# Single-view 3D Reconstruction

Computational Photography

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Some slides from Alyosha Efros, Steve Seitz

# Take-home question

Suppose you have estimated finite three vanishing points corresponding to orthogonal directions:

1) How to solve for intrinsic matrix? (assume K has three parameters)
    - The transpose of the rotation matrix is its inverse
    - Use the fact that the 3D directions are orthogonal
2) How to recover the rotation matrix that is aligned with the 3D axes defined by these points?
    - In homogeneous coordinates, 3d point at infinity is (X, Y, Z, 0)

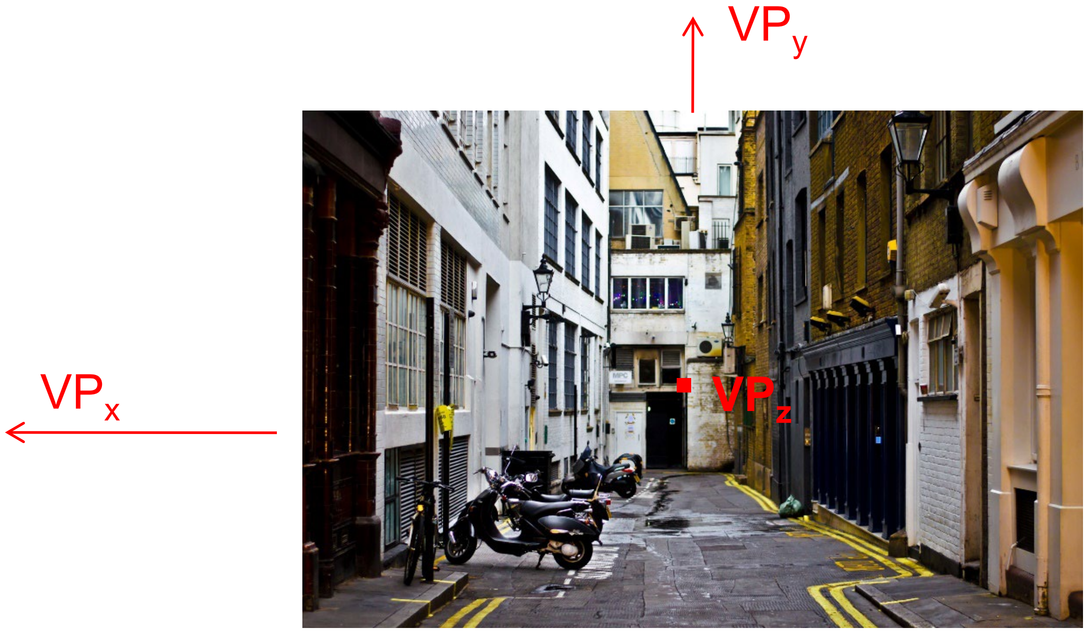


Photo from Garry Knight

# Take-home question

Assume that the man is 6 ft tall.

- What is the height of the front of the building?
- What is the height of the camera?

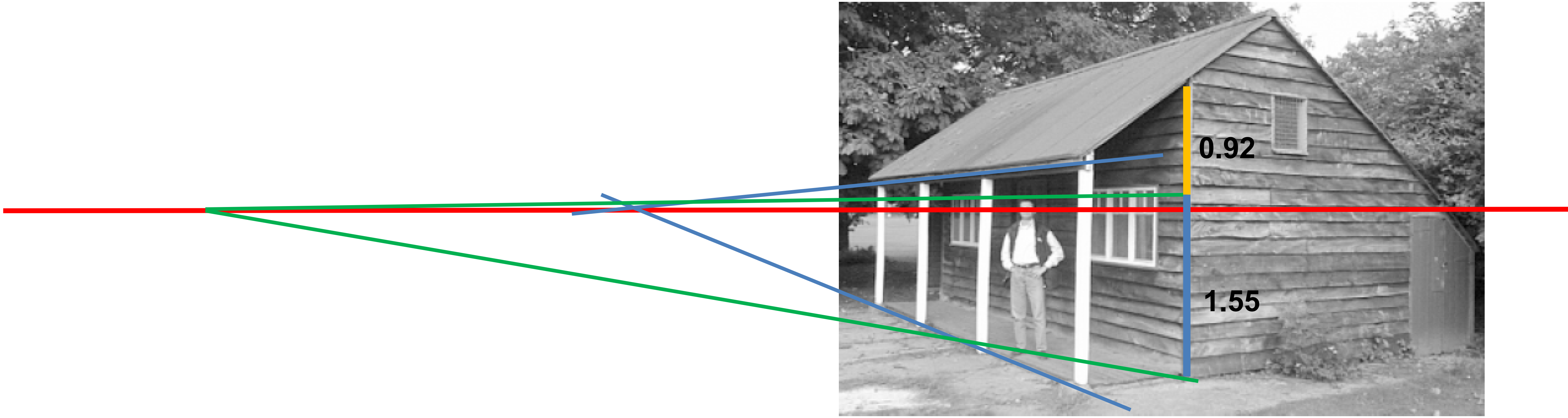
# Take-home question

Assume that the man is 6 ft tall.

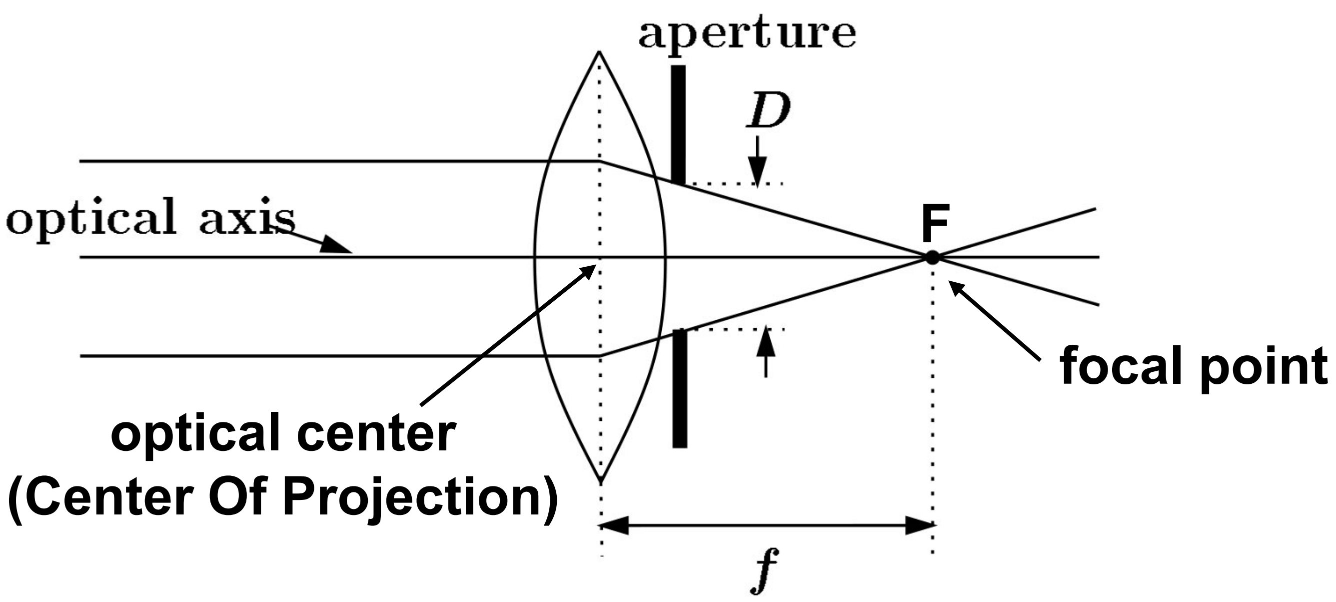
(0.92+1.55)/1.55*6=9.56

- What is the height of the front of the building?
- What is the height of the camera?

~5’7



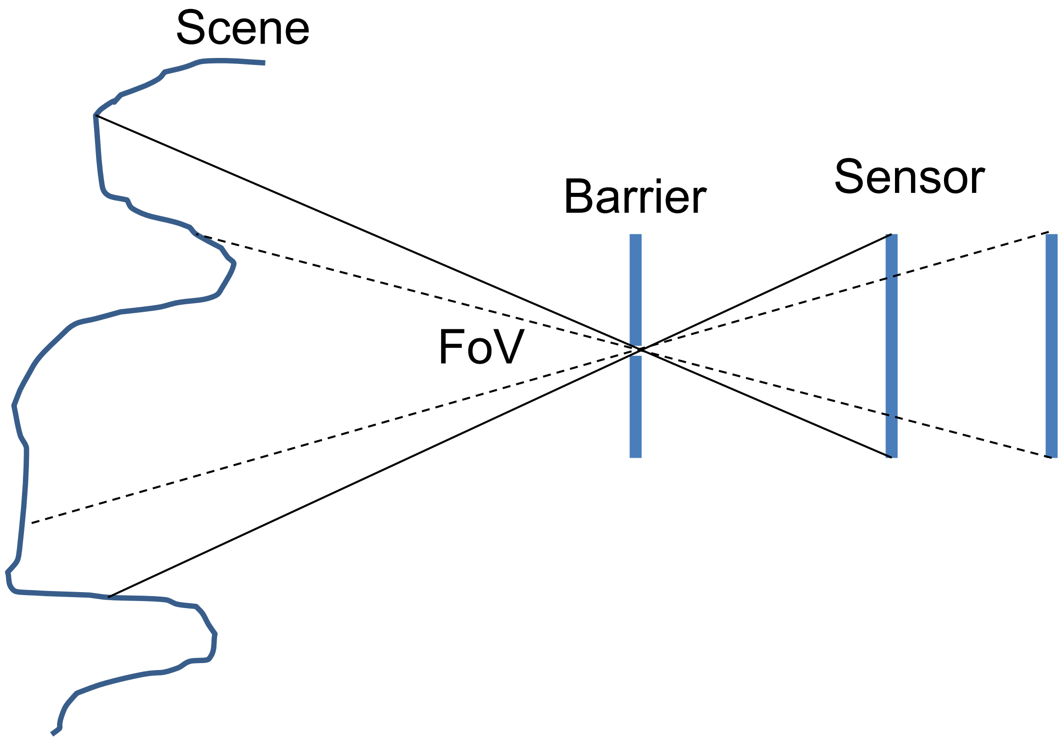
# Focal length, aperture, depth of field



