## Image Stitching



Computational Photography
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## Last Class: Keypoint Matching



$$
d\left(f_{A}, f_{B}\right)<T
$$

1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

## Last Class: Summary

- Keypoint detection: repeatable and distinctive
- Corners, blobs
- Harris, DoG

- Descriptors: robust and selective
- SIFT: spatial histograms of gradient orientation



## Today: Image Stitching

- Combine two or more overlapping images to make one larger image



## Views from rotating camera



Correspondence of rotating camera

- $x=K[R t] X$
- $x^{\prime}=K^{\prime}\left[R^{\prime} \mathrm{t}^{\prime}\right] X$
- $\mathrm{t}=\mathrm{t}^{\prime}=0$

- $x^{\prime}=H x$ where $H=K^{\prime} R^{\prime} R^{-1} K^{-1}$
- Typically only $R$ and $f$ will change (4 parameters), but, in general, H has 8 parameters


## Image Stitching Algorithm Overview

1. Detect keypoints
2. Match keypoints
3. Estimate homography with four matched keypoints (using RANSAC)
4. Project onto a surface and blend

## Image Stitching Algorithm Overview

1. Detect/extract keypoints (e.g., DoG/SIFT)
2. Match keypoints (most similar features, compared to $2^{\text {nd }}$ most similar) $\quad \frac{d 1}{d 2}<$ thresh


## Computing homography

Assume we have four matched points: How do we compute homography $\mathbf{H}$ ?

Direct Linear Transformation (DLT)

$$
\begin{gathered}
\mathbf{x}^{\prime}=\mathbf{H x} \quad \mathbf{x}^{\prime}=\left[\begin{array}{l}
w^{\prime} u^{\prime} \\
w^{\prime} v^{\prime} \\
w^{\prime}
\end{array}\right] \quad \mathbf{H}=\left[\begin{array}{llll}
h_{1} & h_{2} & h_{3} \\
h_{4} & h_{5} & h_{6} \\
h_{7} & h_{8} & h_{9}
\end{array}\right] \\
{\left[\begin{array}{ccccccccc}
-u & -v & -1 & 0 & 0 & 0 & u u^{\prime} & v u^{\prime} & u^{\prime} \\
0 & 0 & 0 & -u & -v & -1 & u v^{\prime} & v v^{\prime} & v^{\prime}
\end{array}\right] \mathbf{h}=\mathbf{0}}
\end{gathered}
$$

## Computing homography

## Direct Linear Transform

$$
\left[\begin{array}{ccccccccc}
-u_{1} & -v_{1} & -1 & 0 & 0 & 0 & u_{1} u_{1}^{\prime} & v_{1} u_{1}^{\prime} & u_{1}^{\prime} \\
0 & 0 & 0 & -u_{1} & -v_{1} & -1 & u_{1} v_{1}^{\prime} & v_{1} v_{1}^{\prime} & v_{1}^{\prime} \\
& & & & \vdots & & & & \\
0 & 0 & 0 & -u_{n} & -v_{n} & -1 & u_{n} v_{n}^{\prime} & v_{n} v_{n}^{\prime} & v_{n}^{\prime}
\end{array}\right] \mathbf{h}=\mathbf{0} \Rightarrow \mathbf{A h}=\mathbf{0}
$$

- Apply SVD: UDV ${ }^{\top}=\boldsymbol{A}$
- $\boldsymbol{h}=\boldsymbol{V}_{\text {smallest }}$ (column of $\boldsymbol{V}$ corr. to smallest singular value)

$$
\mathbf{h}=\left[\begin{array}{c}
h_{1} \\
h_{2} \\
\vdots \\
h_{9}
\end{array}\right] \mathbf{H}=\left[\begin{array}{ccc}
h_{1} & h_{2} & h_{3} \\
h_{4} & h_{5} & h_{6} \\
h_{7} & h_{8} & h_{9}
\end{array}\right] \quad \begin{aligned}
& \text { Matlab } \\
& {[\mathrm{U}, \mathrm{~S}, \mathrm{~V}]=\operatorname{svd}(\mathrm{A}) ;} \\
& \mathrm{h}=\mathrm{V}(:, \text { end }) ;
\end{aligned}
$$

## Computing homography

Assume we have four matched points: How do we compute homography $\mathbf{H}$ ?

