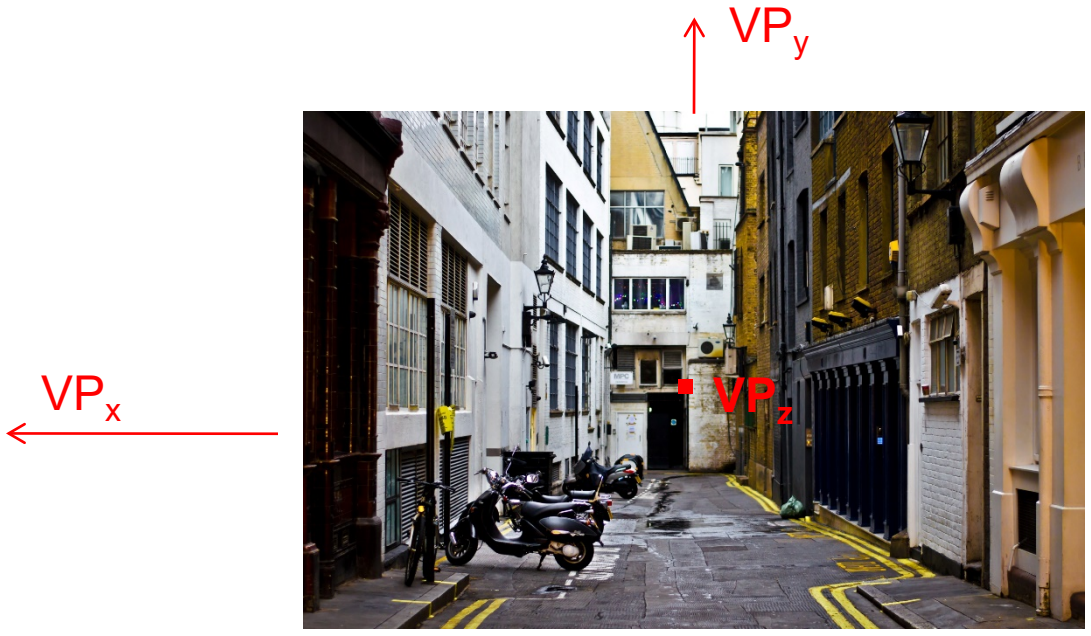


Photo from Garry Knight

Take-home question

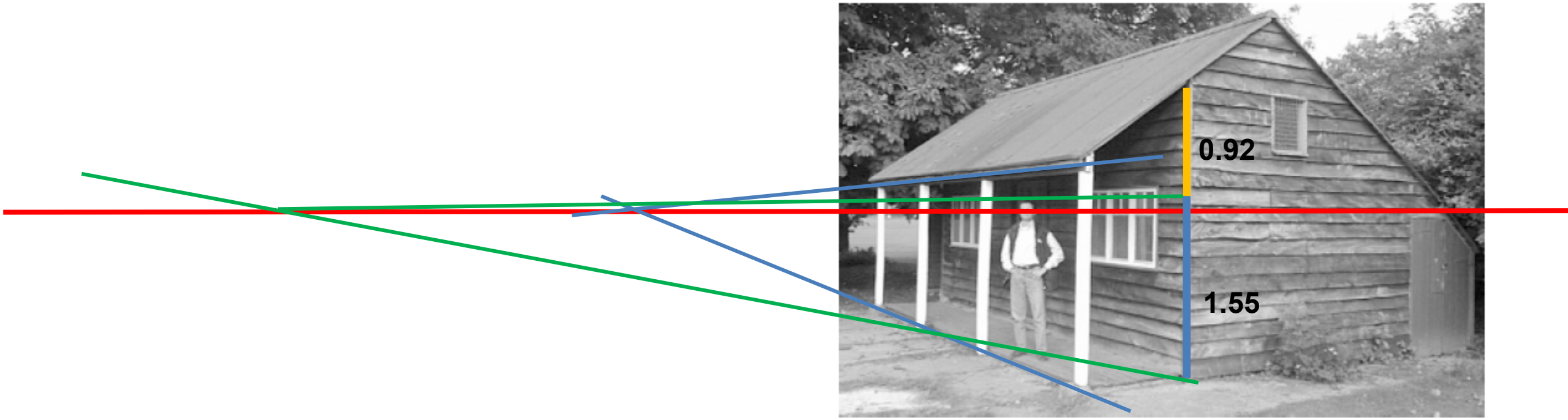
Assume that the man is 6 ft tall.

- What is the height of the front of the building?
- What is the height of the camera?

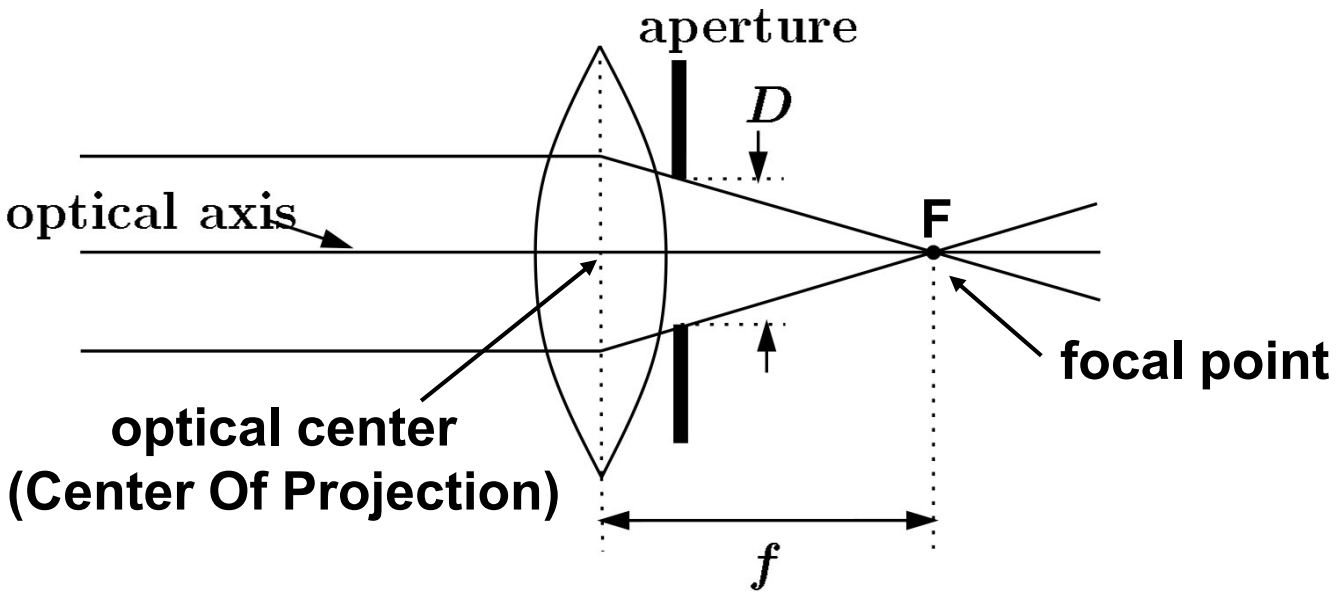
Take-home question

Assume that the man is 6 ft tall. (0.92+1.55)/1.55*6=9.56

- What is the height of the front of the building?
- What is the height of the camera? ~5'7



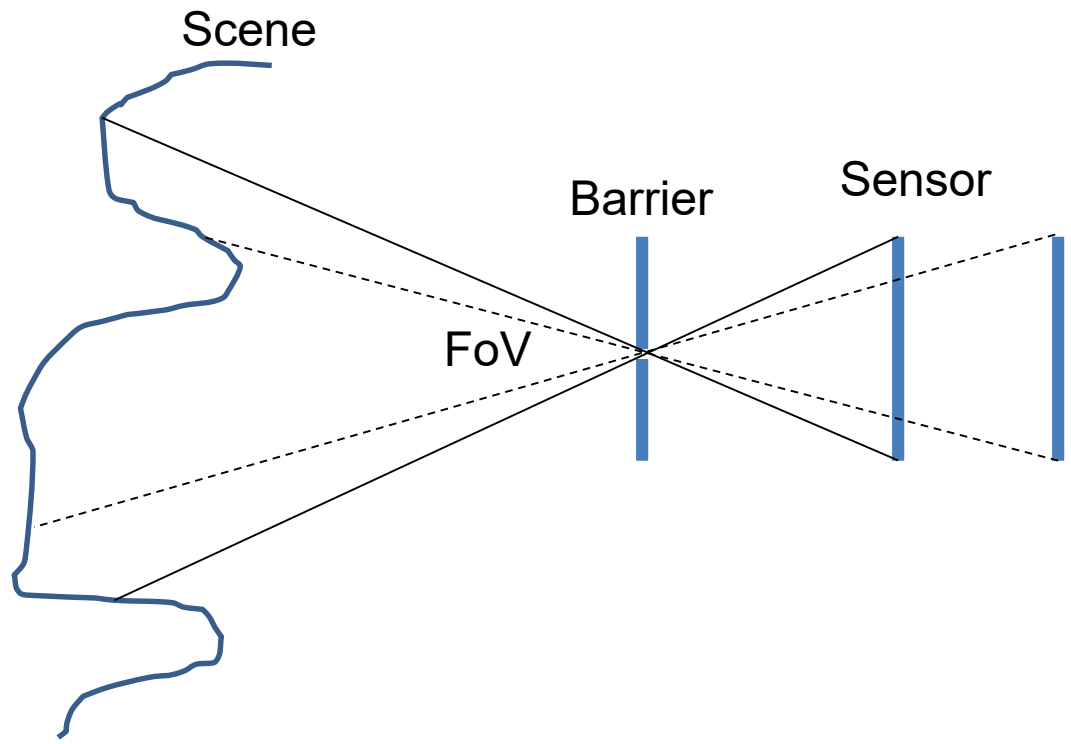
Focal length, aperture, depth of field



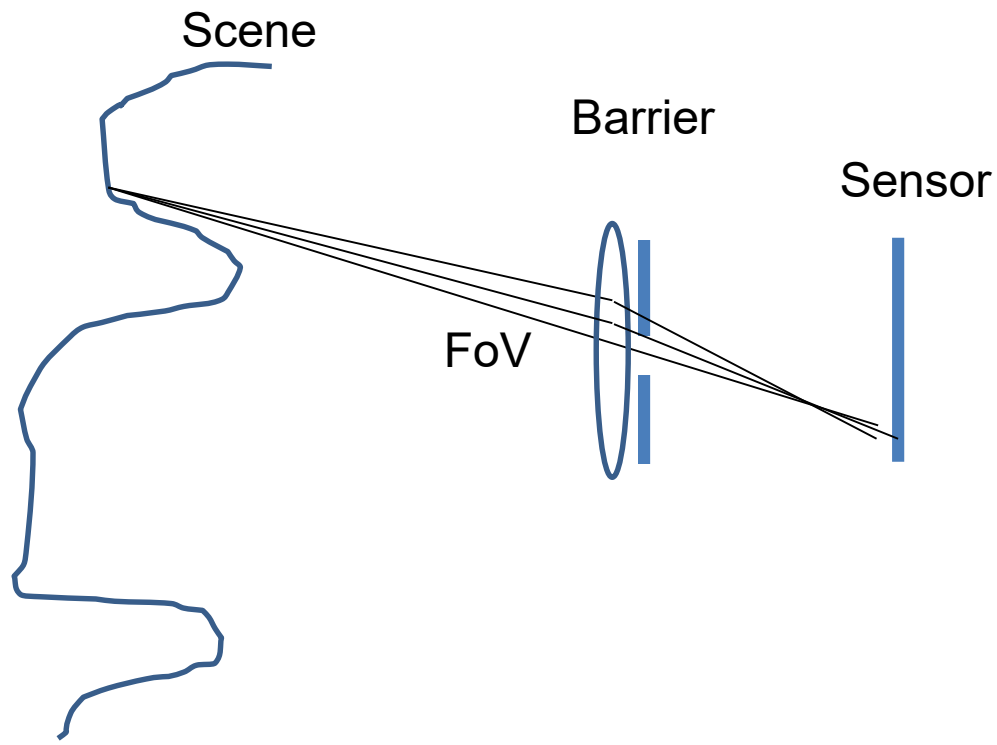
- Increase in focal length “zooms in”, decreasing field of view (and light per pixel), increasing depth of field (less blur)

- Increase in aperture lets more light in but decreases depth of field

Increasing focal length decreases field of view because smaller range of rays to scene can hit sensor



Decreasing aperture increases depth of field because lens refocuses rays from smaller range of angels



Difficulty in macro (close-up) photography

- For close objects, we have a small relative DOF
- Can only shrink aperture so far

How to get both bugs in focus?

Solution: Focus stacking

1. Take pictures with varying focal length

Example from
http://www.wonderfulphotos.com/articles/macro/focus_stacking/

Solution: Focus stacking

1. Take pictures with varying focal length
2. Combine

Focus stacking

http://www.wonderfulphotos.com/articles/macro/focus_stacking/

Focus stacking

How to combine?