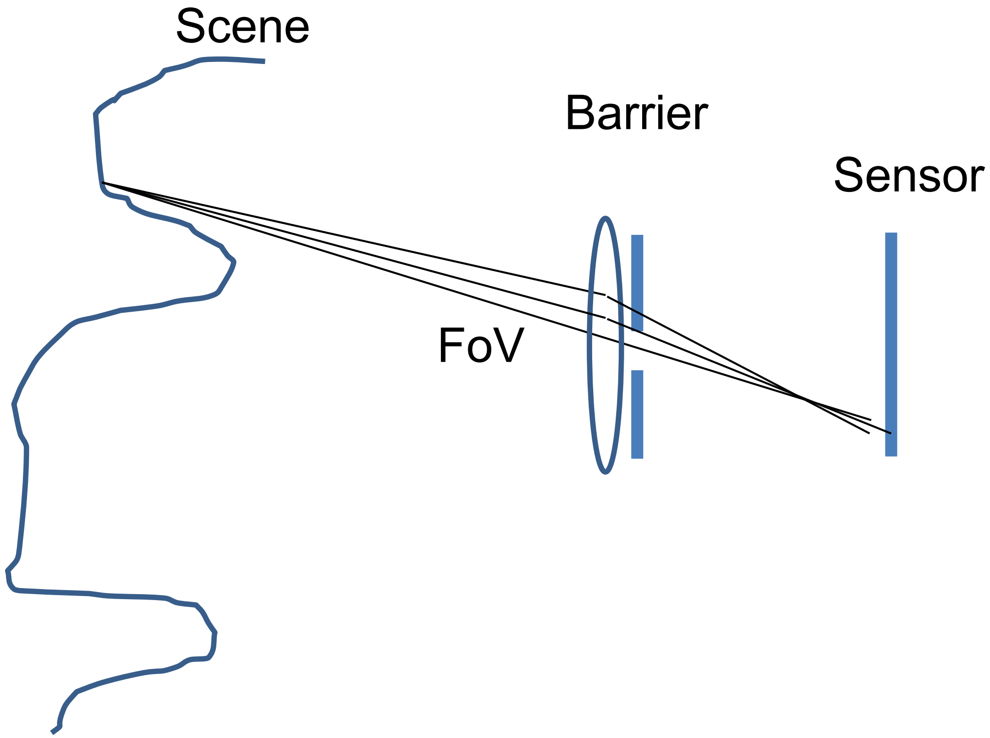
- Increase in focal length “zooms in”, decreasing field of view (and light per pixel), increasing depth of field (less blur)

- Increase in aperture lets more light in but decreases depth of field

# Increasing focal length decreases field of view because smaller range of rays to scene can hit sensor



# Decreasing aperture increases depth of field because lens refocuses rays from smaller range of angles



# Difficulty in macro (close-up) photography

- For close objects, we have a small relative DOF
- Can only shrink aperture so far

How to get both bugs in focus?

# Solution: Focus stacking

1. Take pictures with varying focal length

Example from
http://www.wonderfulphotos.com/articles/macro/focus_stacking/

# Solution: Focus stacking

1. Take pictures with varying focal length
2. Combine

# Focus stacking

http://www.wonderfulphotos.com/articles/macro/focus_stacking/

# Focus stacking

## How to combine?

1. Align images (e.g., using corresponding points)
2. Two ideas
   a) Mask regions by hand and combine with pyramid blend
   b) Gradient domain fusion (mixed gradient) without masking

Automatic solution would make an interesting final project

Recommended Reading:

http://www.digital-photography-school.com/an-introduction-to-focus-stacking

http://www.zen20934.zen.co.uk/photography/Workflow.htm#Focus%20Stacking

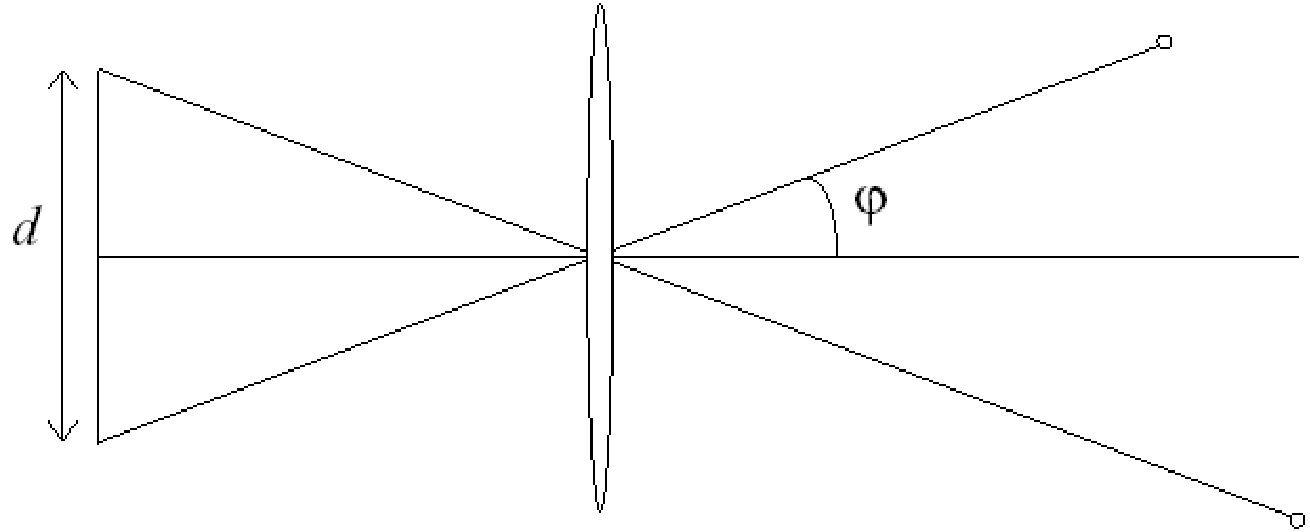
# Relation between field of view and focal length

Field of view (angle width)

Film/Sensor Width

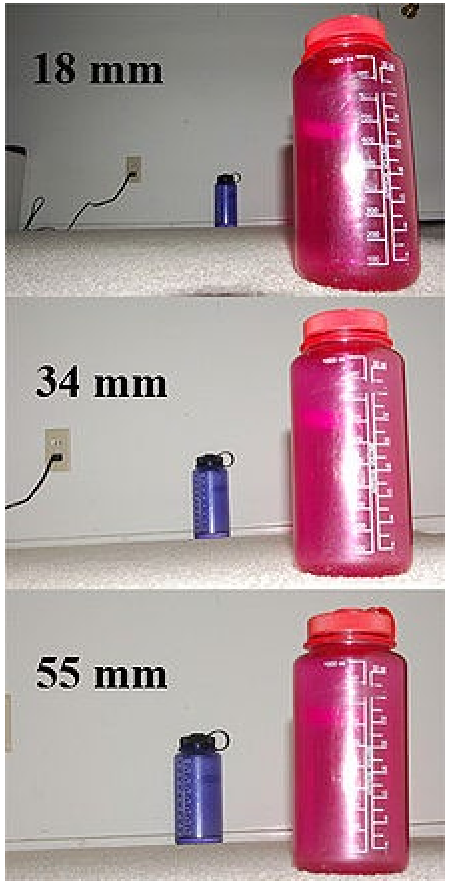
$$fov = 2\tan^{-1}\frac{d}{2f}$$

Focal length



# Dolly Zoom or “Vertigo Effect”

http://www.youtube.com/watch?v=NB4bikrNzMk



How is this done?

Zoom in while
moving away

http://en.wikipedia.org/wiki/Focal_length

# Dolly zoom (or “Vertigo effect”)

Field of view (angle width)