## Normalized DLT

1. Normalize coordinates for each image
a) Translate for zero mean
b) Scale so that $u$ and $v$ are $\sim=1$ on average

$$
\widetilde{\mathbf{x}}=\mathbf{T} \mathbf{x} \quad \widetilde{\mathbf{x}}^{\prime}=\mathbf{T}^{\prime} \mathbf{x}^{\prime}
$$

- This makes problem better behaved numerically (see Hartley and Zisserman p. 107-108)

2. Compute $\widetilde{\mathbf{H}}$ using DLT in normalized coordinates
3. Unnormalize: $\mathbf{H}=\mathbf{T}^{\prime-1} \widetilde{\mathbf{H}} \mathbf{T}$

$$
\mathbf{x}_{i}^{\prime}=\mathbf{H} \mathbf{x}_{i}
$$

## Computing homography

- Assume we have matched points with outliers: How do we compute homography H?

Automatic Homography Estimation with RANSAC

## RANSAC: RANdom SAmple Consensus

Scenario: We've got way more matched points than needed to fit the parameters, but we're not sure which are correct

## RANSAC Algorithm

- Repeat N times

1. Randomly select a sample

- Select just enough points to recover the parameters

2. Fit the model with random sample
3. See how many other points agree

- Best estimate is one with most agreement
- can use agreeing points to refine estimate


## Computing homography

- Assume we have matched points with outliers: How do we compute homography $\mathbf{H}$ ?

Automatic Homography Estimation with RANSAC

1. Choose number of iterations $N$
2. Choose 4 random potential matches
3. Compute $\mathbf{H}$ using normalized DLT
4. Project points from $\mathbf{x}$ to $\mathbf{x}^{\prime}$ for each potentially matching pair: $\mathbf{x}^{p}{ }_{i}=\mathbf{H x}{ }_{i}$
5. Count points with projected distance $<\mathrm{t}$

- E.g., $t=3$ pixels

$$
\sqrt{\left(u_{i}^{\prime}-u_{i}^{p}\right)^{2}+\left(v_{i}^{\prime}-v_{i}^{p}\right)^{2}}<t
$$

6. Repeat steps 2-5 $N$ times

- Choose $\mathbf{H}$ with most inliers


## Automatic Image Stitching

1. Compute interest points on each image
2. Find candidate matches
3. Estimate homography $\mathbf{H}$ using matched points and RANSAC with normalized DLT
4. Project each image onto the same surface and blend

## Choosing a Projection Surface

Many to choose: planar, cylindrical, spherical, cubic, etc.


## Planar Mapping



1) For red image: pixels are already on the planar surface
2) For green image: map to first image plane

## Planar Projection



## Planar Projection

Planar

## Cylindrical Mapping



1) Convert pixel coordinate $\boldsymbol{p}$ to bearing: $\widetilde{\boldsymbol{p}}=K^{-1} \boldsymbol{p}, \widetilde{\boldsymbol{p}}=\widetilde{\boldsymbol{p}} /\|\widetilde{\boldsymbol{p}}\|$
2) Compute theta, h: $\theta=\tan ^{-1}\left(\widetilde{\boldsymbol{p}}_{x} / f\right), h=\widetilde{\boldsymbol{p}}_{y} f$
3) For non-reference image, adjust for relative rotation/height

## Cylindrical Projection

Cylindrical

## Cylindrical Projection

Cylindrical

## Planar vs. Cylindrical Projection



## Automatically choosing images to stitch

## Recognizing Panoramas



## Recognizing Panoramas

Input: N images

1. Extract SIFT points, descriptors from all images
2. Find K-nearest neighbors for each point $(K=4)$
3. For each image
a) Select $M$ candidate matching images by counting matched keypoints ( $\mathrm{M}=6$ )
b) Solve homography $\mathbf{H}_{\mathrm{ij}}$ for each matched image