1. Align images (e.g., using corresponding points)
2. Two ideas
   a) Mask regions by hand and combine with pyramid blend
   b) Gradient domain fusion (mixed gradient) without masking

Automatic solution would make an interesting final project

Recommended Reading:

http://www.digital-photography-school.com/an-introduction-to-focus-stacking

http://www.zen20934.zen.co.uk/photography/Workflow.htm#Focus%20Stacking

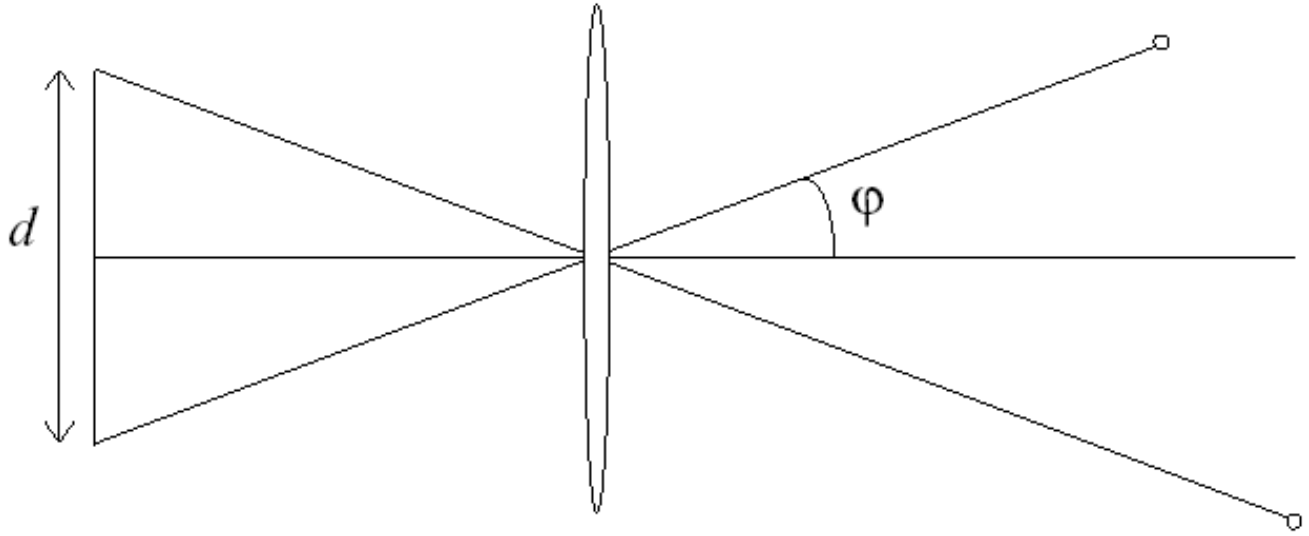
Relation between field of view and focal length

Field of view (angle width)

Film/Sensor Width

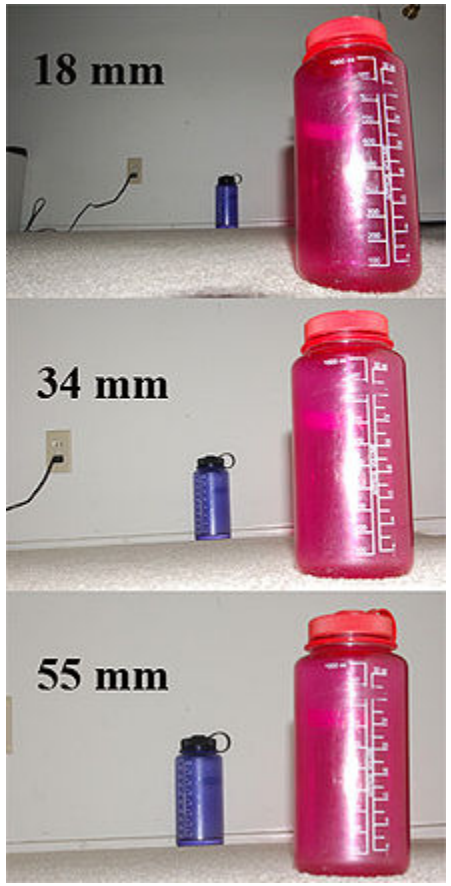
$$fov = 2\tan^{-1}\frac{d}{2f}$$

Focal length



Dolly Zoom or “Vertigo Effect”

http://www.youtube.com/watch?v=NB4bikrNzMk



How is this done?

Zoom in while moving away

http://en.wikipedia.org/wiki/Focal_length

Dolly zoom (or “Vertigo effect”)

Field of view (angle width)

Film/Sensor Width

$$fov = 2\tan^{-1}\frac{d}{2f}$$

Focal length

width of object

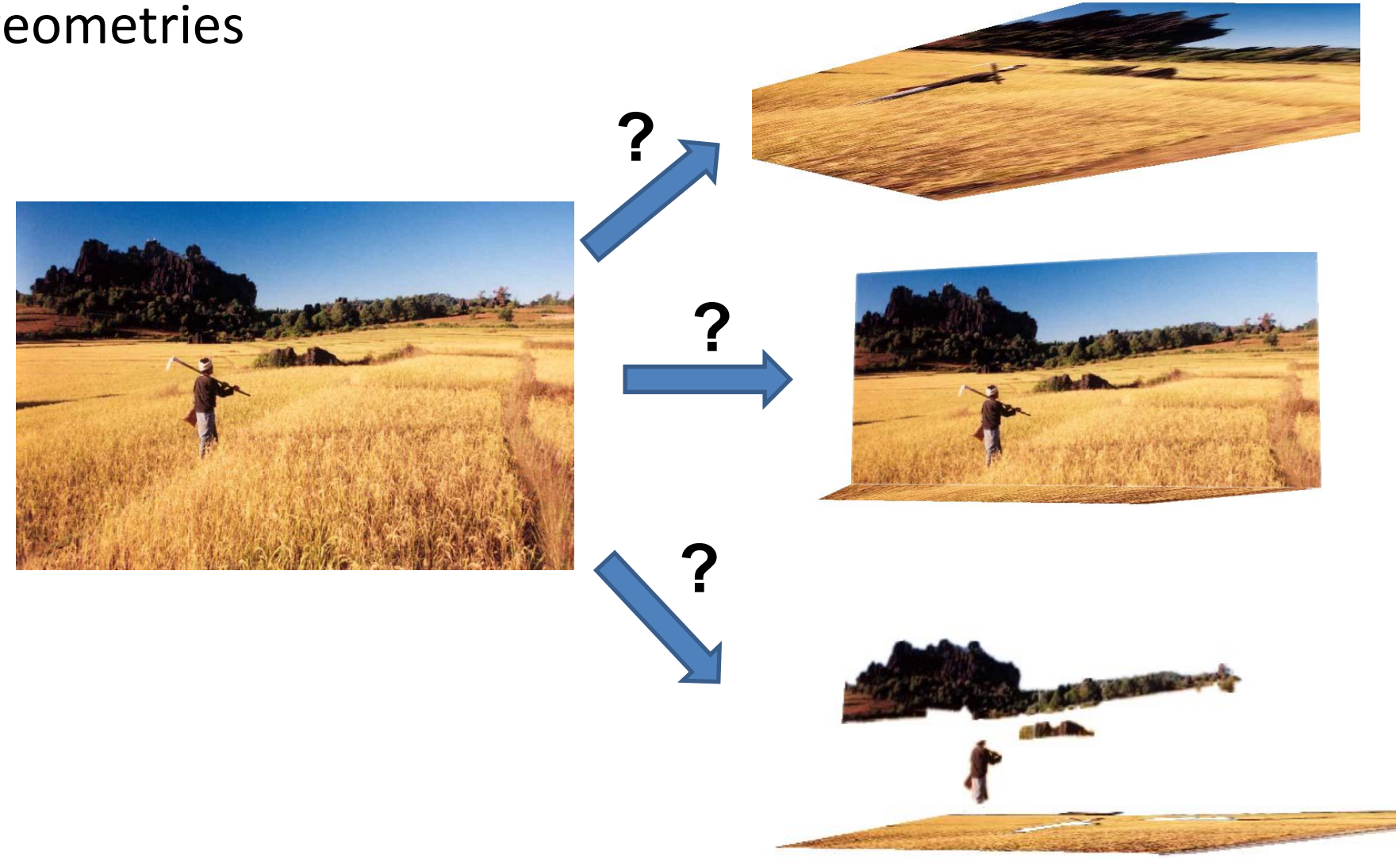
$$2\tan\frac{fov}{2} = \frac{width}{\text{distance}}$$

Distance between object and camera

Today’s class: 3D Reconstruction

The challenge

One 2D image could be generated by an infinite number of 3D geometries



The solution

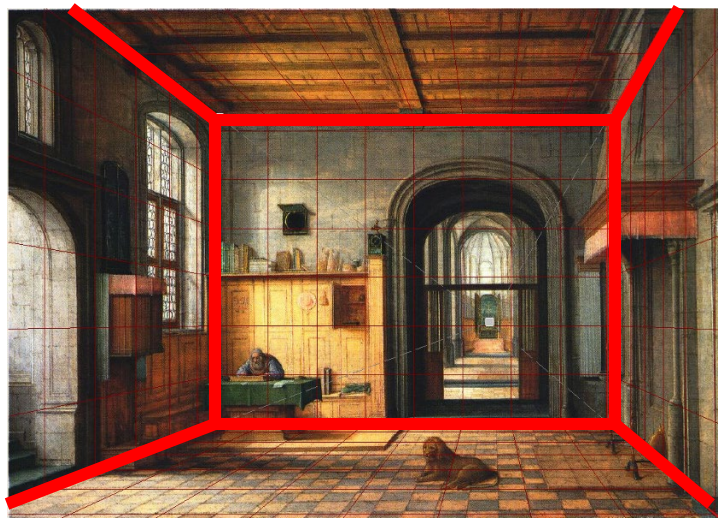
Make simplifying assumptions about 3D geometry

Unlikely

Likely

Today’s class: Two Models

- Box + frontal billboards



- Ground plane + non-frontal billboards

“Tour into the Picture” (Horry et al. SIGGRAPH ’97)

Create a 3D “theatre stage” of five planes

Specify foreground objects through bounding polygons

Use camera transformations to navigate through the scene

Following slides modified from Efros

The idea

Many scenes can be represented as an axis-aligned box volume (i.e. a stage)

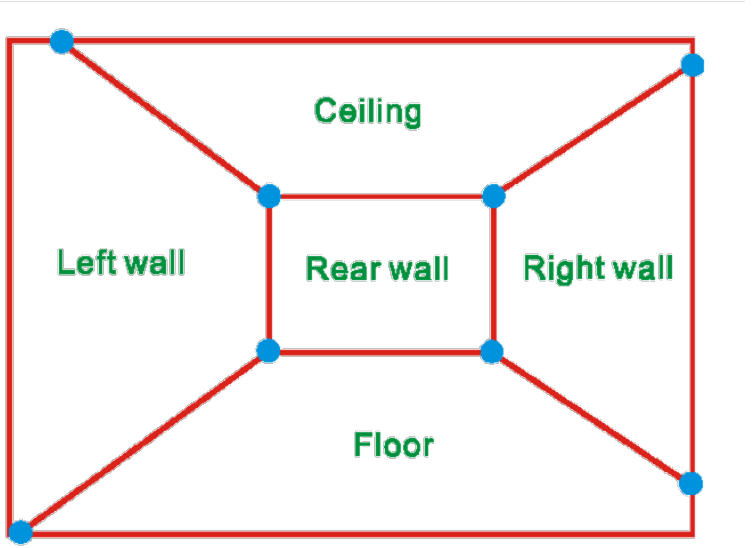
Key assumptions

- All walls are orthogonal
- Camera view plane is parallel to back of volume

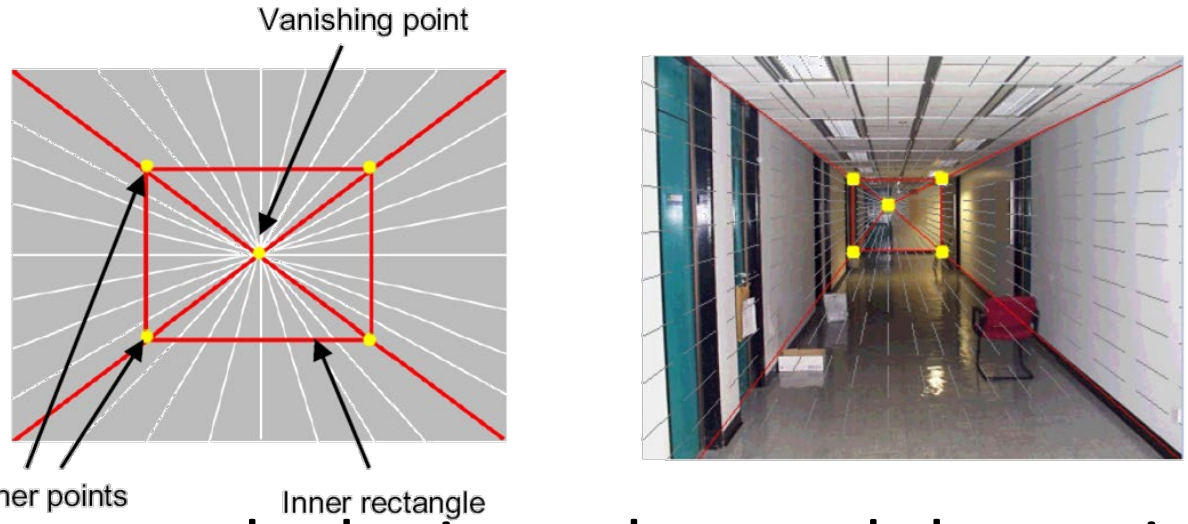
How many vanishing points does the box have?