Film/Sensor Width

$$fov = 2\tan^{-1}\frac{d}{2f}$$

Focal length

width of object

Given fov, calculate distance to keep object width in frame constant

$$\text{distance} = \frac{width}{2\tan fov/2}$$

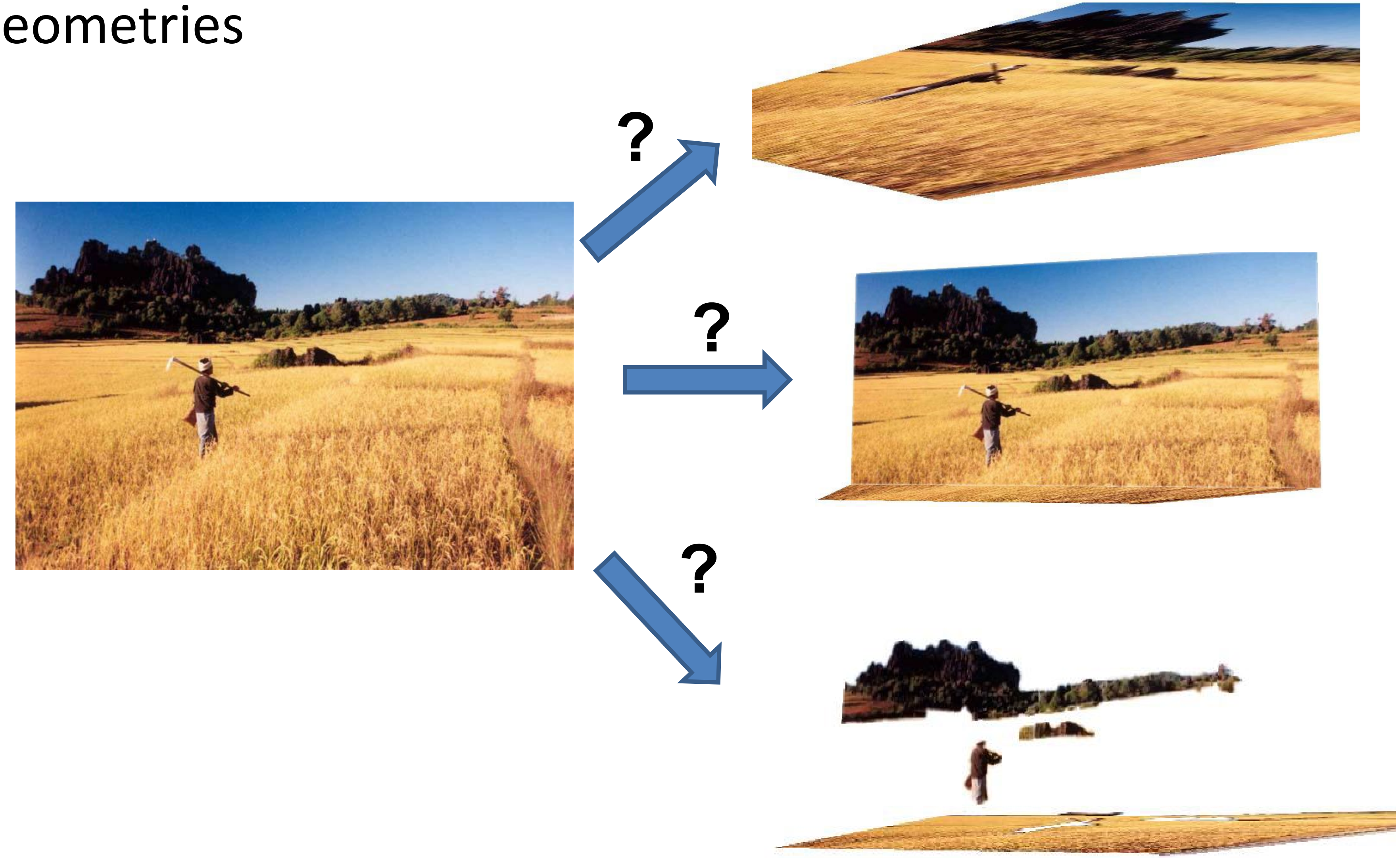
Distance between object and camera

Camera fov

# Today’s class: Single View 3D Scene Reconstruction

# The challenge

One 2D image could be generated by an infinite number of 3D geometries



# The solution

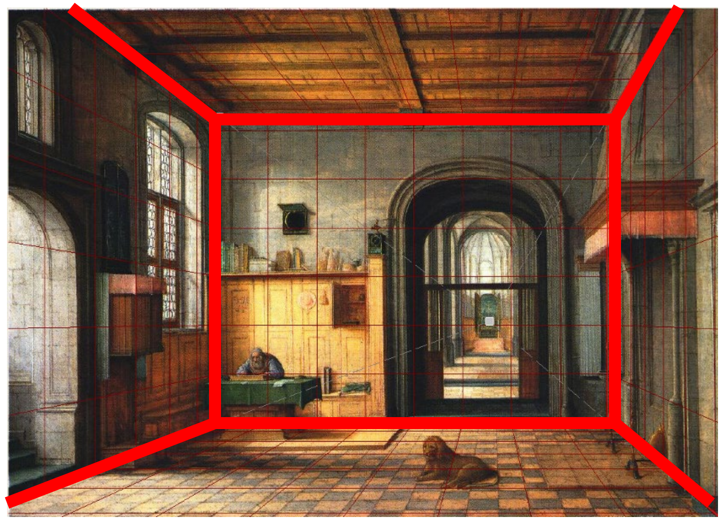
Make simplifying assumptions about 3D geometry

Unlikely

Likely

# Today’s class: Two Models

- Box + frontal billboards

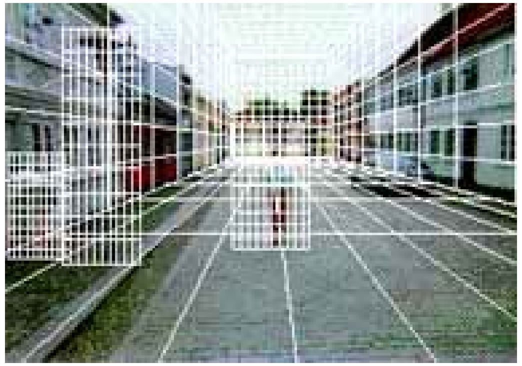


- Ground plane + non-frontal billboards

# “Tour into the Picture” (Horry et al. SIGGRAPH ’97)

Create a 3D “theater stage” of five planes

Specify foreground objects through bounding polygons



Use camera transformations to navigate through the scene

Following slides modified from Efros

# The idea

Many scenes can be represented as an axis-aligned box volume (i.e. a stage)

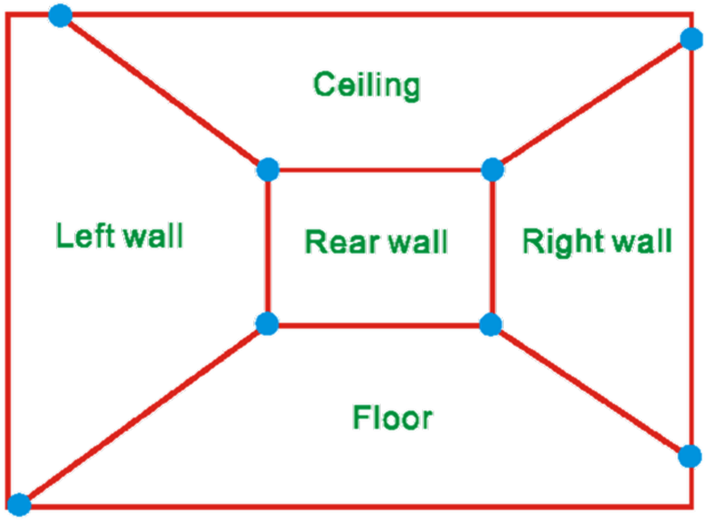
Key assumptions

- All walls are orthogonal
- Camera view plane is parallel to back of volume

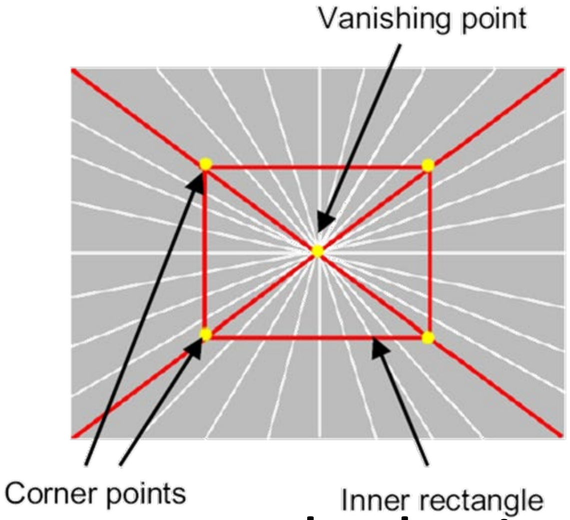
How many vanishing points does the box have?

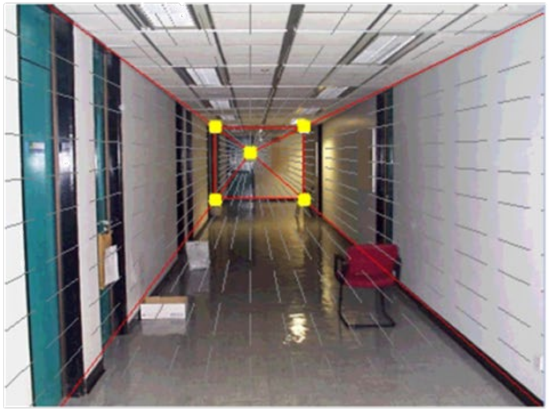
- Three, but two at infinity
- Single-point perspective