## Recognizing Panoramas

## Input: N images

1. Extract SIFT points, descriptors from all images
2. Find $K$-nearest neighbors for each point $(K=4)$
3. For each image
a) Select M candidate matching images by counting matched keypoints ( $M=6$ )
b) Solve homography $\mathbf{H}_{\mathrm{ij}}$ for each matched image
c) Decide if match is valid $\left(\mathrm{n}_{\mathrm{i}}>8+0.3 \mathrm{n}_{\mathrm{f}}\right)$


## RANSAC for Homography



## RANSAC for Homography



## Verification



## RANSAC for Homography



## Recognizing Panoramas (cont.)

(now we have matched pairs of images)
4. Find connected components

## Finding the panoramas



## Finding the panoramas



## Finding the panoramas



## Recognizing Panoramas (cont.)

(now we have matched pairs of images)
4. Find connected components
5. For each connected component
a) Perform bundle adjustment to solve for rotation $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$ and focal length $f$ of all cameras
b) Project to a surface (plane, cylinder, or sphere)
c) Render with multiband blending

## Bundle adjustment for stitching

- Non-linear minimization of re-projection error

$$
\mathbf{R}_{i}=e^{\left[\boldsymbol{\theta}_{i}\right] \times},\left[\boldsymbol{\theta}_{i}\right]_{\times}=\left[\begin{array}{ccc}
0 & -\theta_{i 3} & \theta_{i 2} \\
\theta_{i 3} & 0 & -\theta_{i 1} \\
-\theta_{i 2} & \theta_{i 1} & 0
\end{array}\right]
$$

- $\hat{\mathbf{x}}^{\prime}=\mathbf{H x}$ where $\mathbf{H}=\mathbf{K}^{\prime} \mathbf{R}^{\prime} \mathbf{R}^{-1} \mathbf{K}^{-1}$

$$
\mathbf{K}_{i}=\left[\begin{array}{ccc}
f_{i} & 0 & 0 \\
0 & f_{i} & 0 \\
0 & 0 & 1
\end{array}\right]
$$

$$
\operatorname{error}=\sum_{1}^{N} \sum_{j}^{M_{i}} \sum_{k} \operatorname{dist}\left(\mathbf{x}^{\prime}, \hat{\mathbf{x}}^{\prime}\right)
$$

- Solve non-linear least squares (LevenbergMarquardt algorithm)
- See paper for details


## Bundle Adjustment

New images initialized with rotation, focal length of the best matching image


## Bundle Adjustment

New images initialized with rotation, focal length of the best matching image


## Details to make it look good



- Choosing seams
- Blending


## Choosing seams

- Easy method
- Assign each pixel to image with nearest center



## Choosing seams

- Easy method
- Assign each pixel to image with nearest center
- Create a mask:
- mask (y, x) = 1 iff pixel should come from im1
- Smooth boundaries (called "feathering"):
- mask_sm = imfilter(mask, gausfil);
- Composite
- imblend = im1_c.*mask + im2_c.*(1-mask);



## Choosing seams

- Better method: dynamic program to find seam along well-matched regions



## Gain compensation

- Simple gain adjustment
- Compute average RGB intensity of each image in overlapping region
- Normalize intensities by ratio of averages



## Multi-band (aka Laplacian Pyramid) Blending

- Burt \& Adelson 1983
- Blend frequency bands over range $\propto \lambda$



## Multiband Blending with Laplacian Pyramid

- At low frequencies, blend slowly
- At high frequencies, blend quickly



## Multiband blending

1. Compute Laplacian pyramid of images and mask
2. Create blended image at each level of pyramid
3. Reconstruct complete image

(a) Original inuges and blevded reult

(b) Baxd 1 (scale 0 to $\sigma$ )