- Three, but two at infinity
- Single-point perspective

Can use the vanishing point to fit the box to the particular scene



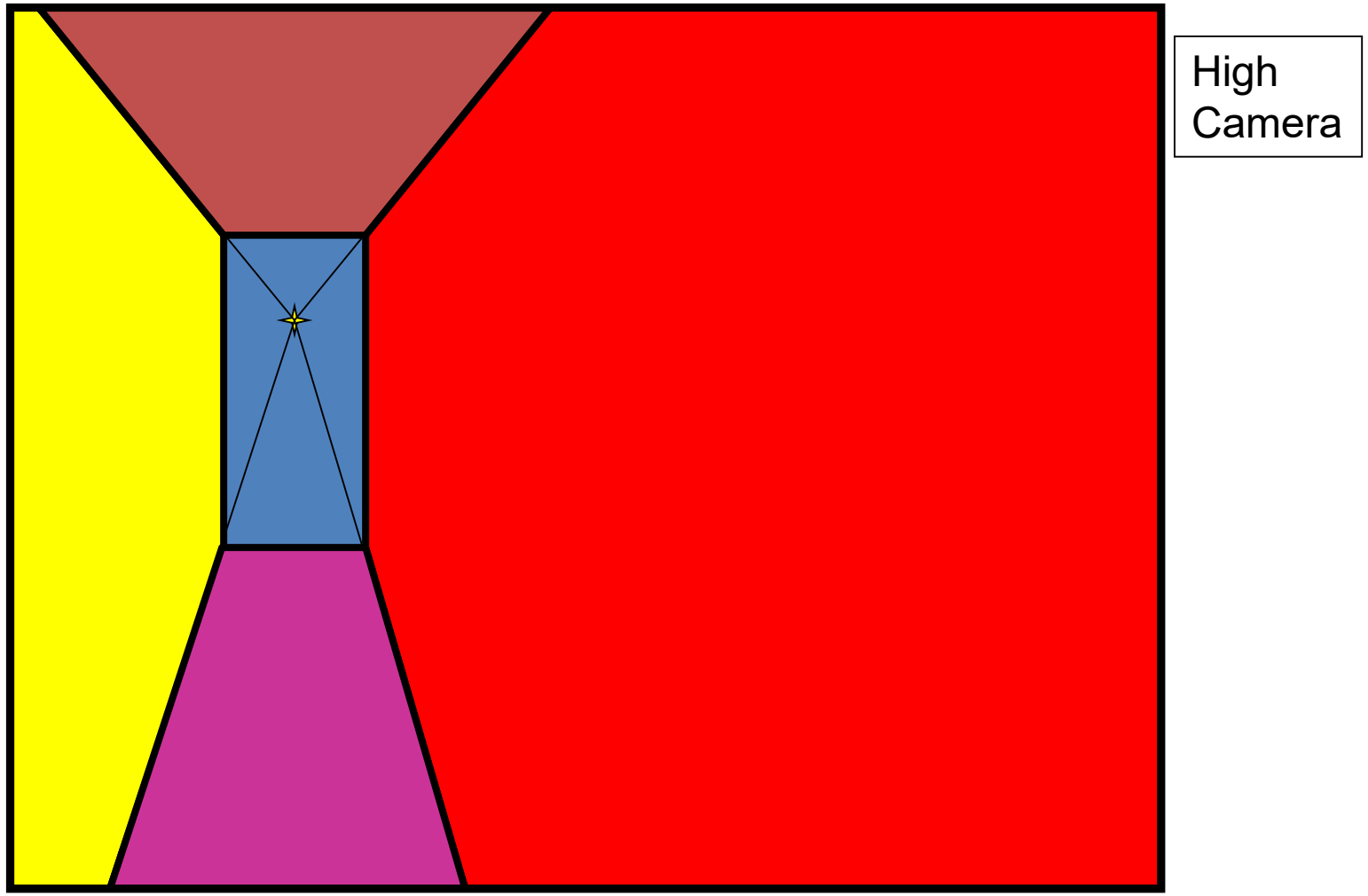
Step 1: specify scene geometry



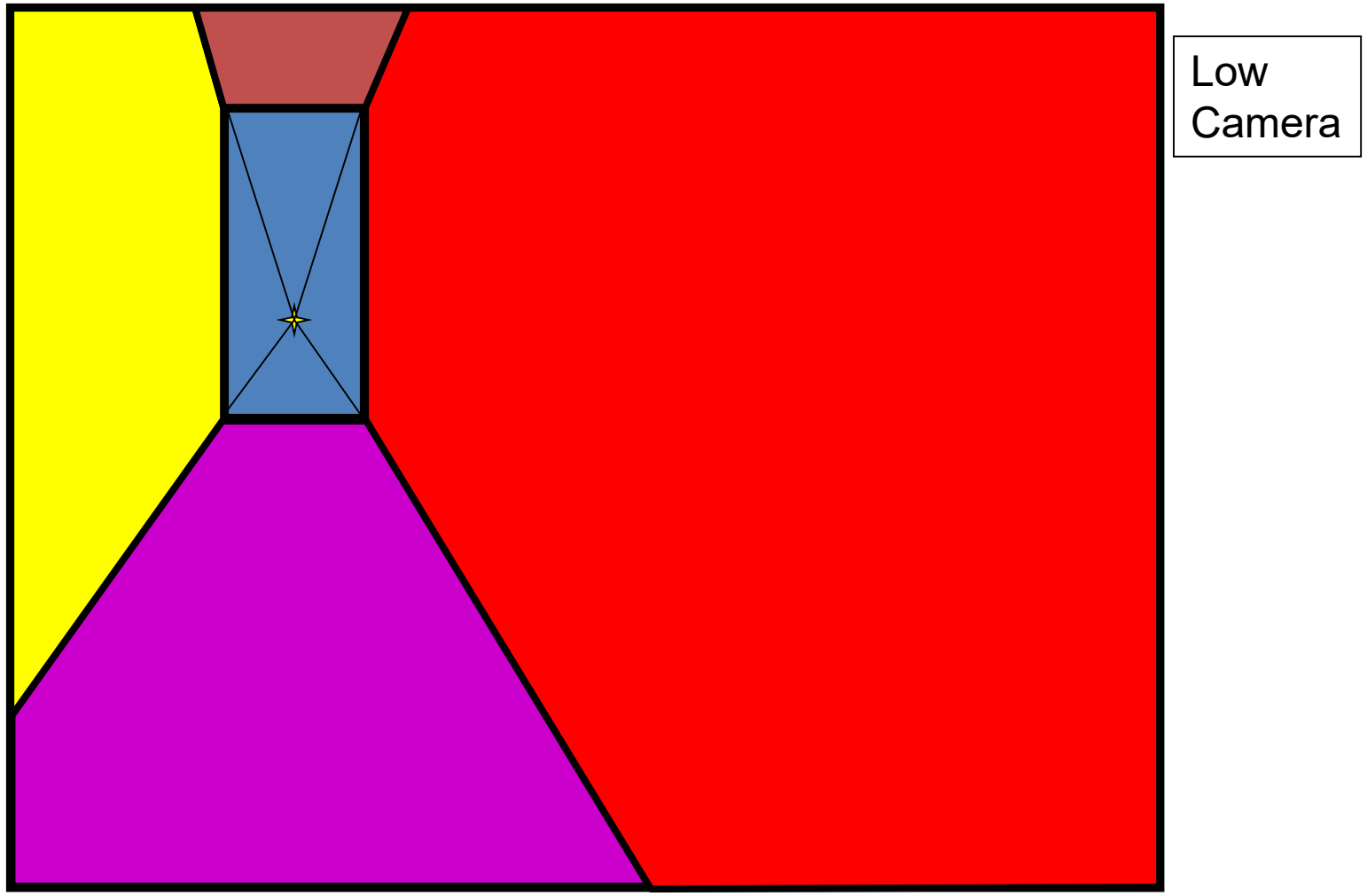
- User controls the inner box and the vanishing point placement (# of DOF?)

- Q: If we assume camera is looking straight at back wall, what camera parameter(s) does the vanishing point position provide?
- A: Vanishing point direction is perpendicular to image plane, so the vp is the principal point

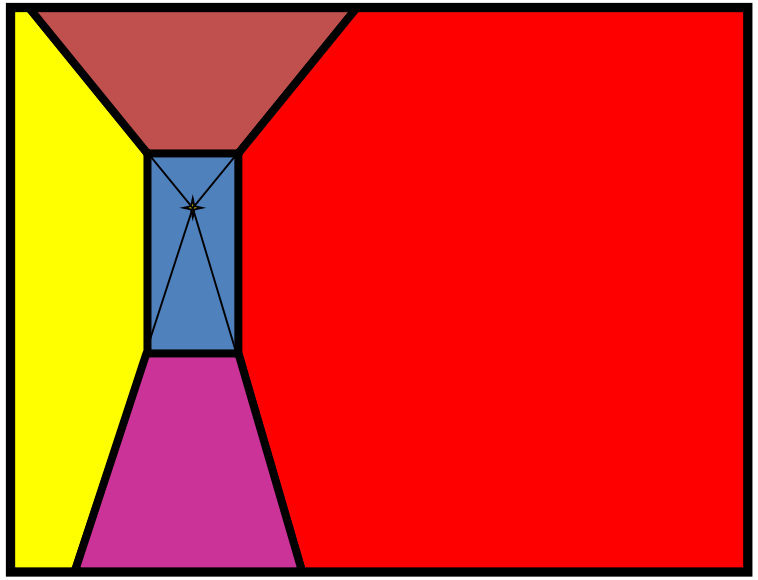
Example of user input: vanishing point and back face of view volume are defined



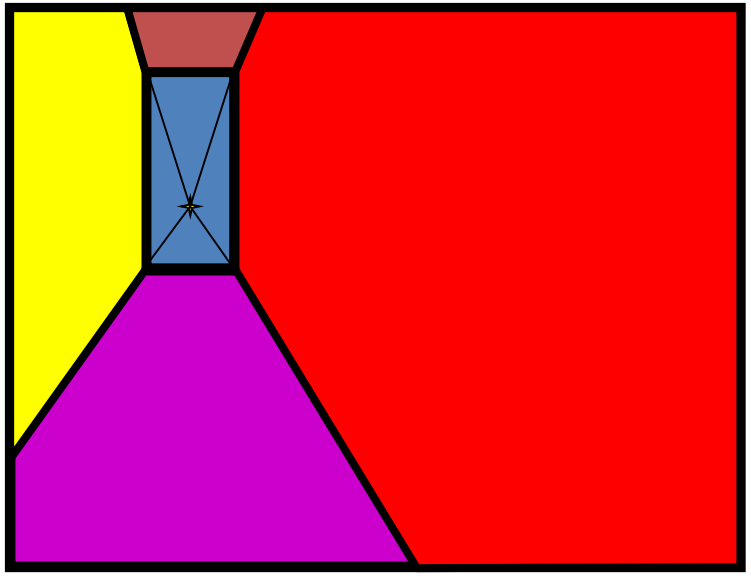
Example of user input: vanishing point and back face of view volume are defined



Comparison of how image is subdivided based on two different camera positions. You should see how moving the box corresponds to moving the eyepoint in the 3D world.