Can use the vanishing point to fit the box to the particular scene



# Step 1: specify scene geometry

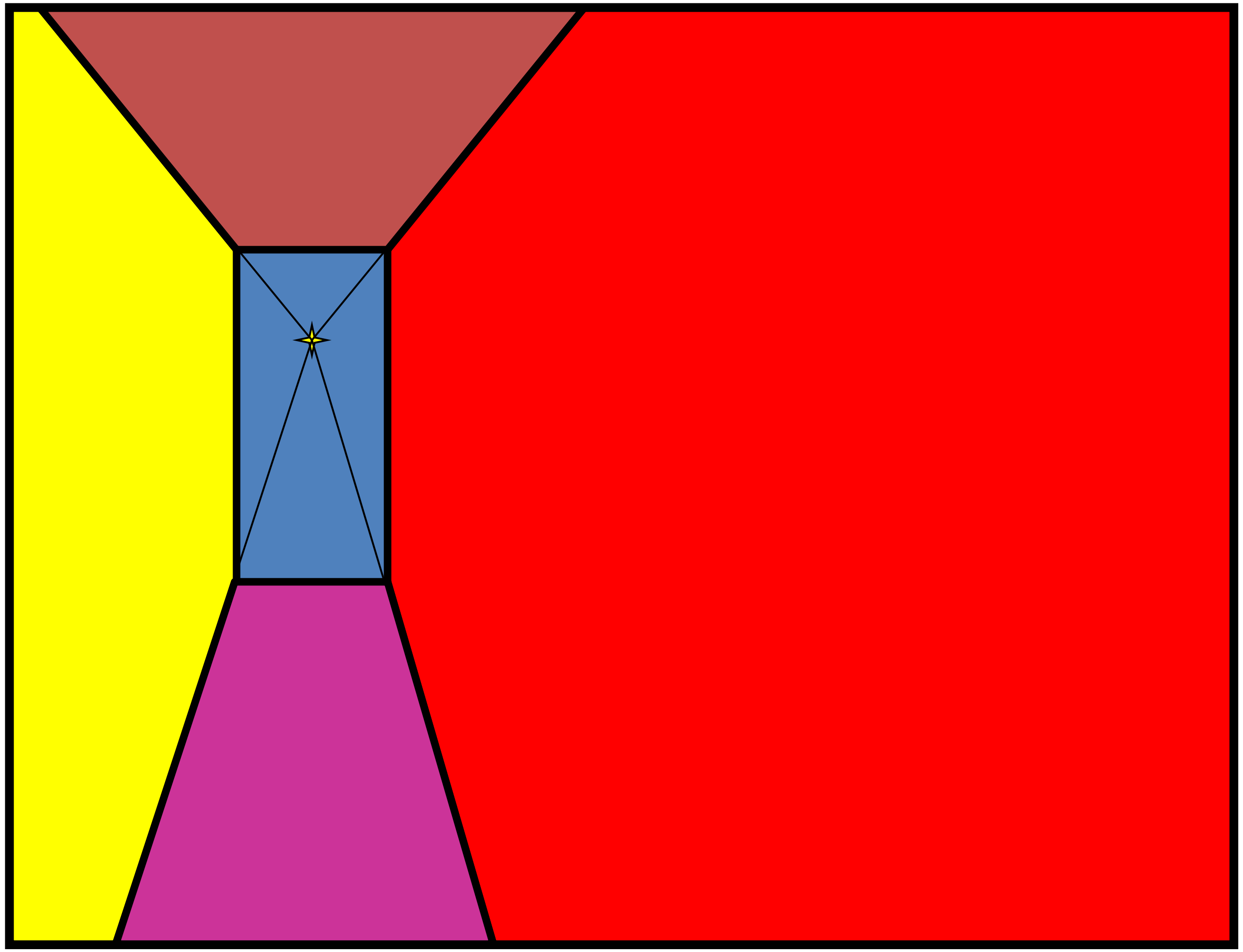




- User controls the inner box and the vanishing point placement (# of DOF?)

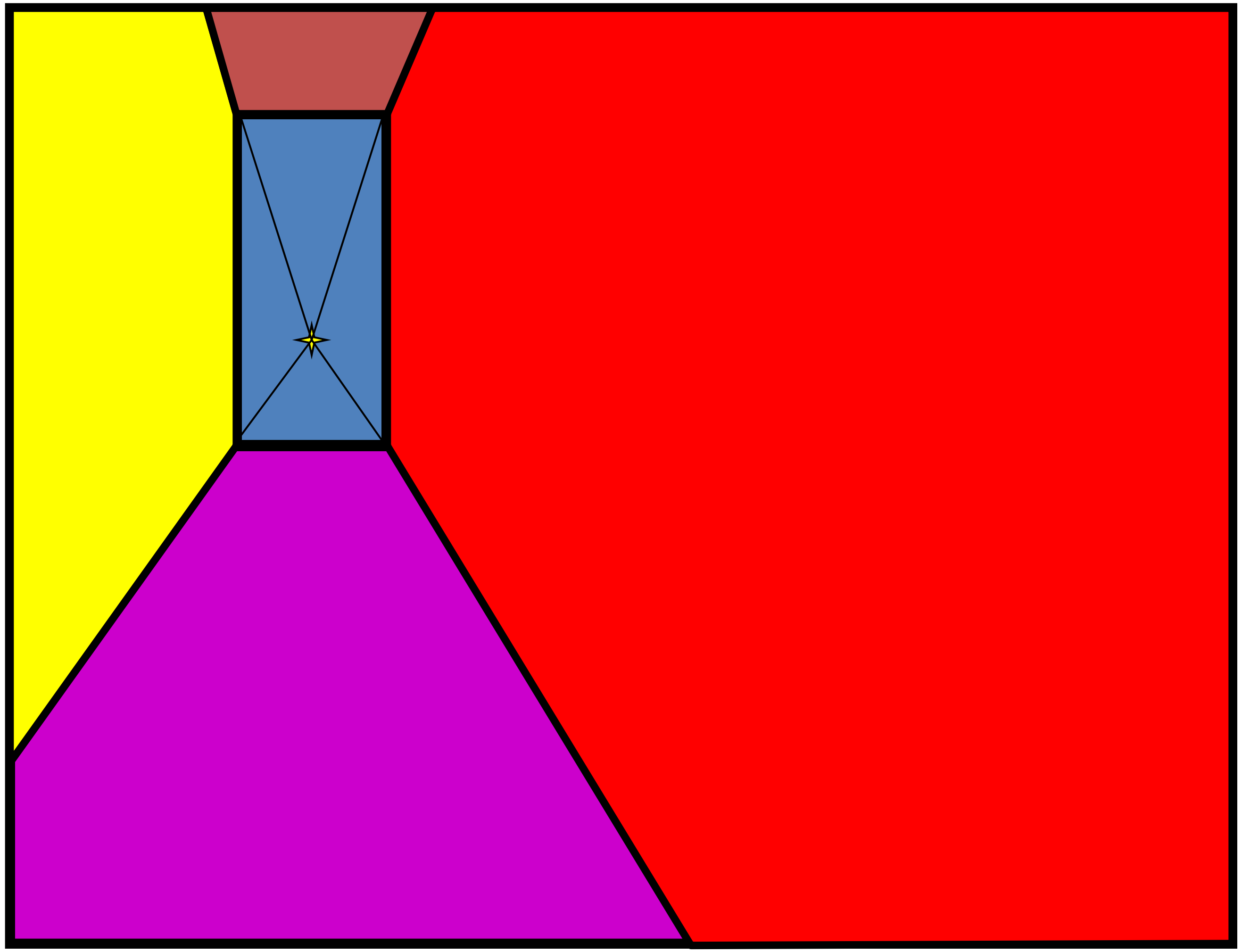
- Q: If we assume camera is looking straight at back wall, what camera parameter(s) does the vanishing point position provide?
- A: Vanishing point direction is perpendicular to image plane, so the vp is the principal point