(c) Bavd 2 (scale $\sigma$ to $2 \sigma$ )


## Blending comparison (IJCV 2007)


(a) Linear blending

(b) Multi-band blending

## Blending Comparison


(b) Without gain compensation

(c) With gain compensation

(d) With gain compensation and multi-band blending

## Straightening

Rectify images so that "up" is vertical

(a)

(b)

(c)

(a) Without automatic straightening


## Further reading

Harley and Zisserman: Multi-view Geometry book

- DLT algorithm: HZ p. 91 (alg 4.2), p. 585
- Normalization: HZ p. 107-109 (alg 4.2)
- RANSAC: HZ Sec 4.7, p. 123, alg 4.6
- Tutorial: http://users.cecs.anu.edu.au/~hartley/Papers/CVPR99tutorial/tut 4up.pdf
- Recognising Panoramas: Brown and Lowe, IJCV 2007 (also bundle adjustment)


## How does iphone panoramic stitching work?

- Capture images at 30 fps
- Stitch the central $1 / 8$ of a selection of images
- Select which images to stitch using the accelerometer and frame-toframe matching
- Faster and avoids radial distortion that often occurs towards corners of images
- Alignment
- Initially, perform cross-correlation of small patches aided by accelerometer to find good regions for matching
- Register by matching points (KLT tracking or RANSAC with FAST (similar to SIFT) points) or correlational matching
- Blending
- Linear (or similar) blending, using a face detector to avoid blurring face regions and choose good face shots (not blinking, etc)

[^0]
## Project 5

## Input video: <br> https://www.youtube.com/watch?v=agl5za gHHU

Aligned frames:
https://www.youtube.com/watch?v=Uahy6kPotaE

Background:
https://www.youtube.com/watch?v=Vt9vv1zCnLA

Foreground:
https://www.youtube.com/watch?v=OICkKNndEt4

## Tips and Photos from Russ Hewett

## Capturing Panoramic Images

- Tripod vs Handheld
- Help from modern cameras
- Leveling tripod
- Gigapan
- Or wing it
- Image Sequence
- Requires a reasonable amount of overlap (at least 15-30\%)
- Enough to overcome lens distortion
- Exposure
- Consistent exposure between frames
- Gives smooth transitions
- Manual exposure
- Makes consistent exposure of dynamic scenes easier
- But scenes don't have constant intensity everywhere
- Caution
- Distortion in lens (Pin Cushion, Barrel, and Fisheye)
- Polarizing filters
- Sharpness in image edge / overlap region

Pike's Peak Highway, CO


Pike's Peak Highway, CO

360 Degrees, Tripod Leveled


Howth, Ireland

Handheld Camera


## Handheld Camera

## Les Diablerets, Switzerland

Macro


## Side of Laptop

## Considerations For Stitching

- Variable intensity across the total scene
- Variable intensity and contrast between frames
- Lens distortion
- Pin Cushion, Barrel, and Fisheye
- Profile your lens at the chosen focal length (read from EXIF)
- Or get a profile from LensFun
- Dynamics/Motion in the scene
- Causes ghosting
- Once images are aligned, simply choose from one or the other
- Misalignment
- Also causes ghosting
- Pick better control points
- Visually pleasing result
- Super wide panoramas are not always 'pleasant' to look at
- Crop to golden ratio, 10:3, or something else visually pleasing

Ghosting and Variable Intensity



## Ghosting From Motion



## Motion Between Frames




Gibson City, IL


## Mount Blanca, CO




Mount Blanca, CO


## Things to remember

- Homography relates rotating cameras
- Homography is plane to plane mapping
- Recover homography using RANSAC and normalized DLT
- Can choose surface of projection: cylinder, plane, and sphere are most common
- Refinement methods (blending, straightening, etc.)

Next class

- Object recognition and retrieval


[^0]:    http://www.patentlyapple.com/patently-apple/2012/11/apples-cool-iphone-5-panorama-app-revealed-in-5-patents.html