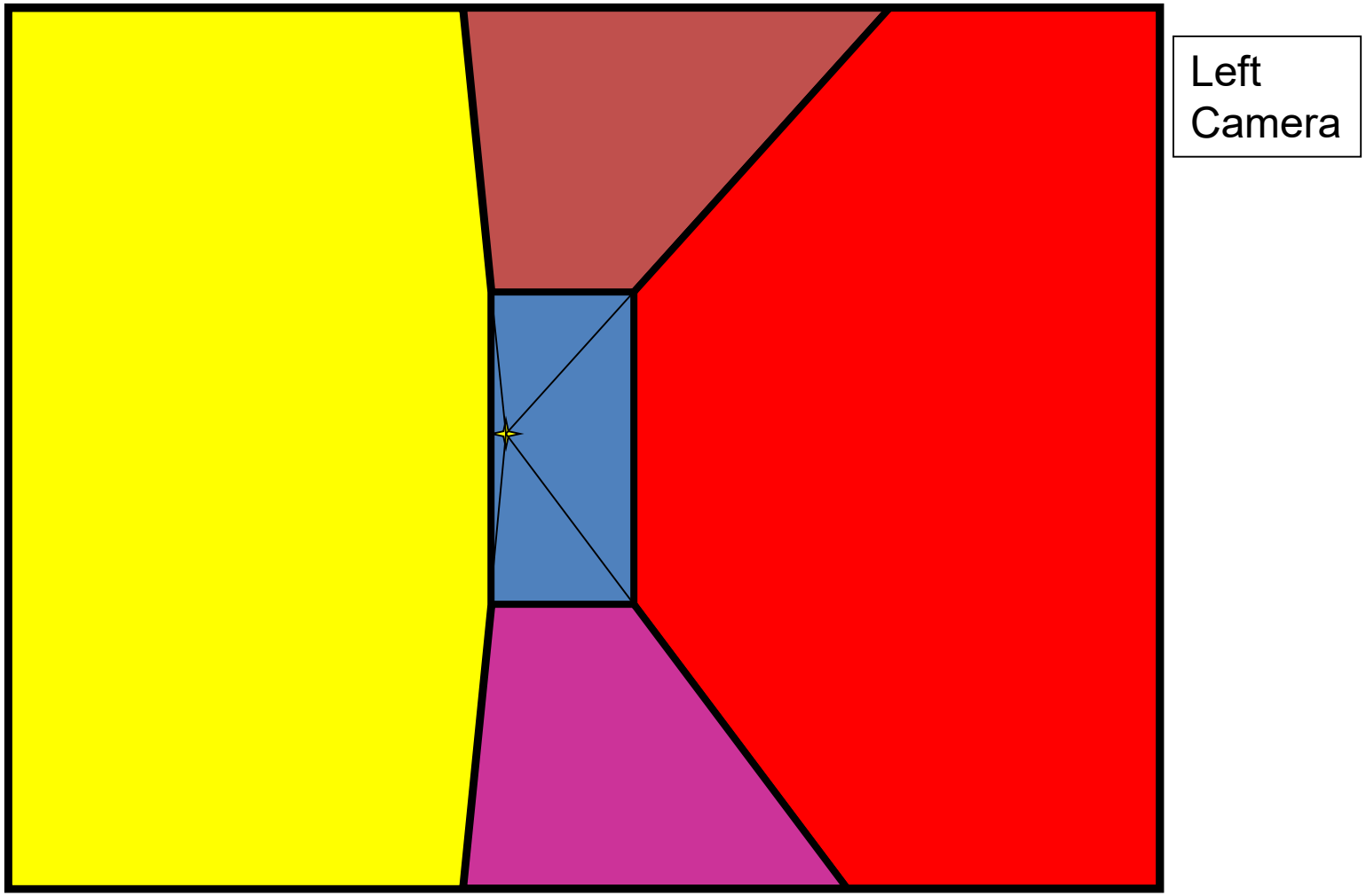


High Camera

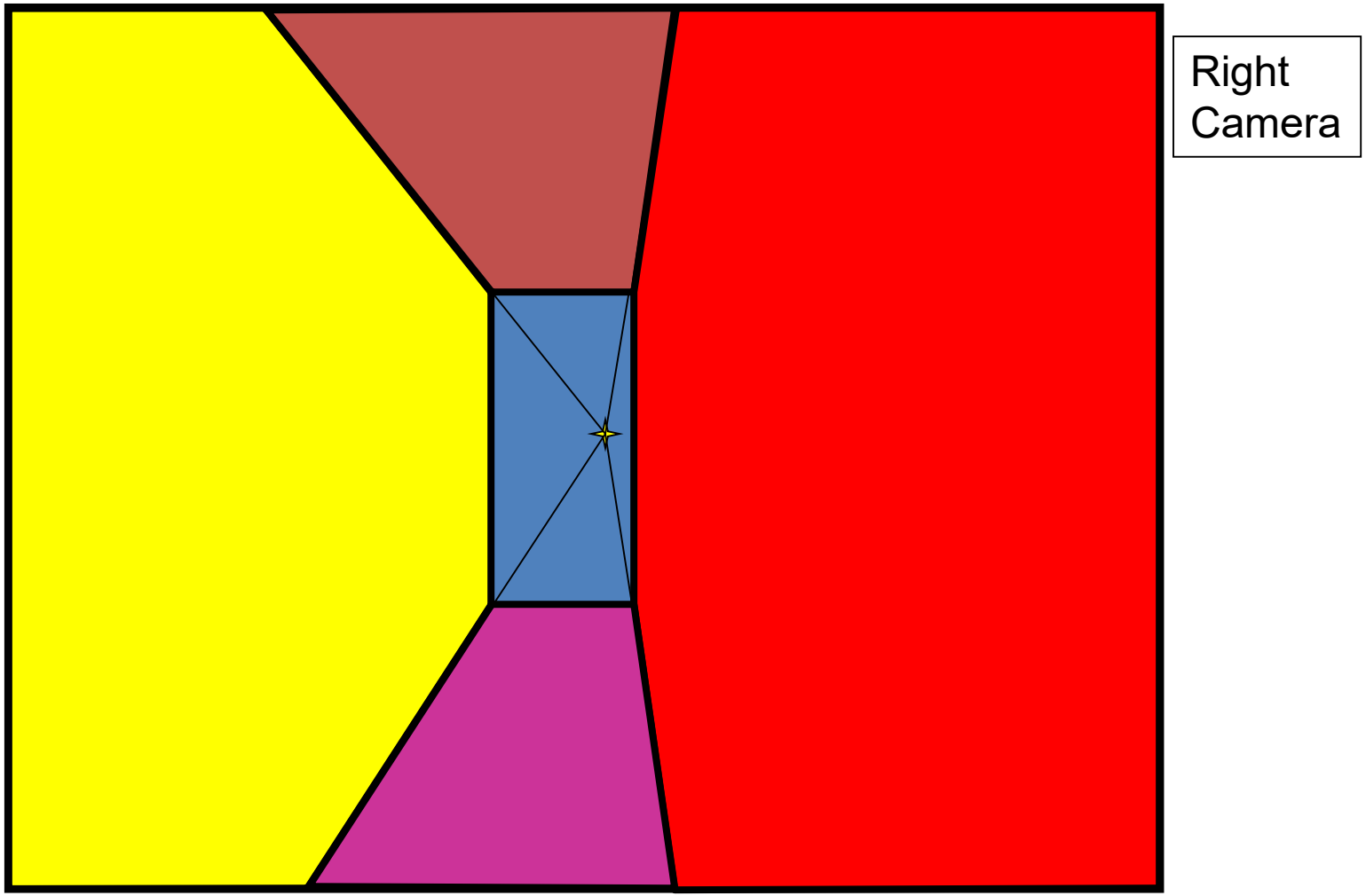


Low Camera

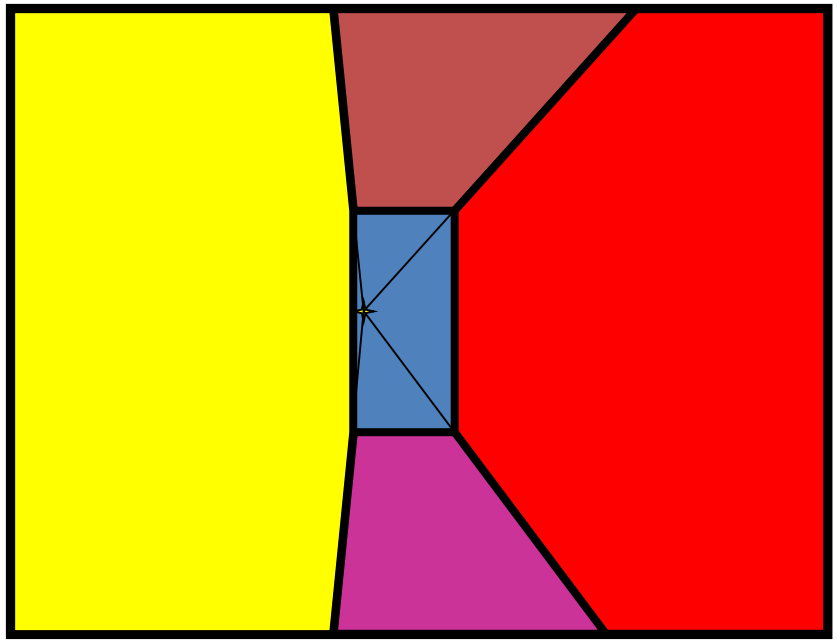
Another example of user input: vanishing point and back face of view volume are defined



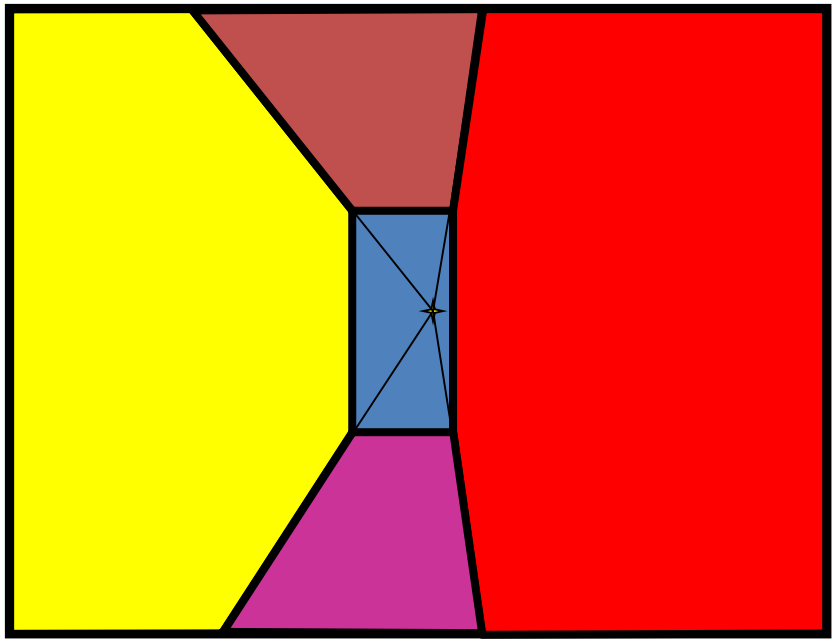
Another example of user input: vanishing point and back face of view volume are defined



Comparison of two camera placements – left and right.
Corresponding subdivisions match view you would see if you looked down a hallway.



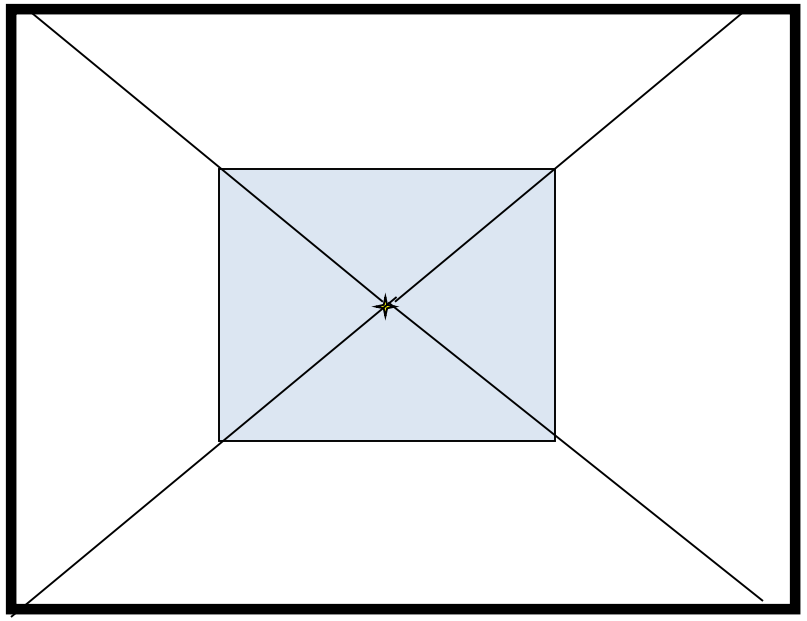
Left Camera



Right Camera

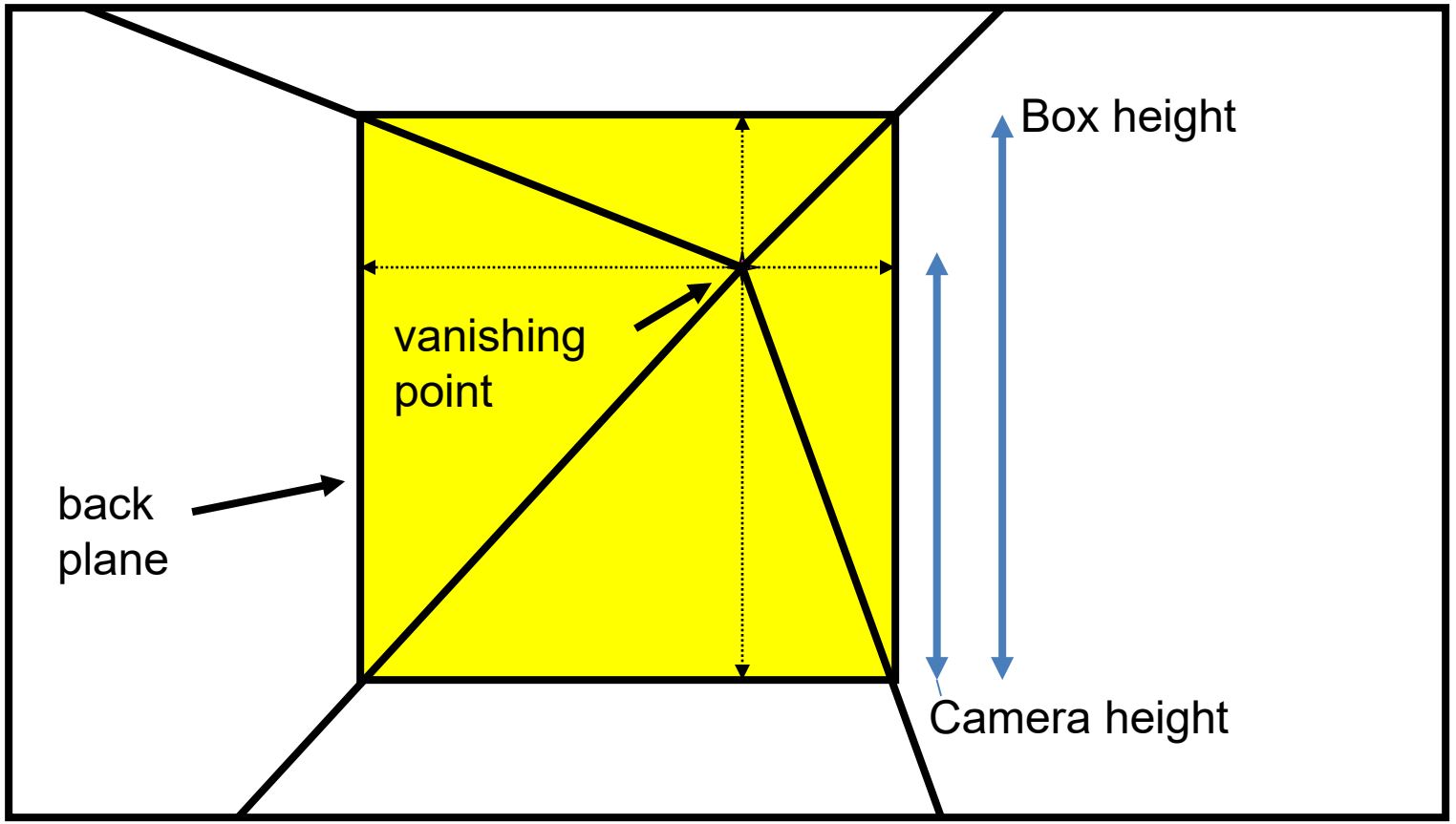
Question

- Think about the camera center and image plane…
  - What happens when we move the box?
  - What happens when we move the vanishing point?



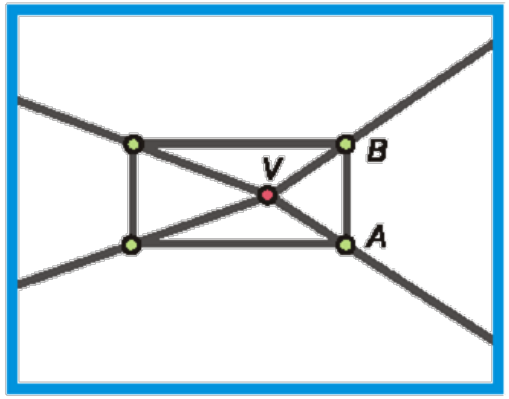
2D to 3D conversion

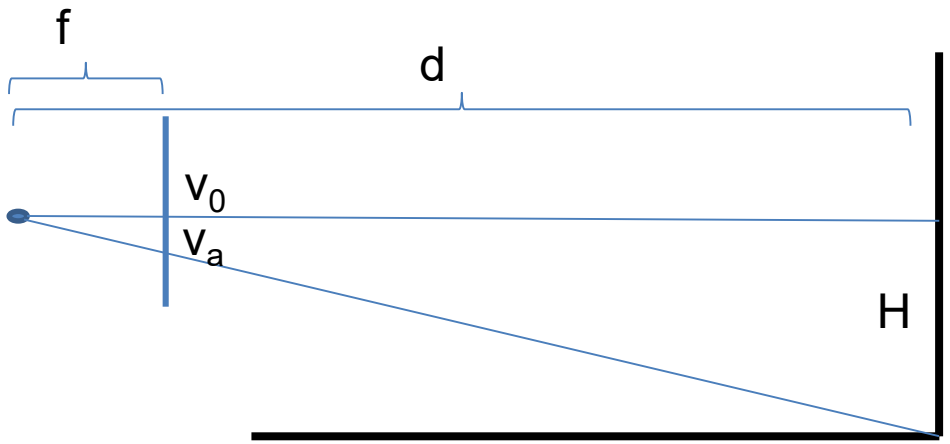
- Use ratios



Box width / height in 3D is proportional to width over height in the image because back plane is parallel to image plane

Get depth using similar triangles



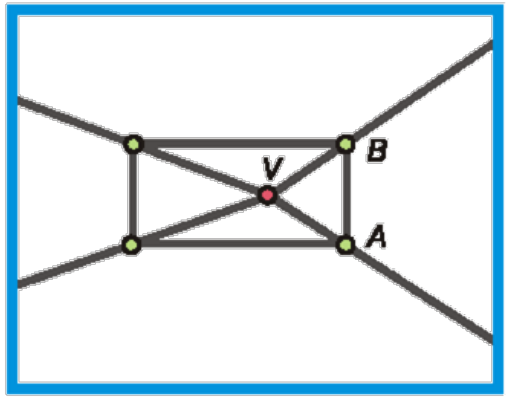


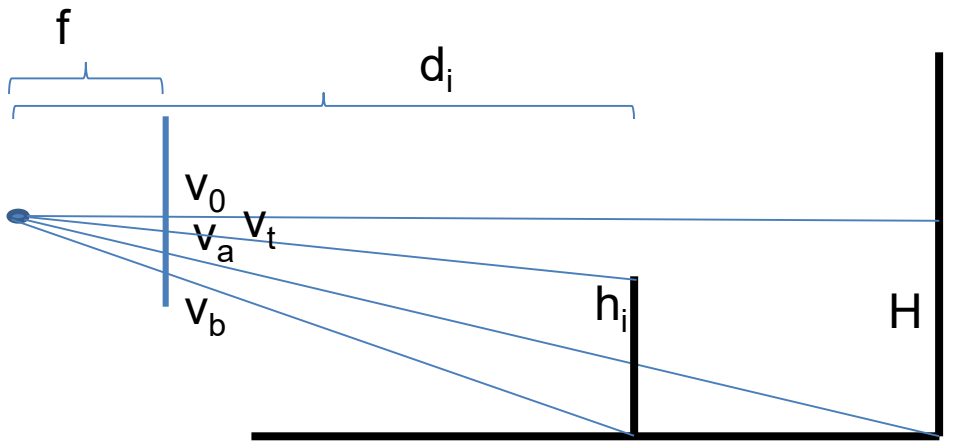
$f / (v_a - v_0) = d / H$

- Can compute by similar triangles (CVA vs. CV'A')
- Need to know focal length f (or FoV)

- Note: can compute position of any object on the ground
  - Simple unprojection
  - What about things off the ground?

Get depth using similar triangles





$f / (v_a - v_0) = d / h$

$h_i = (v_t - v_b) / (v_0 - v_a) * H$

$f / (v_b - v_t) = d_i / h_i$

- Can compute by similar triangles (CVA vs. CV'A')
- Need to know focal length f (or FoV)

- Can compute 3D position of any object on the ground w/ unprojection
  - What about things off the ground?