Example of user input: vanishing point and back face of view volume are defined



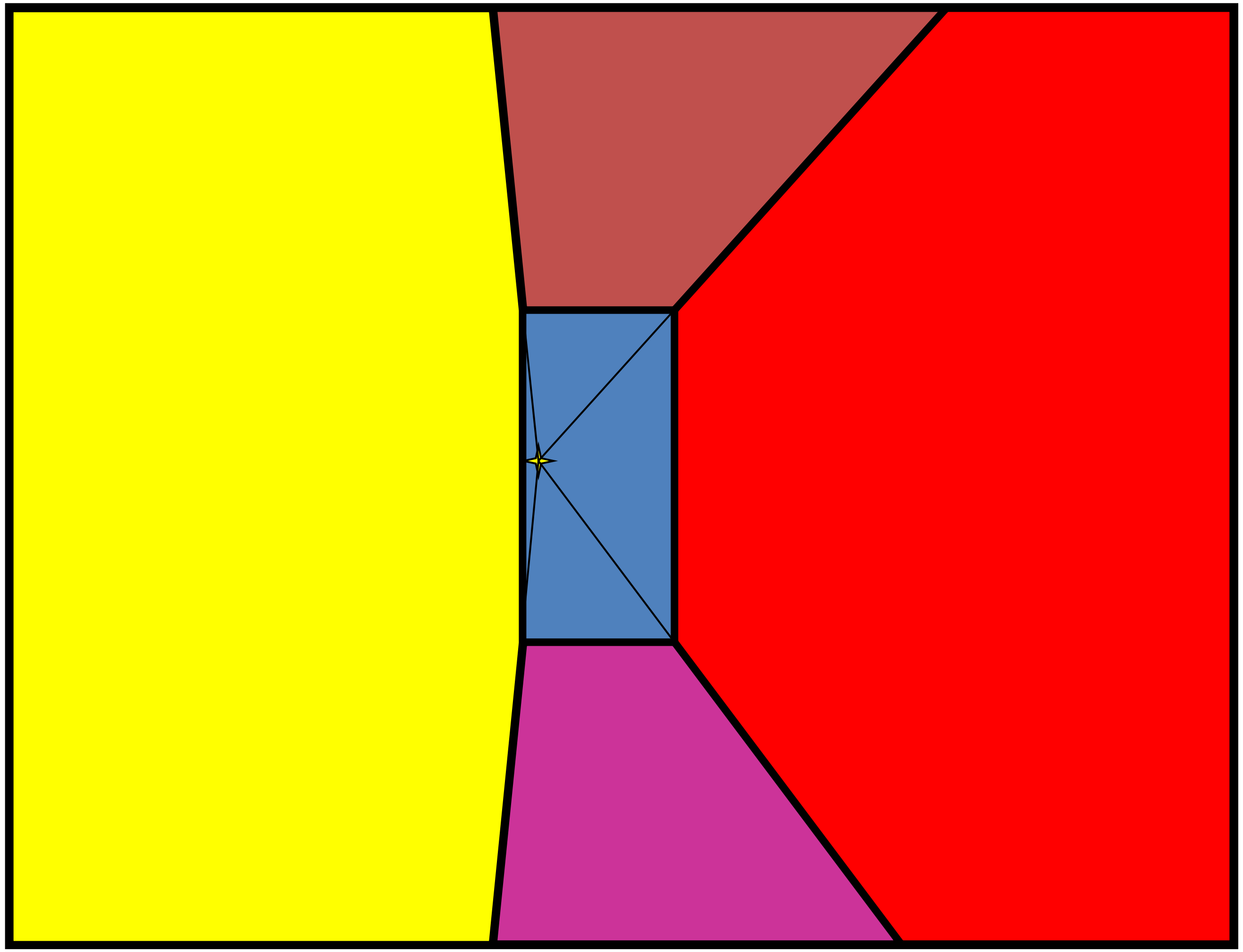
High Camera

Example of user input: vanishing point and back face of view volume are defined



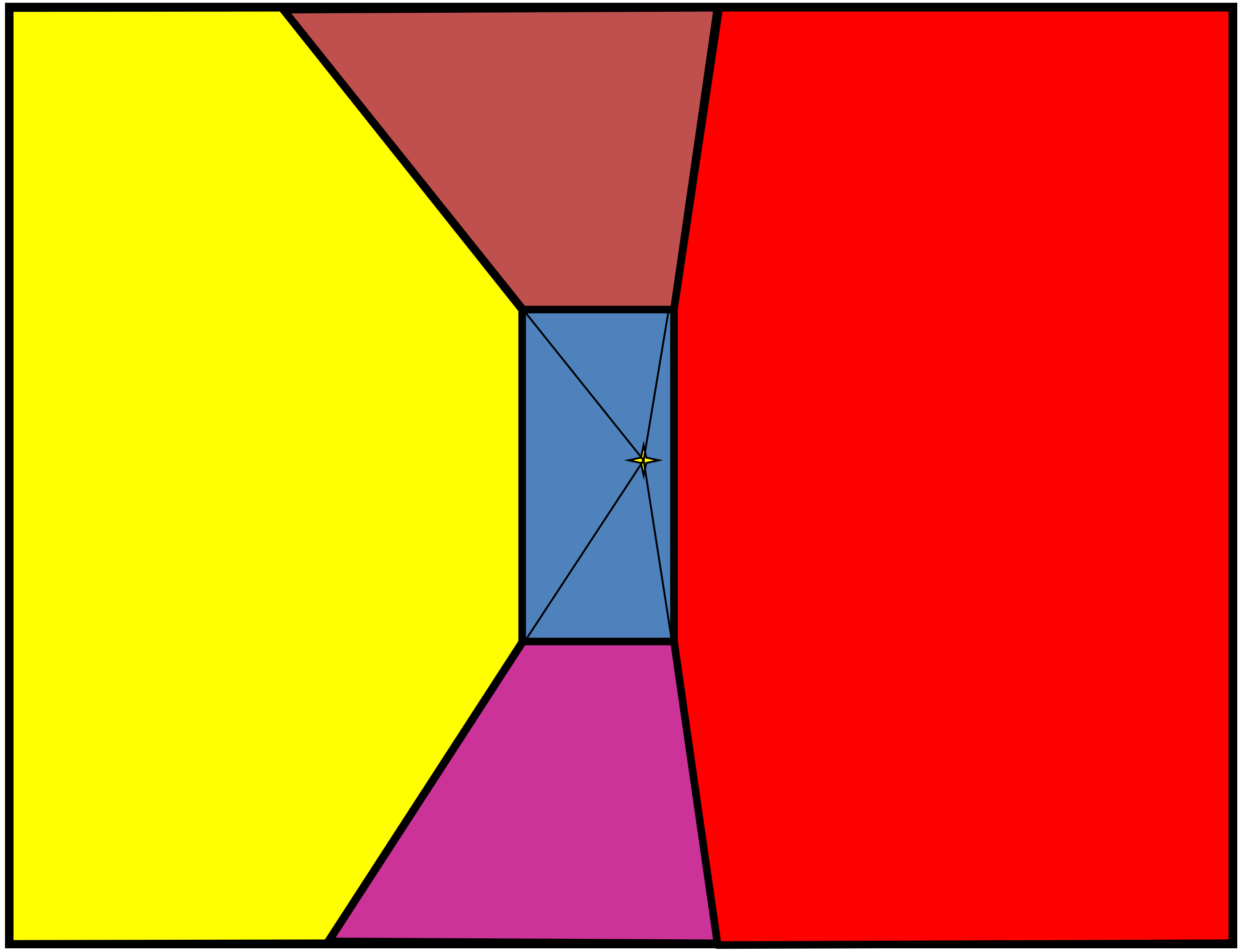
Low Camera

# Another example of user input: vanishing point and back face of view volume are defined



Left Camera

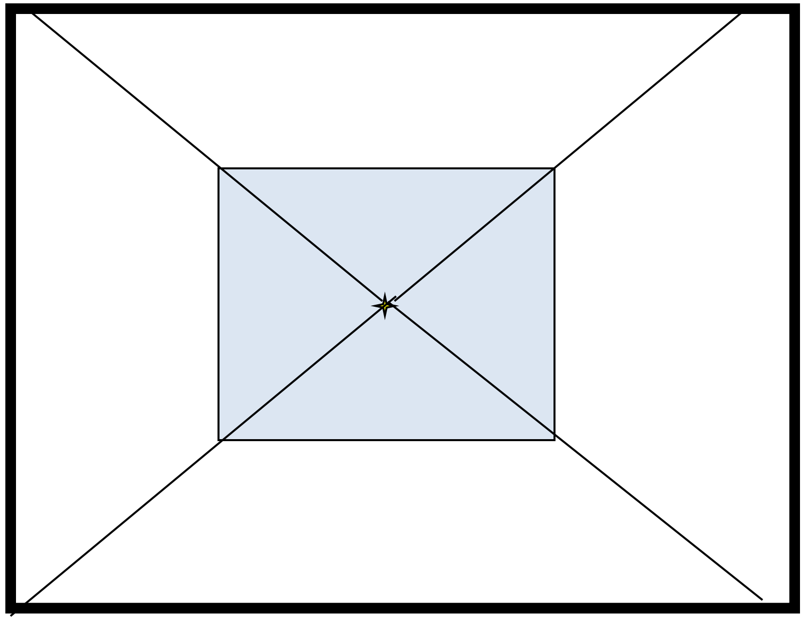
Another example of user input: vanishing point and back face of view volume are defined



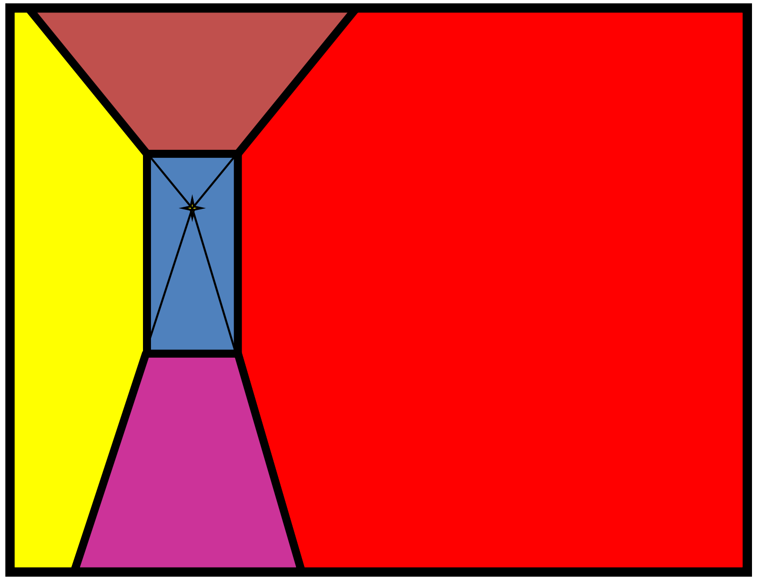
Right Camera

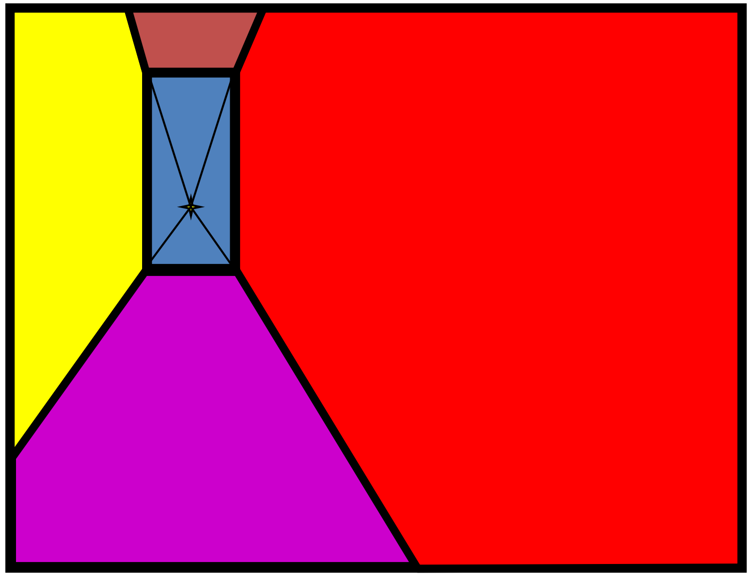
# Question

- Think about the camera center and image plane…
  - – What happens when we move the box?
  - – What happens when we move the vanishing point?

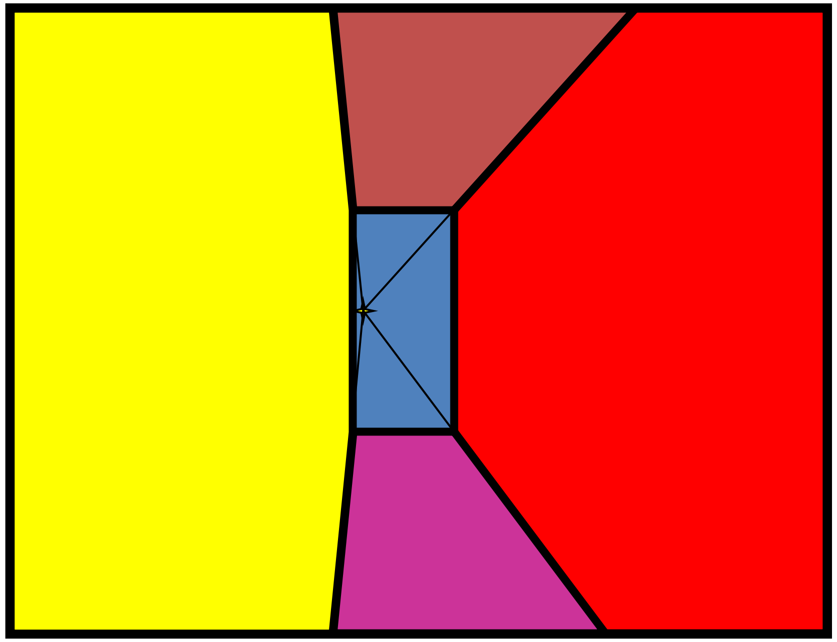


Moving the box corresponds to changing the position of the camera.