Step 2: map image textures into frontal view

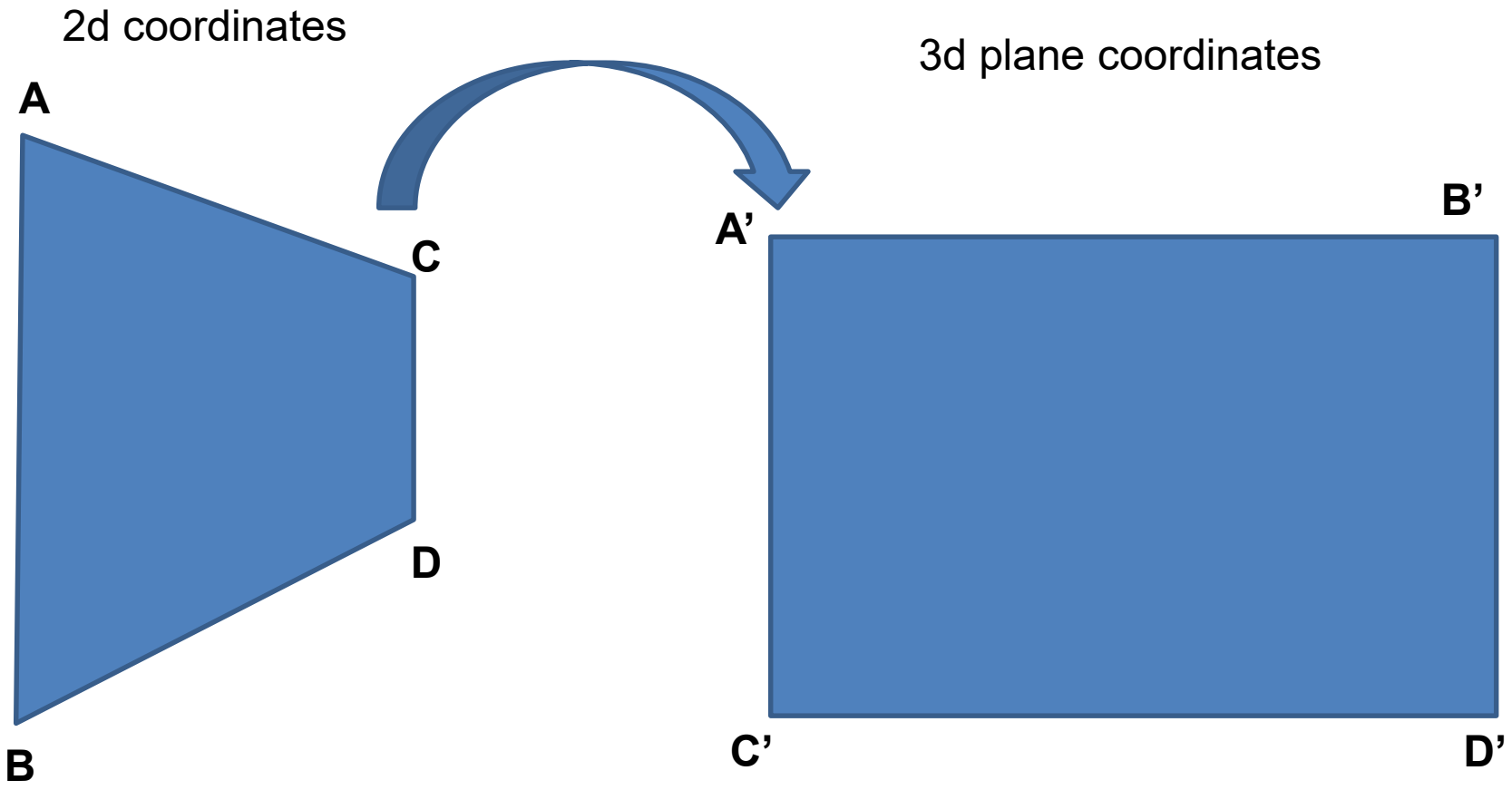
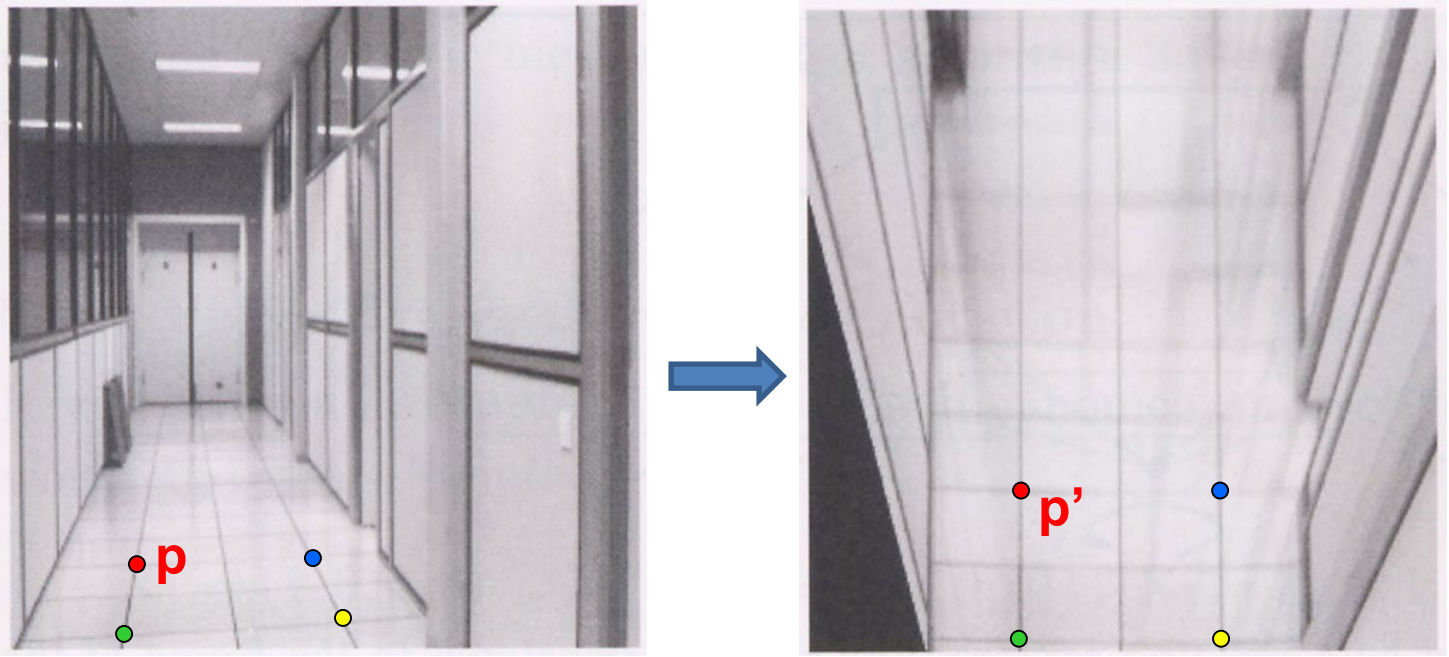


Image rectification by homography



To unwarp (rectify) an image solve for homography **H** given **p** and **p’**: w**p’=Hp**

Computing homography

Assume we have four matched points: How do we compute homography **H**?

Direct Linear Transformation (DLT)

$$\mathbf{p}' = \mathbf{H}\mathbf{p} \qquad \mathbf{p}' = \begin{bmatrix} w'u' \\ w'v' \\ w' \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

$$\begin{bmatrix} -u & -v & -1 & 0 & 0 & 0 & uu' & vu' & u' \\ 0 & 0 & 0 & -u & -v & -1 & uv' & vv' & v' \end{bmatrix} \mathbf{h} = \mathbf{0} \qquad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix}$$

Computing homography

Direct Linear Transform

$$\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1u_1' & v_1u_1' & u_1' \\ 0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1v_1' & v_1v_1' & v_1' \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & -u_n & -v_n & -1 & u_nv_n' & v_nv_n' & v_n' \end{bmatrix} \mathbf{h} = \mathbf{0} \Rightarrow \mathbf{Ah} = \mathbf{0}$$

- Apply SVD: $\boldsymbol{USV}^T = \boldsymbol{A}$
- $\boldsymbol{h} = \boldsymbol{V}_{\text{smallest}}$ (column of $\boldsymbol{V}^T$ corresponds to smallest singular value)

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

Python

```
U, S, Vt = scipy.linalg.svd(A)
# last column corr. to smallest singular value
h = Vt[:,-1];
```

Explanation of SVD, solving systems of linear equations, derivation of solution here

Solving for homography (another formulation)

$$\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1u_1' & v_1u_1' & u_1' \\ 0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1v_1' & v_1v_1' & v_1' \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & -u_n & -v_n & -1 & u_nv_n' & v_nv_n' & v_n' \end{bmatrix} \mathbf{h} = \mathbf{0} \Rightarrow \mathbf{Ah} = \mathbf{0}$$

$\mathbf{A}$ ($2n \times 9$) $\quad \mathbf{h}$ (9) $\quad \mathbf{0}$ ($2n$)

Defines a least squares problem: $\text{minimize } \|\mathbf{Ah} - 0\|^2$

- Since **h** is only defined up to scale, solve for unit vector $\hat{\mathbf{h}}$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^\top\mathbf{A}$ with smallest eigenvalue
  - Can derive using Lagrange multipliers method
- Works with 4 or more points

Tour into the picture algorithm

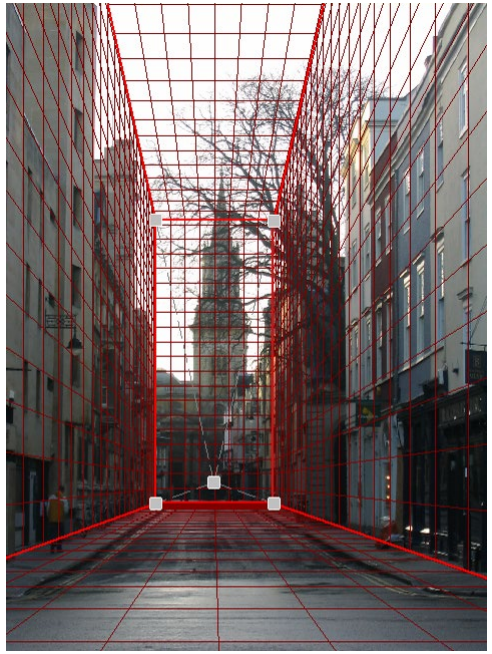
1. Set the box corners

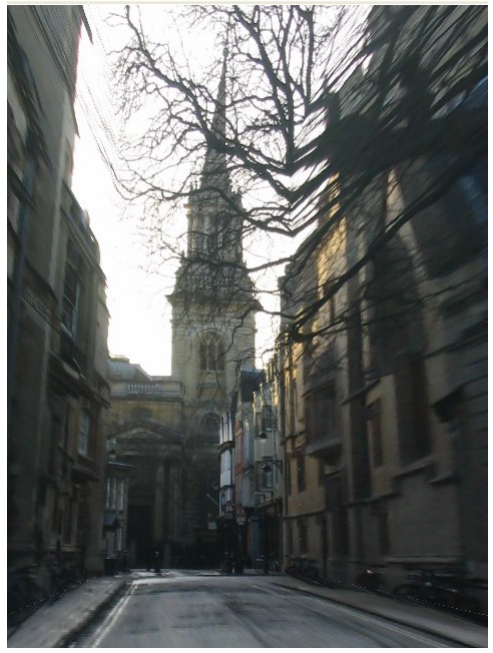
Tour into the picture algorithm

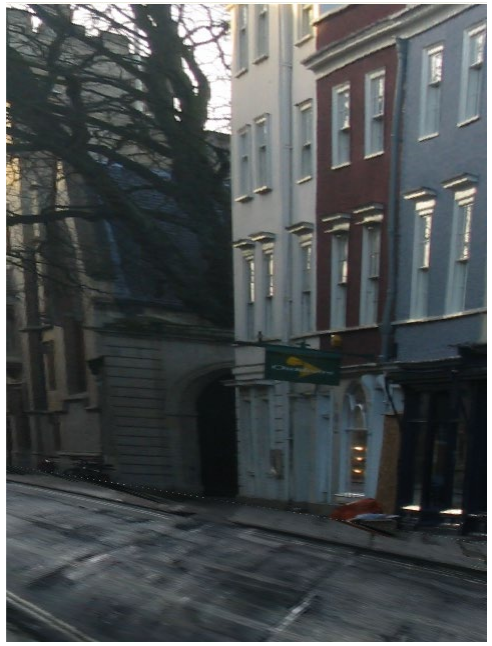
1. Set the box corners
2. Set the VP
3. Get 3D coordinates
   - Compute height, width, and depth of box
4. Get texture maps
   - homographies for each face
5. Create file to store plane coordinates and texture maps

Result

Render from new views







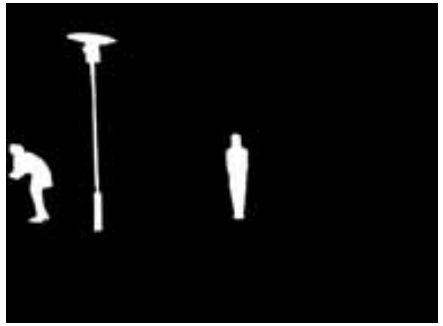
http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15463-f08/www/proj5/www/dmillett/

Foreground Objects

Use separate billboard for each

For this to work, three separate images used:

- Original image.
- Mask to isolate desired foreground images.
- Background with objects removed

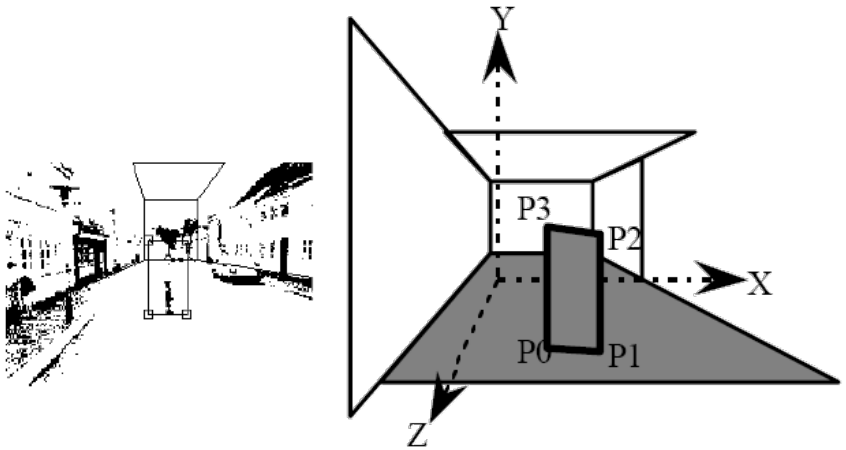


Foreground Objects

Add vertical rectangles for each foreground object

Can compute 3D coordinates P0, P1 since they are on known plane.