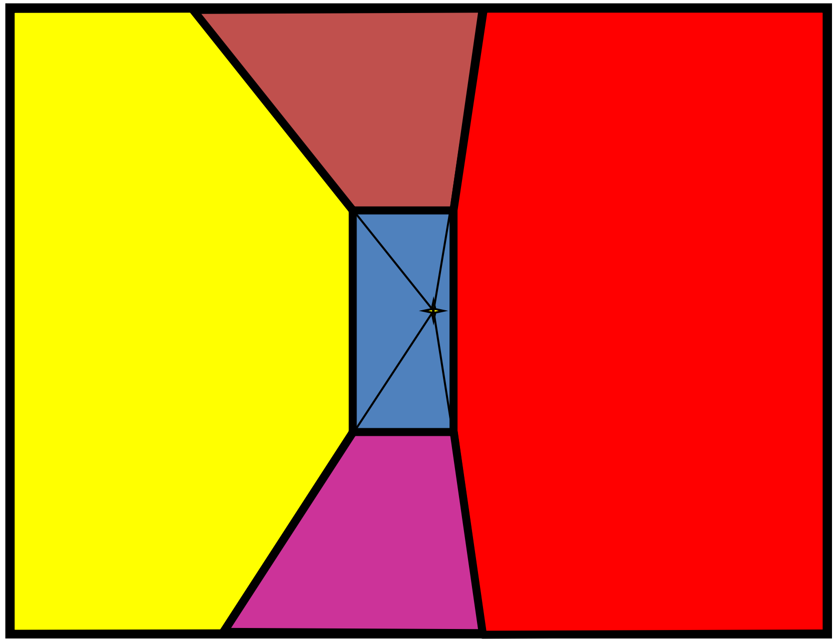

High Camera


Low Camera

Moving the vanishing point changes the orientation of the room relative to the camera.



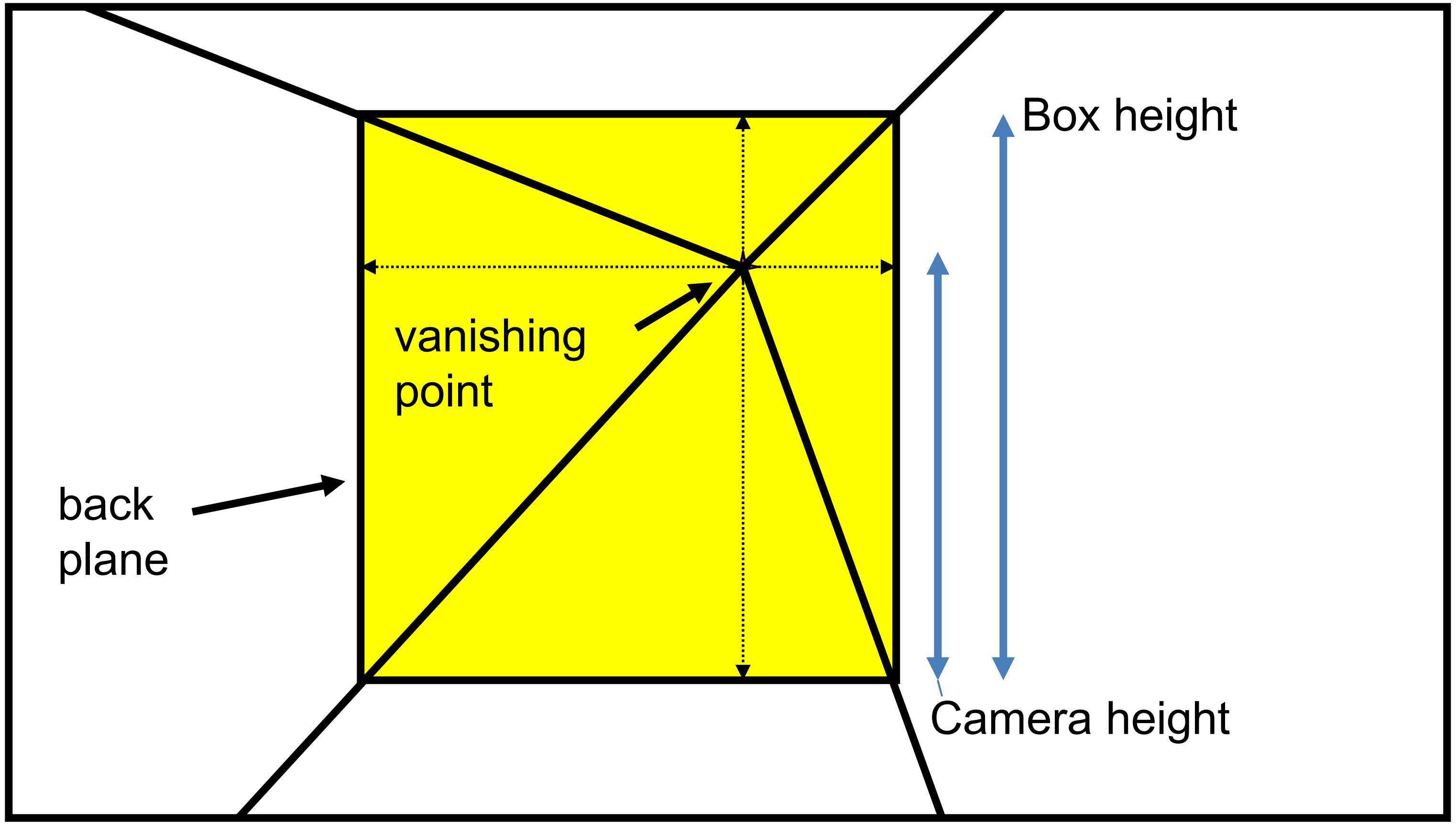
Left Camera



Right Camera

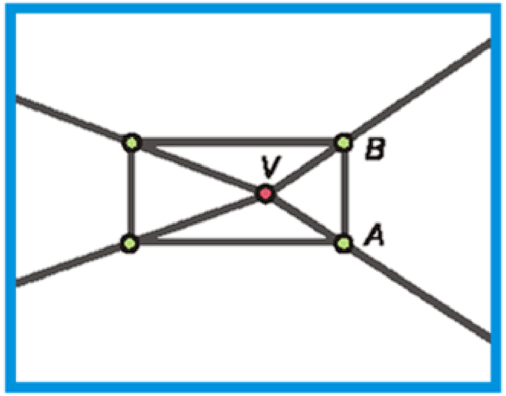
# 2D to 3D conversion

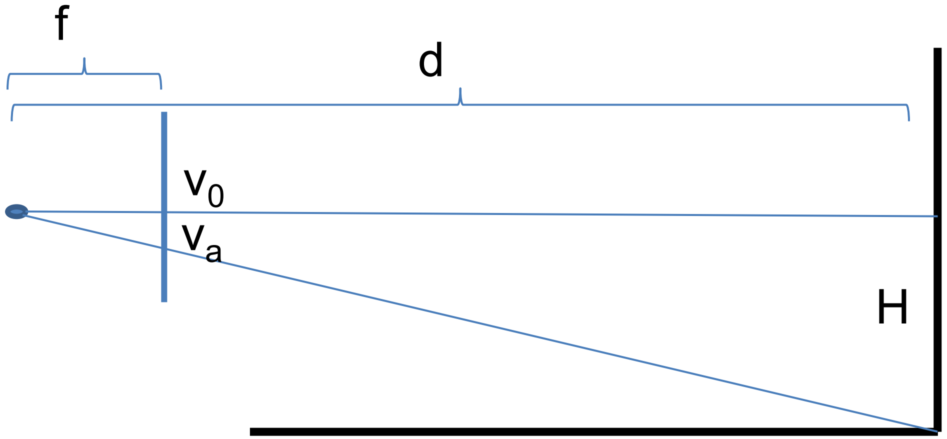
- Use ratios



Box width / height in 3D is proportional to width over height in the image because back plane is parallel to image plane

# Get depth using similar triangles



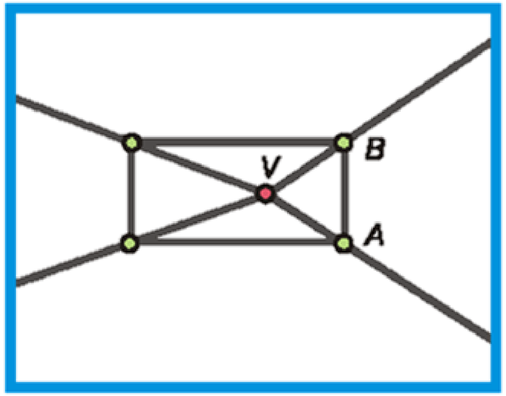


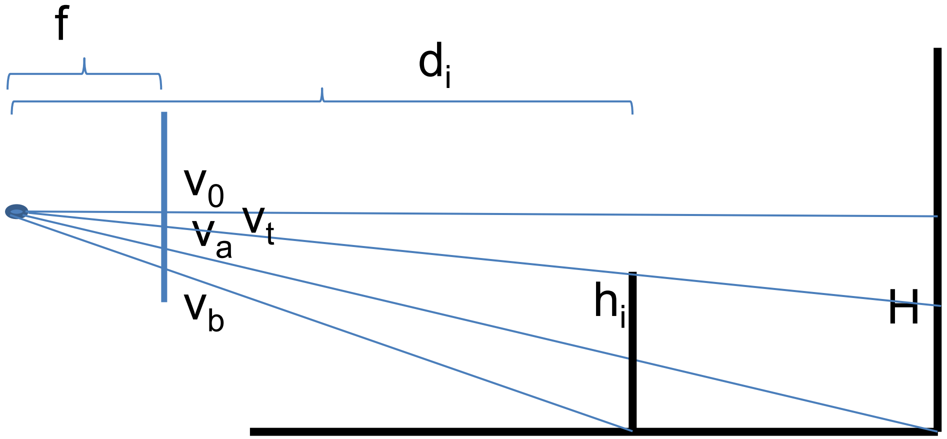
$f / (v_a - v_0) = d / H$

- Can compute by similar triangles
- Need to know focal length f (or FoV)

- Note: can compute position of any object on the ground
  - Simple unprojection
  - What about things off the ground?

# Get depth using similar triangles





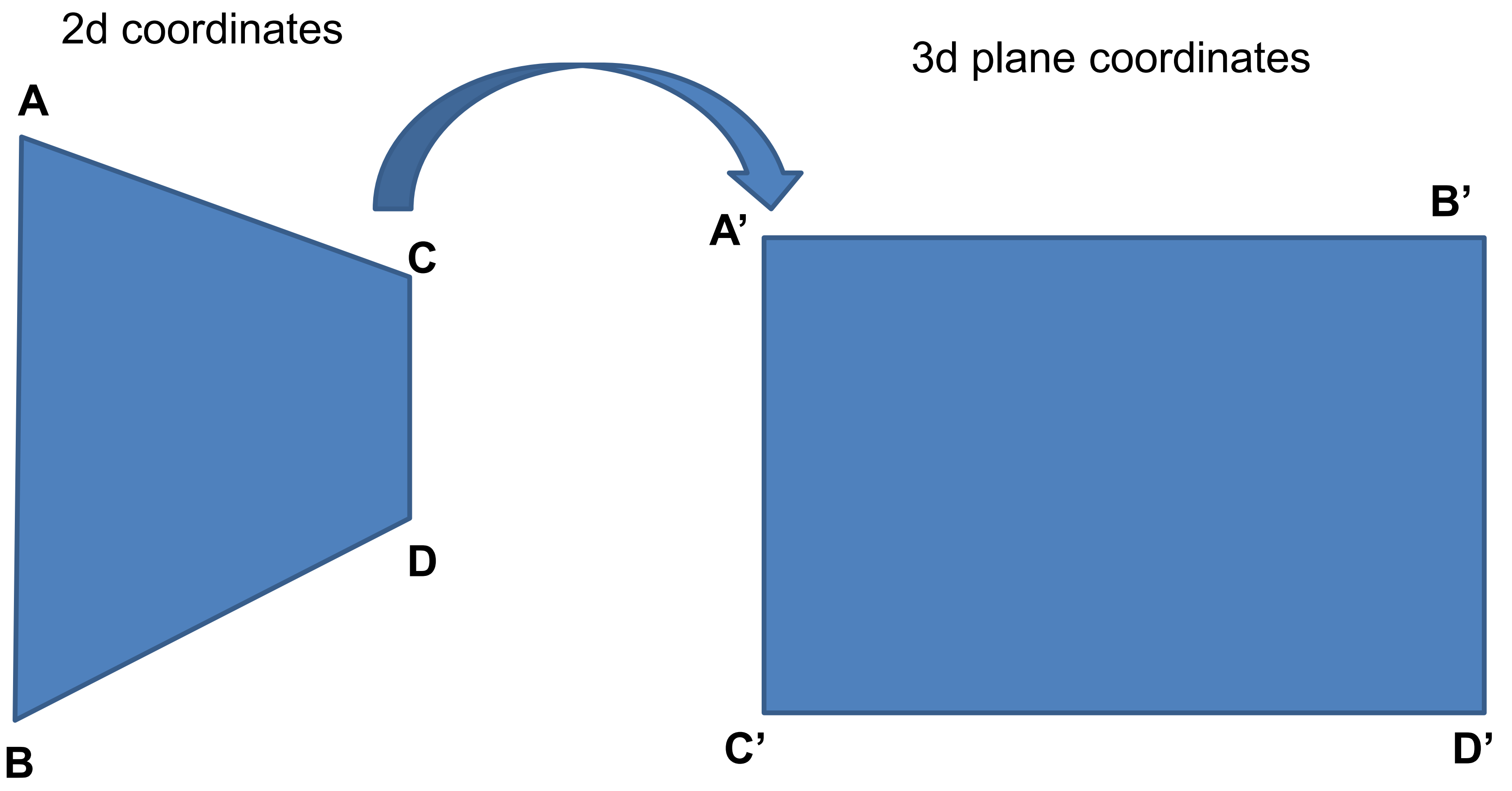
$h_i = H (v_a - v_t)/(v_a - v_0)$

$d_i = f\, h_i / (v_b - v_t)$

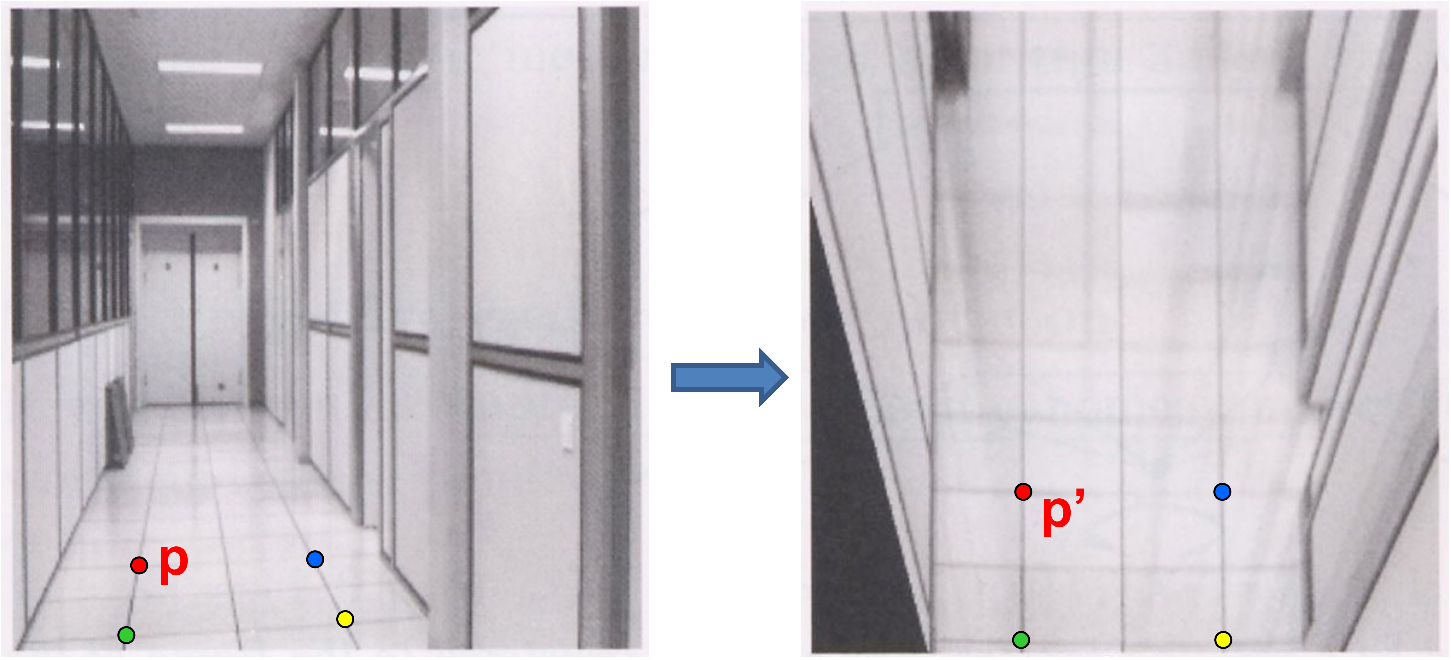
- Can compute by similar triangles (CVA vs. CV’A’)
- Need to know focal length f (or FoV)

- Can compute 3D position of any object on the ground w/ unprojection
  - What about things off the ground?

# Step 2: map image textures into frontal view



# Image rectification by homography



To unwarp (rectify) an image solve for homography **H** given **p** and **p'**: $w\mathbf{p}' = \mathbf{H}\mathbf{p}$

# Computing homography

Assume we have four matched points: How do we compute homography **H**?

Direct Linear Transformation (DLT)

$$\mathbf{p}' = \mathbf{H}\mathbf{p} \qquad \mathbf{p}' = \begin{bmatrix} w'u' \\ w'v' \\ w' \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

$$\begin{bmatrix} -u & -v & -1 & 0 & 0 & 0 & uu' & vu' & u' \\ 0 & 0 & 0 & -u & -v & -1 & uv' & vv' & v' \end{bmatrix} \mathbf{h} = \mathbf{0} \qquad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix}$$

# Computing homography

## Direct Linear Transform

$$\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1u_1' & v_1u_1' & u_1' \\ 0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1v_1' & v_1v_1' & v_1' \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & -u_n & -v_n & -1 & u_nv_n' & v_nv_n' & v_n' \end{bmatrix} \mathbf{h} = \mathbf{0} \Rightarrow \mathbf{Ah} = \mathbf{0}$$

- Apply SVD: $\boldsymbol{USV^T} = \boldsymbol{A}$
- $\boldsymbol{h} = \boldsymbol{V}_{\text{smallest}}$ (column of $\boldsymbol{V}^T$ corresponds to smallest singular value)

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

### Python

```
U, S, Vt = scipy.linalg.svd(A)
# last column of V corresp. to smallest singular value
h = Vt[-1, :];
```

Explanation of SVD, solving systems of linear equations, derivation of solution here

# Solving for homography (another formulation)

$$\underset{2n \times 9}{\underset{\mathbf{A}}{\begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1 u_1' & v_1 u_1' & u_1' \\ 0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1 v_1' & v_1 v_1' & v_1' \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & -u_n & -v_n & -1 & u_n v_n' & v_n v_n' & v_n' \end{bmatrix}}} \underset{9}{\underset{\mathbf{h}}{\mathbf{h}}} = \underset{2n}{\underset{\mathbf{0}}{\mathbf{0}}} \Rightarrow \mathbf{A}\mathbf{h} = \mathbf{0}$$

Defines a least squares problem:

$$\text{minimize } \|\mathbf{A}\mathbf{h} - \mathbf{0}\|^2$$

- Since **h** is only defined up to scale, solve for unit vector $\hat{\mathbf{h}}$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
  - Can derive using Lagrange multipliers method
- Works with 4 or more points