P2, P3 can be computed as before (similar triangles)

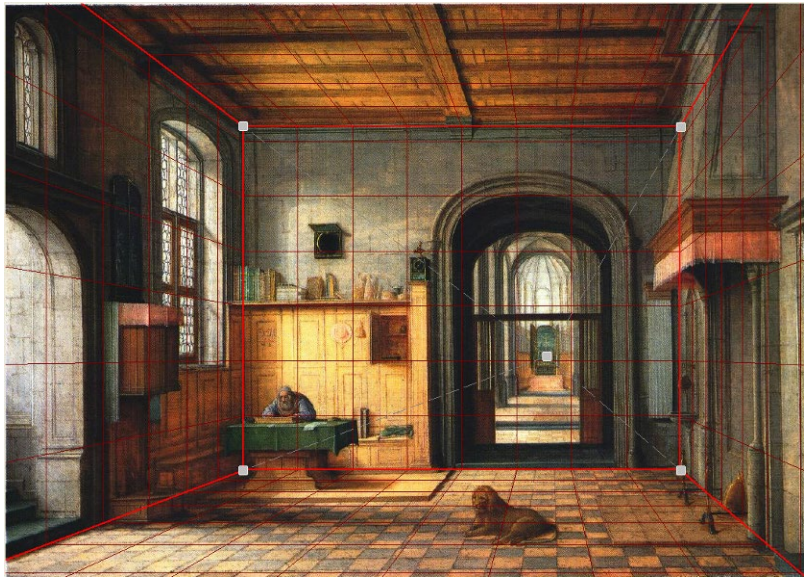


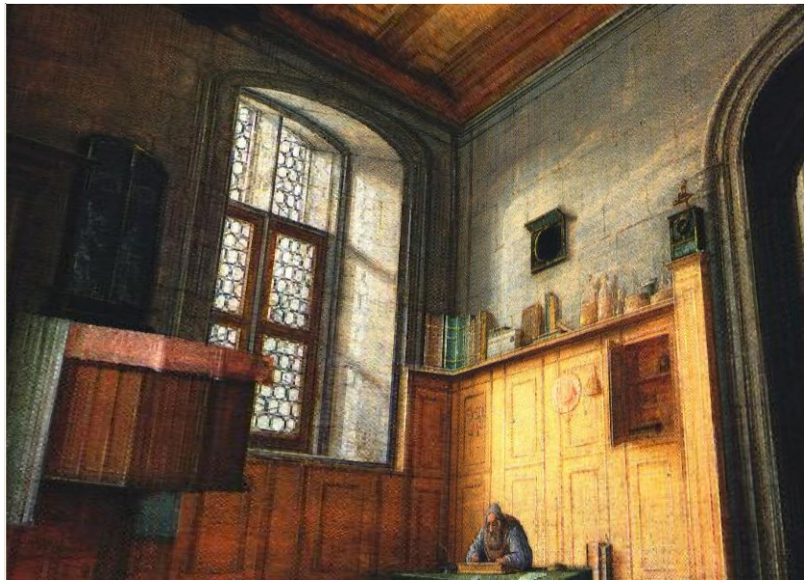
(a) Specifying of a foreground object

(b) Estimating the vertices of the foreground object model

(c) Three foreground object models

Foreground Result





Video from CMU class:
http://www.youtube.com/watch?v=dUAtdmGwcuM

Automatic Photo Pop-up

Input Geometric Labels Cut'n'Fold 3D Model

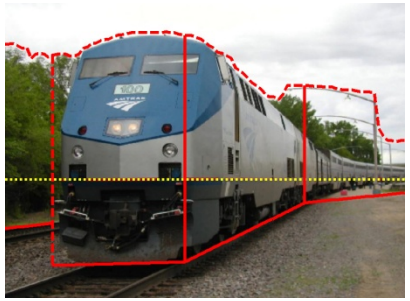
Image

Learned Models

Ground

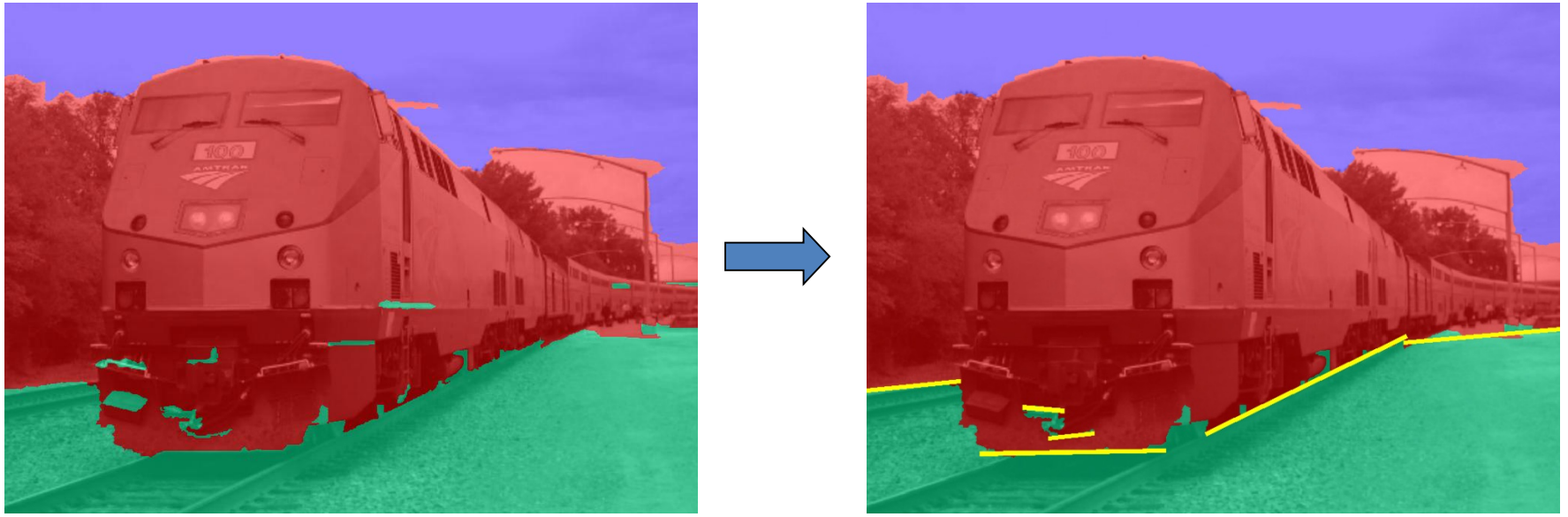
Vertical

Sky



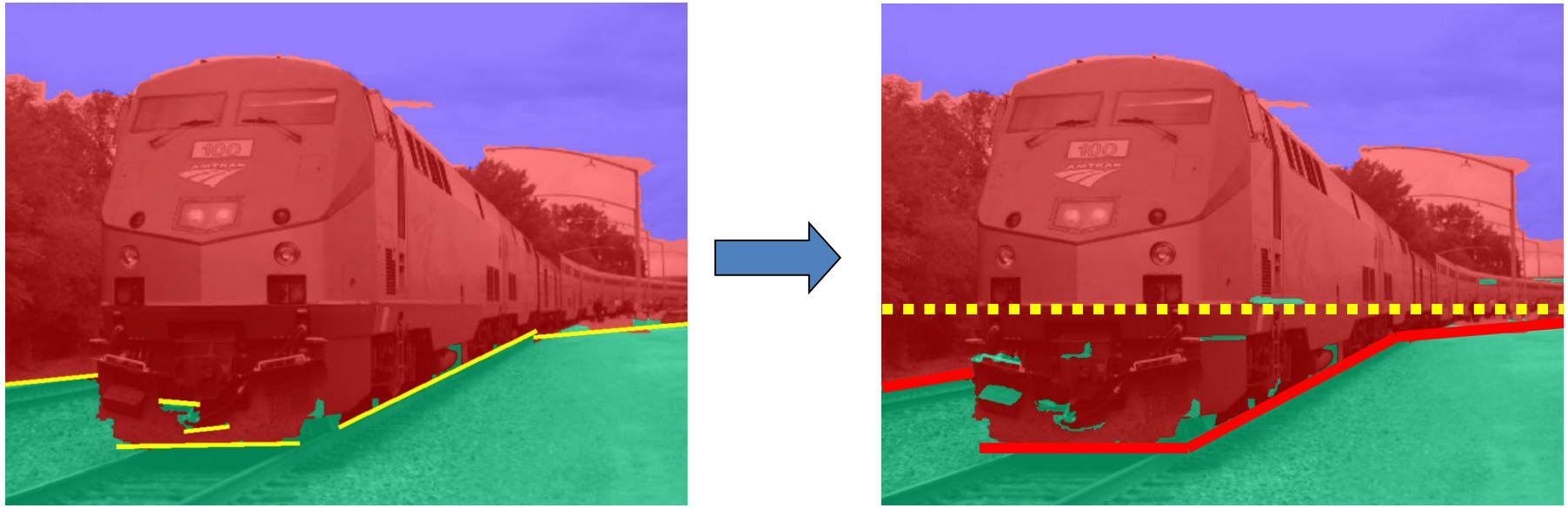
Hoiem et al. 2005

Cutting and Folding



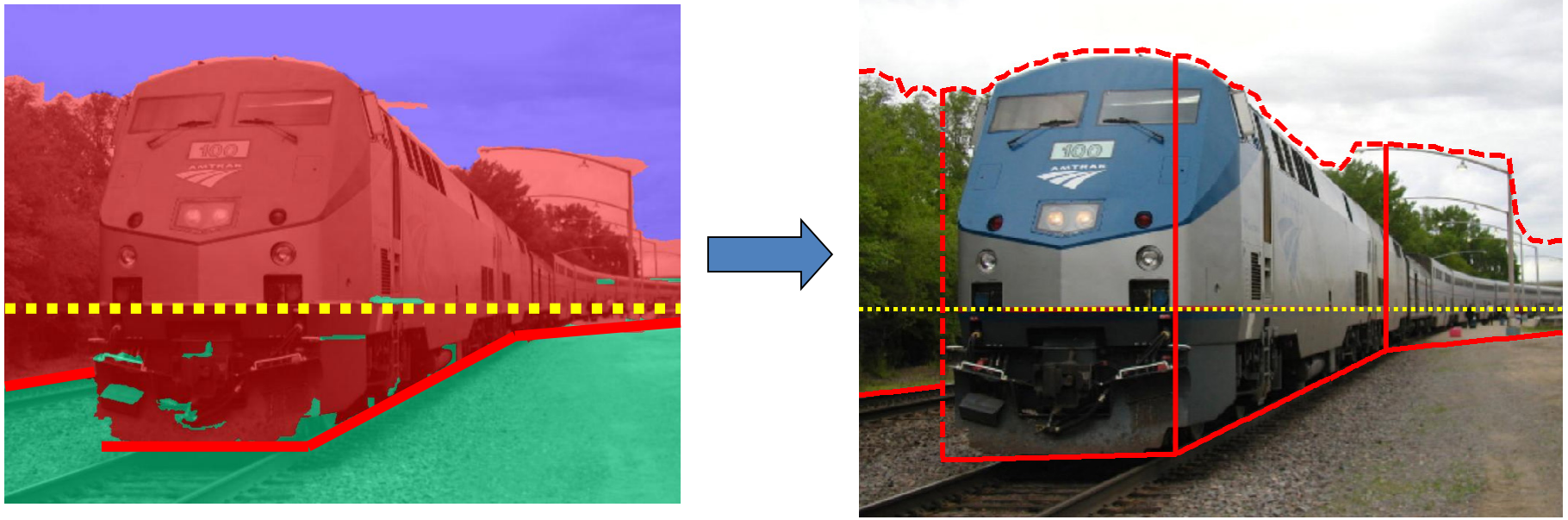
- Fit ground-vertical boundary
  - Iterative Hough transform

Cutting and Folding



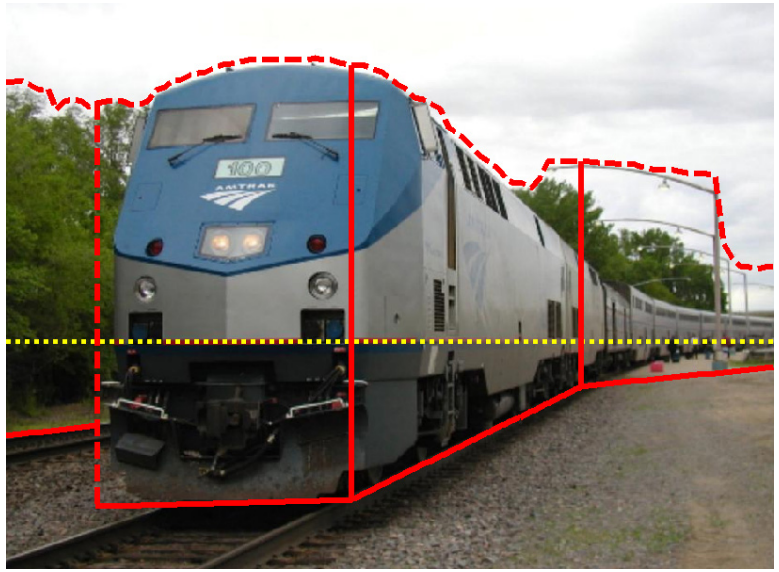
- Form polylines from boundary segments
  - Join segments that intersect at slight angles
  - Remove small overlapping polylines
- Estimate horizon position from perspective cues

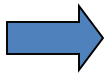
Cutting and Folding



- ``Fold'' along polylines and at corners
- ``Cut'' at ends of polylines and along vertical-sky boundary

Cutting and Folding



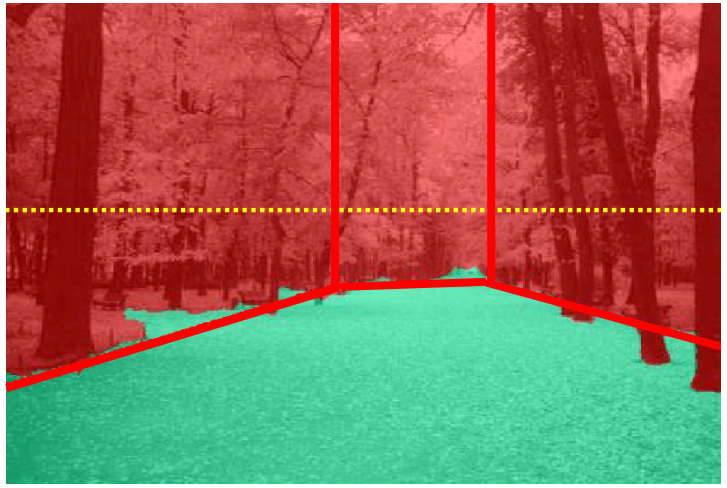


- Construct 3D model
- Texture map

Results

http://www.cs.illinois.edu/homes/dhoiem/projects/popup/

Input Image



Cut and Fold

Automatic Photo Pop-up

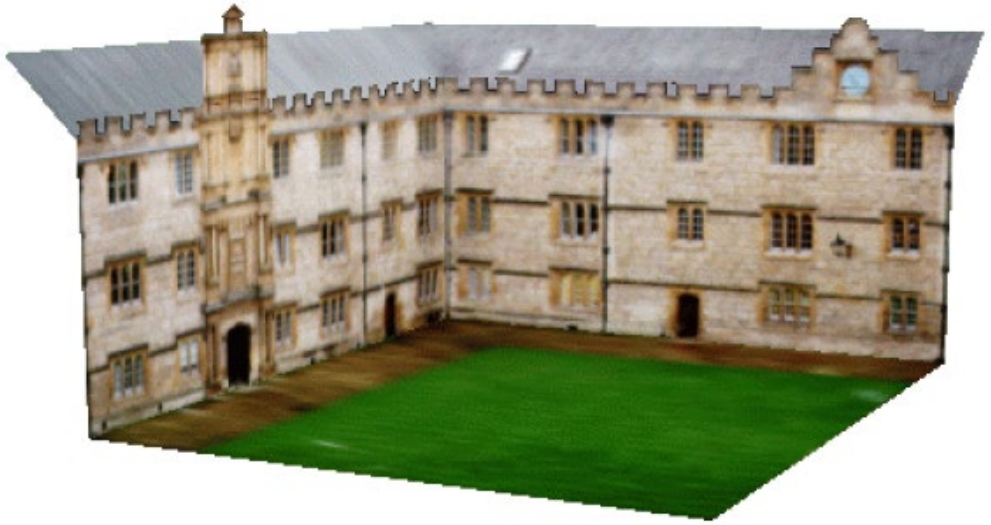
Results

Input Image

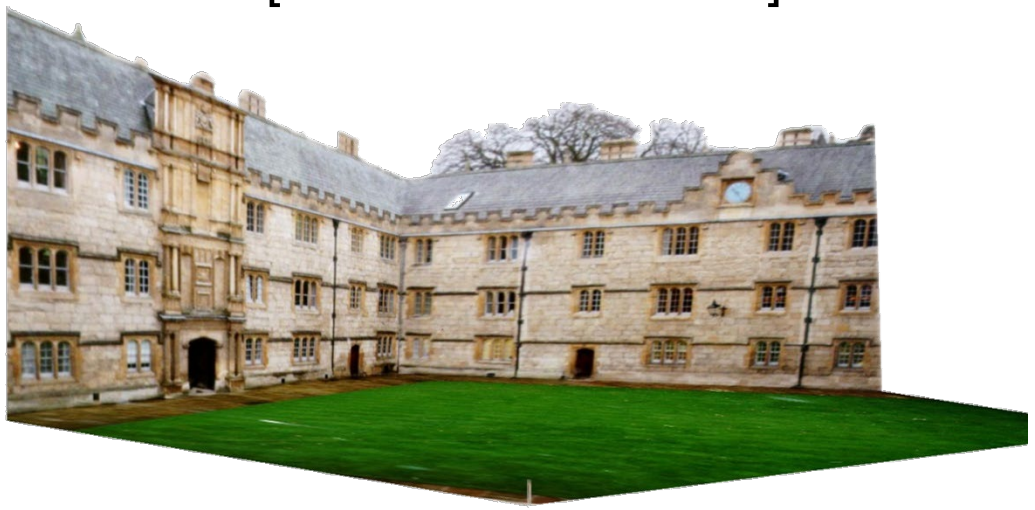
Automatic Photo Pop-up

Comparison with Manual Method

Input Image



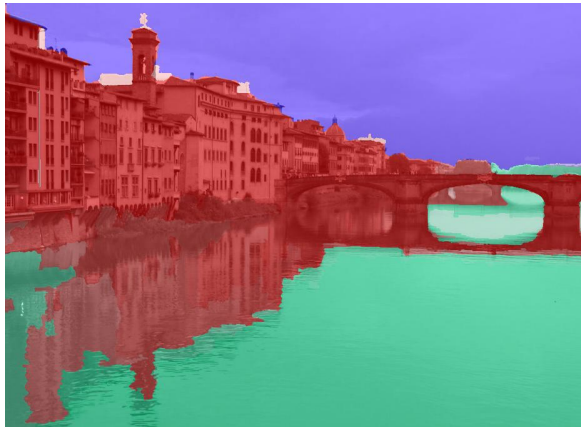
[Liebowitz et al. 1999]



Automatic Photo Pop-up (15 sec)!

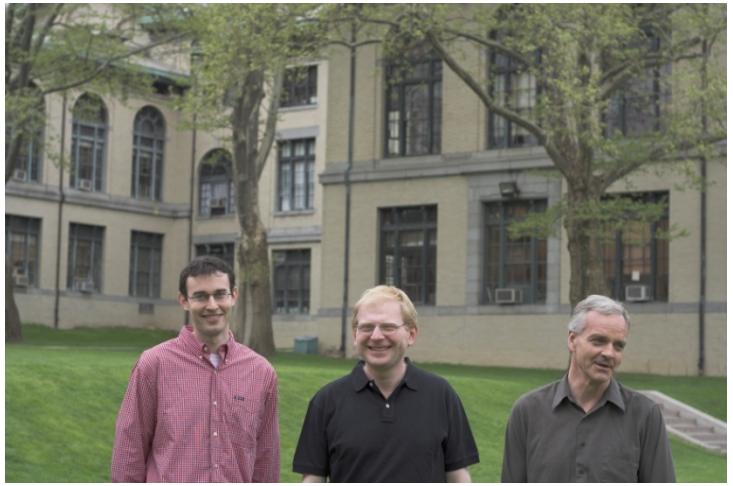
Failures

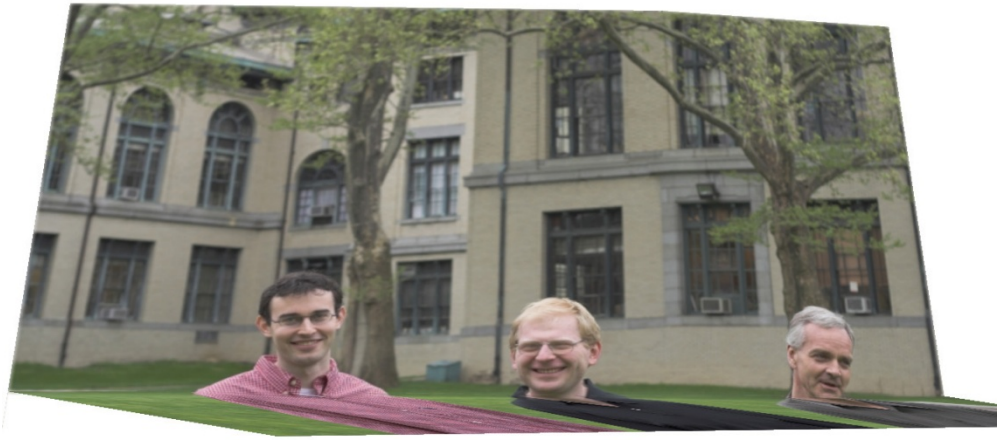
Labeling Errors



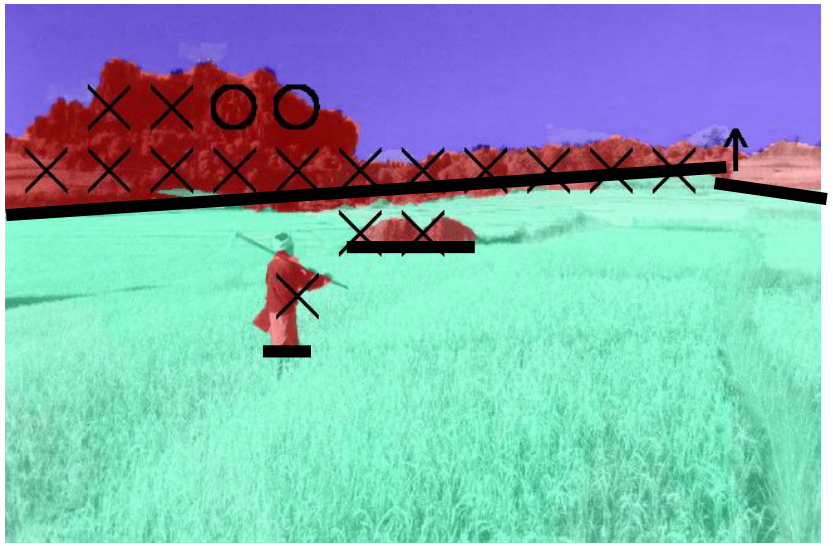
Failures

Foreground Objects

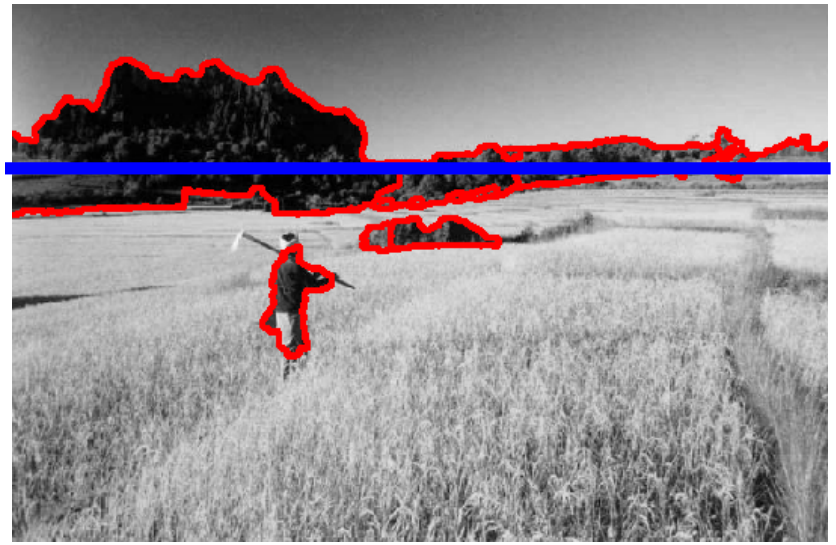




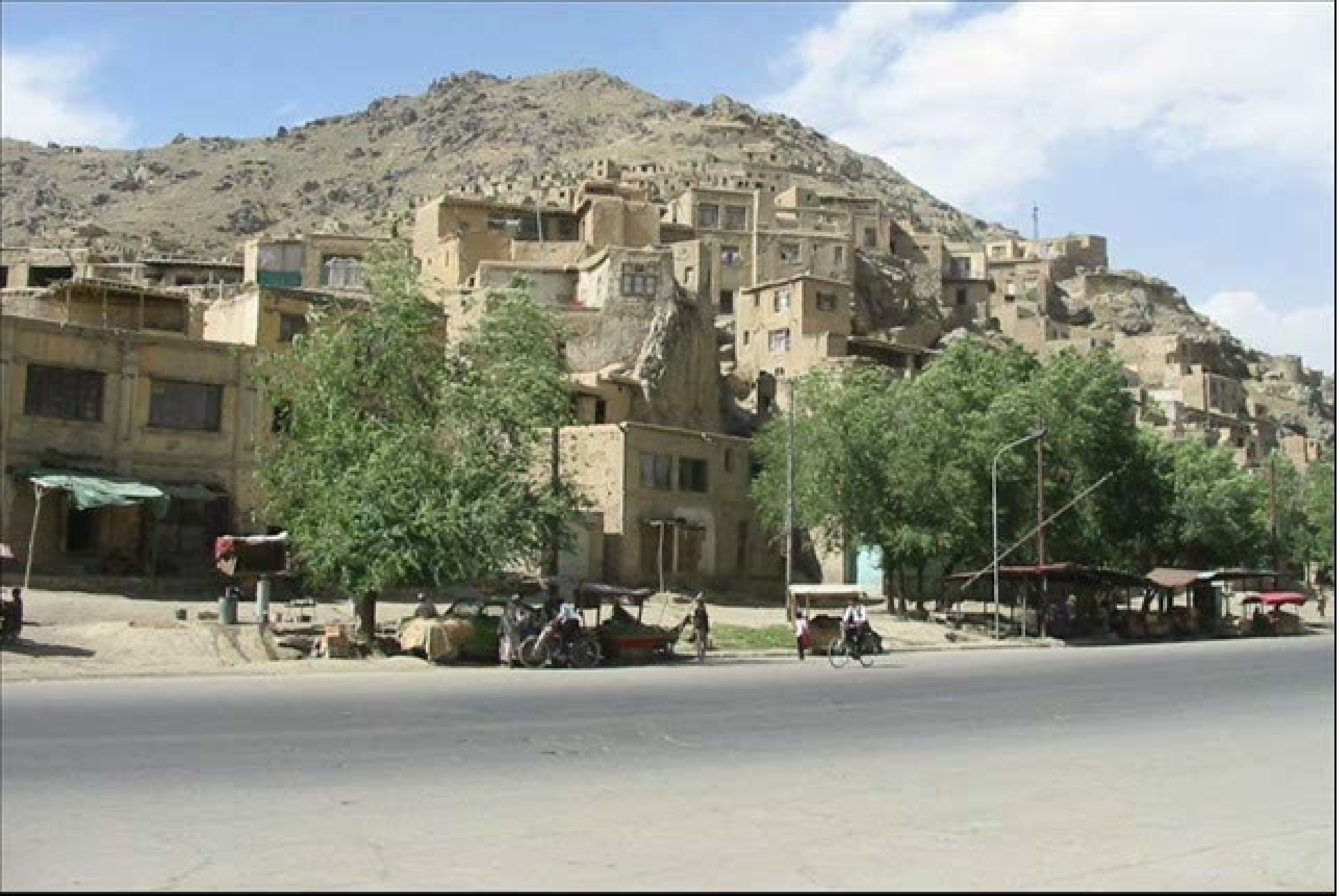
Adding Foreground Labels



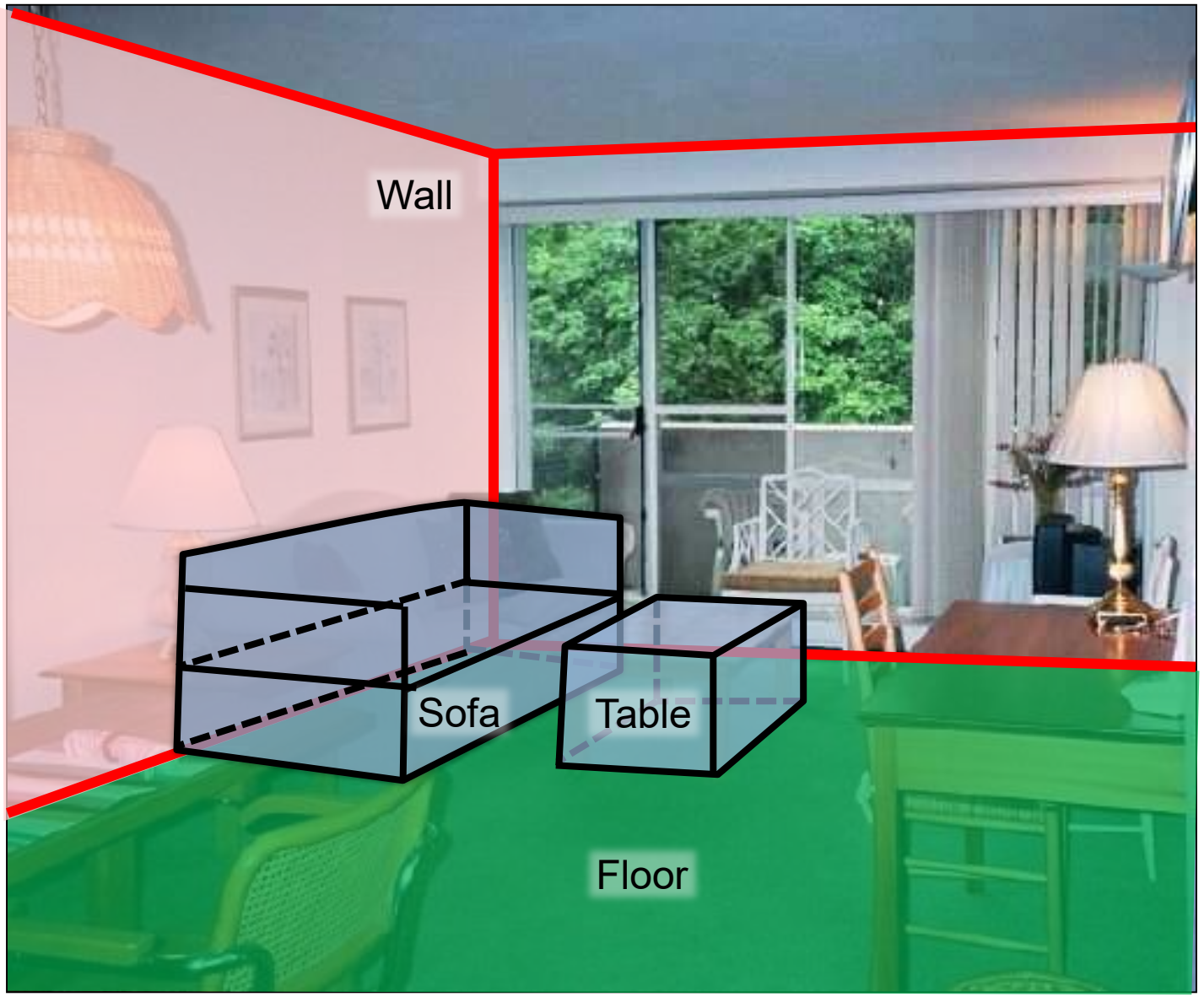
Recovered Surface Labels +
Ground-Vertical Boundary Fit



Object Boundaries + Horizon

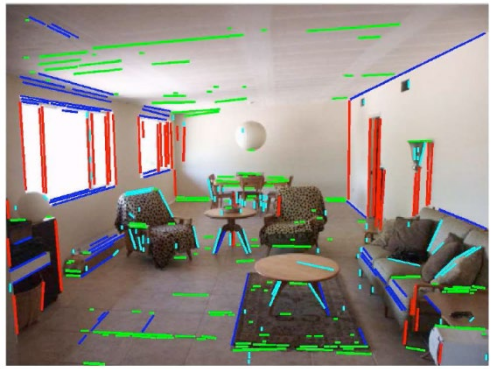


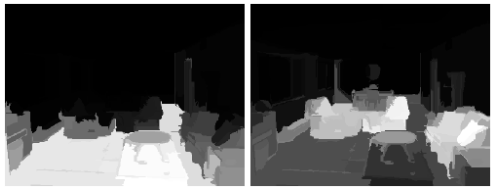
Fitting boxes to indoor scenes



Box Layout Algorithm

Hedau et al. 2010





1. Detect edges
2. Estimate 3 orthogonal vanishing points
3. Apply region classifier to label pixels with visible surfaces
   - Boosted decision trees on region based on color, texture, edges, position
4. Generate box candidates by sampling pairs of rays from VPs
5. Score each box based on edges and pixel labels
   - Learn score via structured learning
6. Jointly refine box layout and pixel labels to get final estimate

Experimental results

Detected Edges

Surface Labels

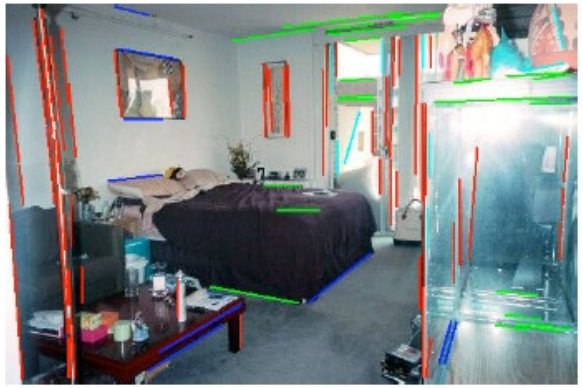
Box Layout

Detected Edges

Surface Labels

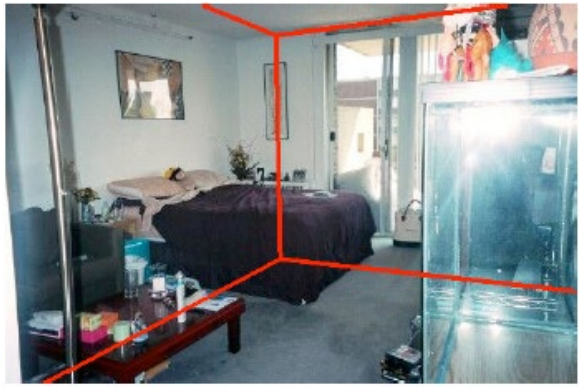
Box Layout

Experimental results

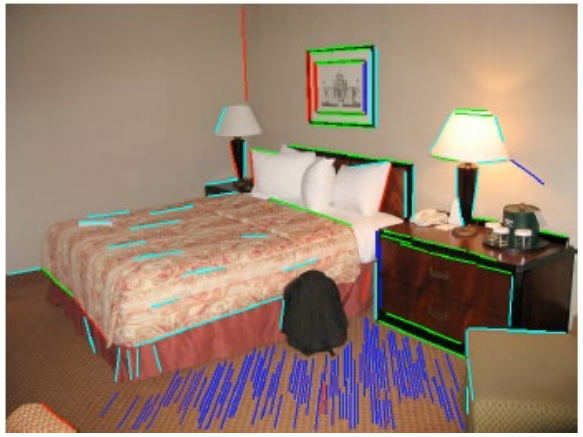


Detected Edges

Surface Labels



Box Layout

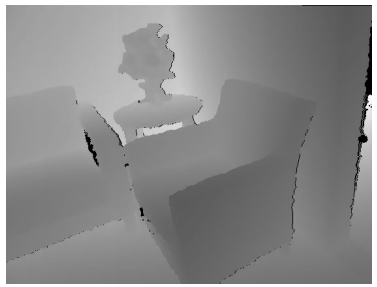


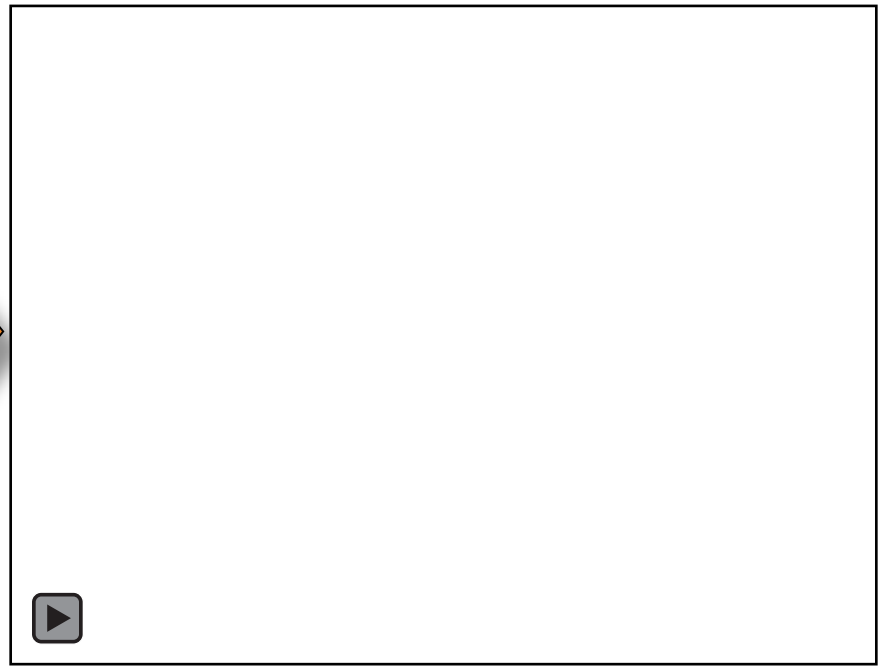
Detected Edges

Surface Labels

Box Layout

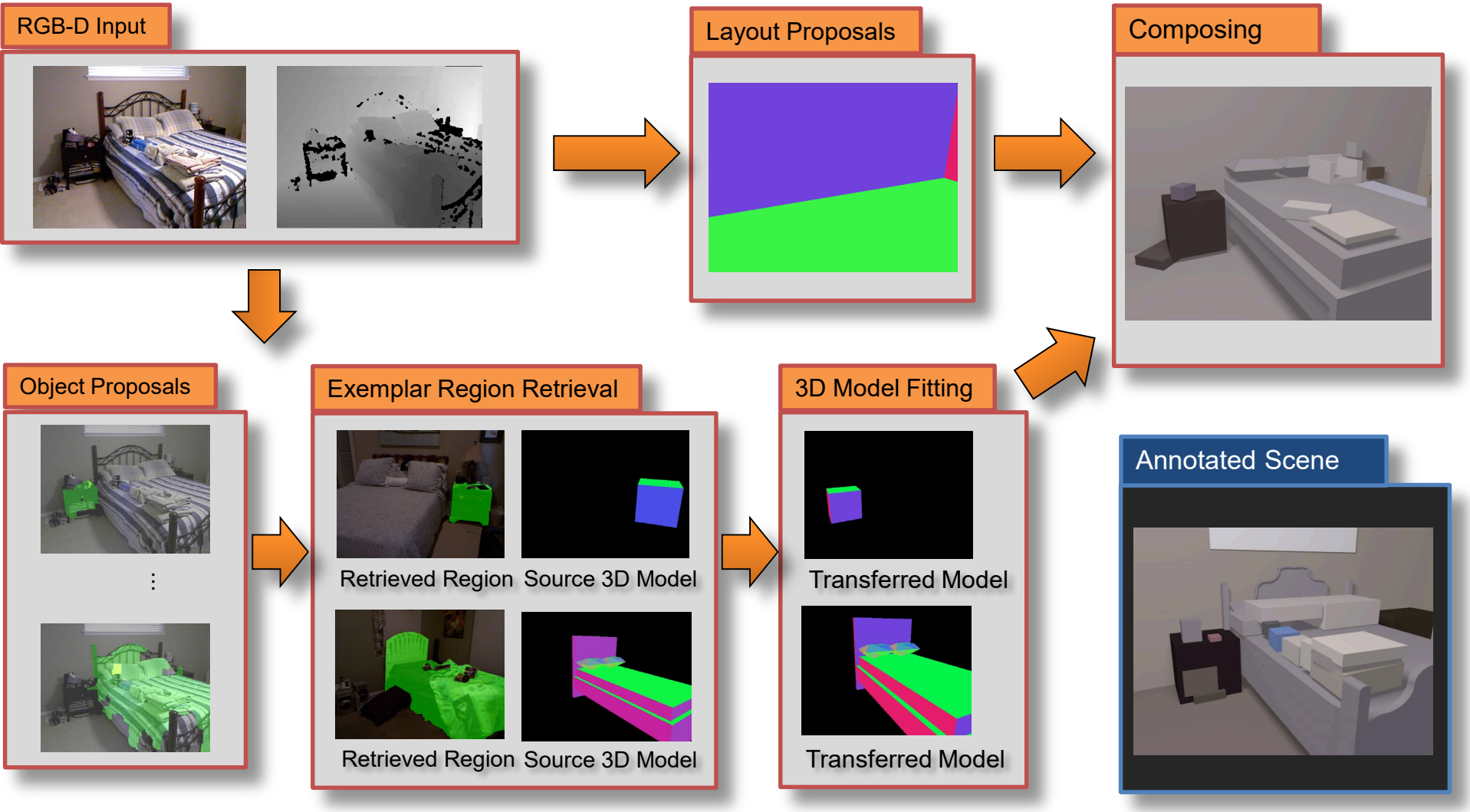
Complete 3D from RGBD



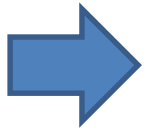


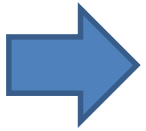
Zou et al. IJCV 2018

Complete 3D from RGBD



Complete 3D from RGBD





Final project ideas

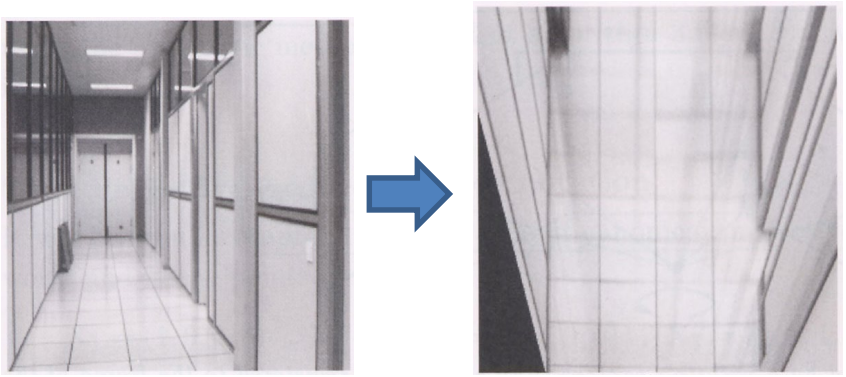
- Interactive program to make 3D model from an image (e.g., output in VRML, or draw path for animation)
  - Add tools for cutting out foreground objects and automatic hole-filling

Summary

- 2D→3D is mathematically impossible
  (but we do it without even thinking)

- Need right assumptions about the world geometry

- Important tools
  - Vanishing points
  - Camera matrix
  - Homography



Next Week

- Project 3 is due Tuesday

- Next three classes: image-based lighting
  - How to model light
  - Recover HDR image from multiple LDR images
  - Recover lighting model from an image
  - Render object into a scene with correct lighting and geometry