# Tour into the picture algorithm

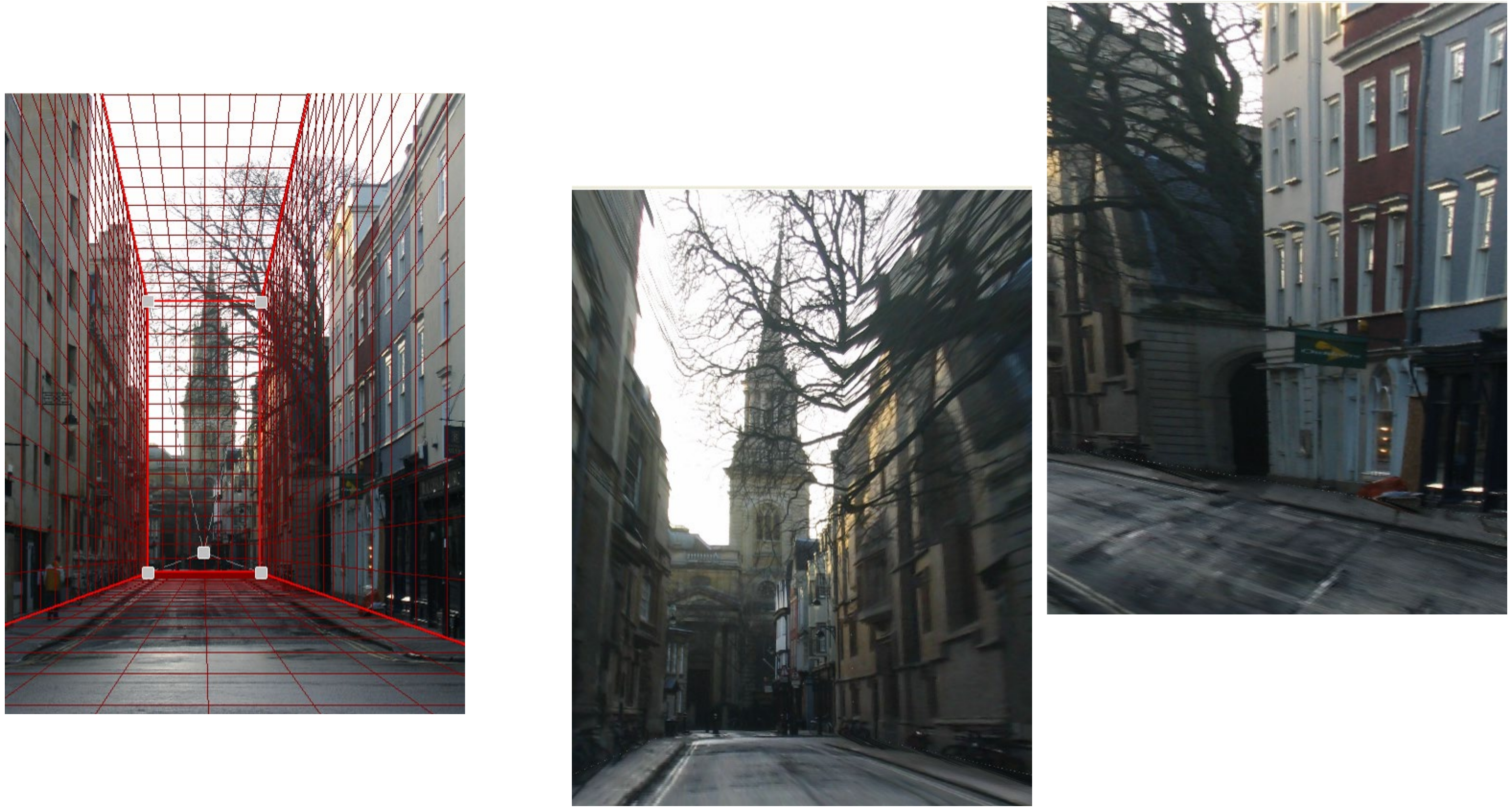
1. Set the box corners

# Tour into the picture algorithm

1. Set the box corners
2. Set the VP
3. Get 3D coordinates
   - Compute height, width, and depth of box
4. Get texture maps
   - homographies for each face
5. Create file to store plane coordinates and texture maps

# Result

Render from new views



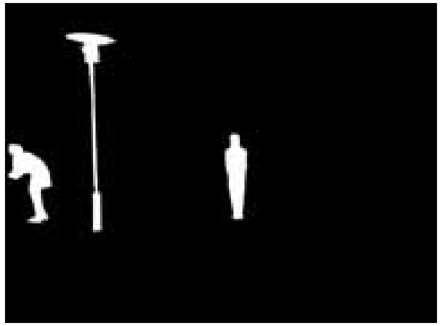
http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15463-f08/www/proj5/www/dmillett/

# Foreground Objects

Use separate billboard for each

For this to work, three separate images used:

- Original image.
- Mask to isolate desired foreground images.
- Background with objects removed

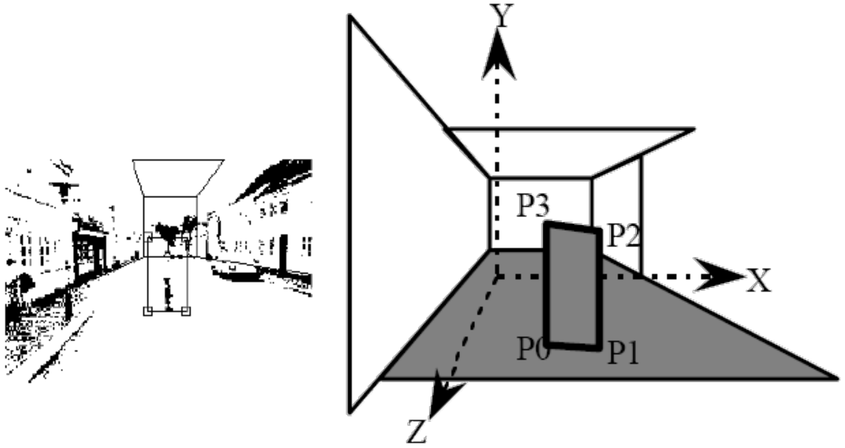


# Foreground Objects

Add vertical rectangles for each foreground object

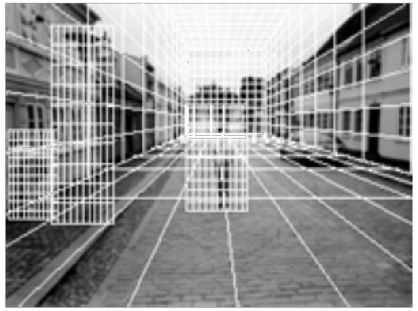
Can compute 3D coordinates P0, P1 since they are on known plane.

P2, P3 can be computed as before (similar triangles)



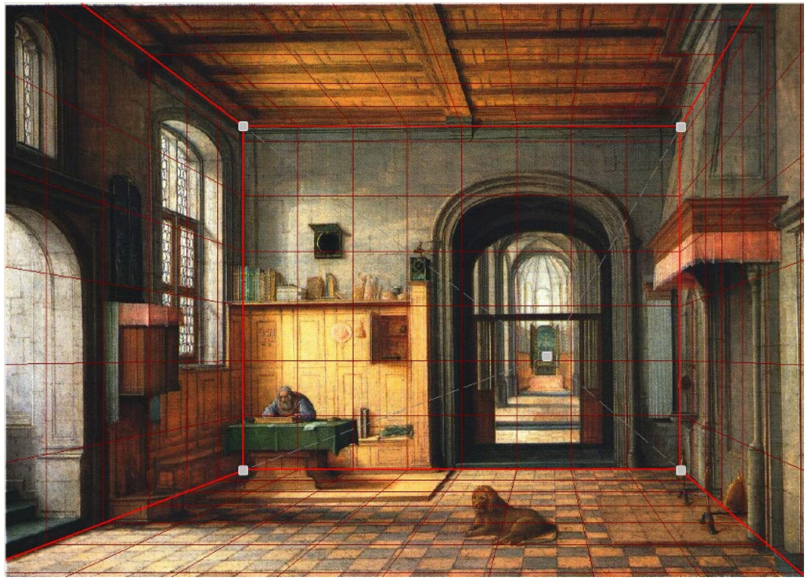
(a) Specifying of a foreground object

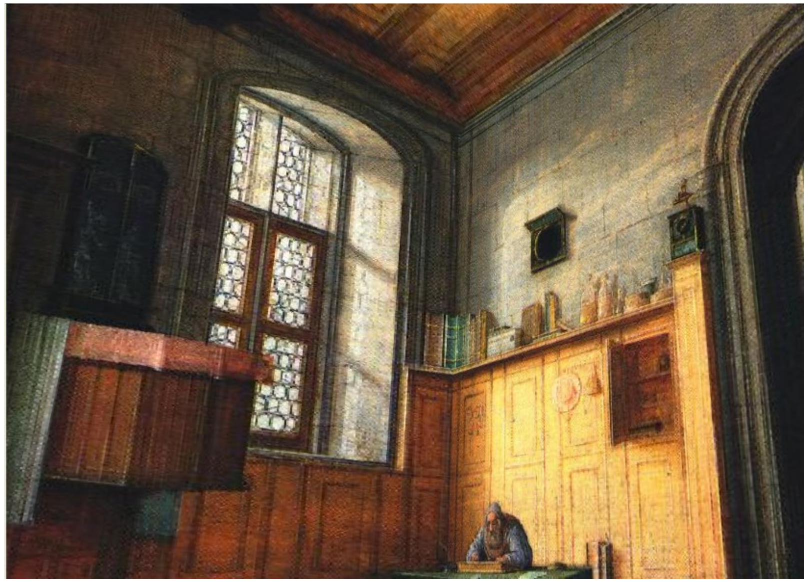
(b) Estimating the vertices of the foreground object model



(c) Three foreground object models

# Foreground Result

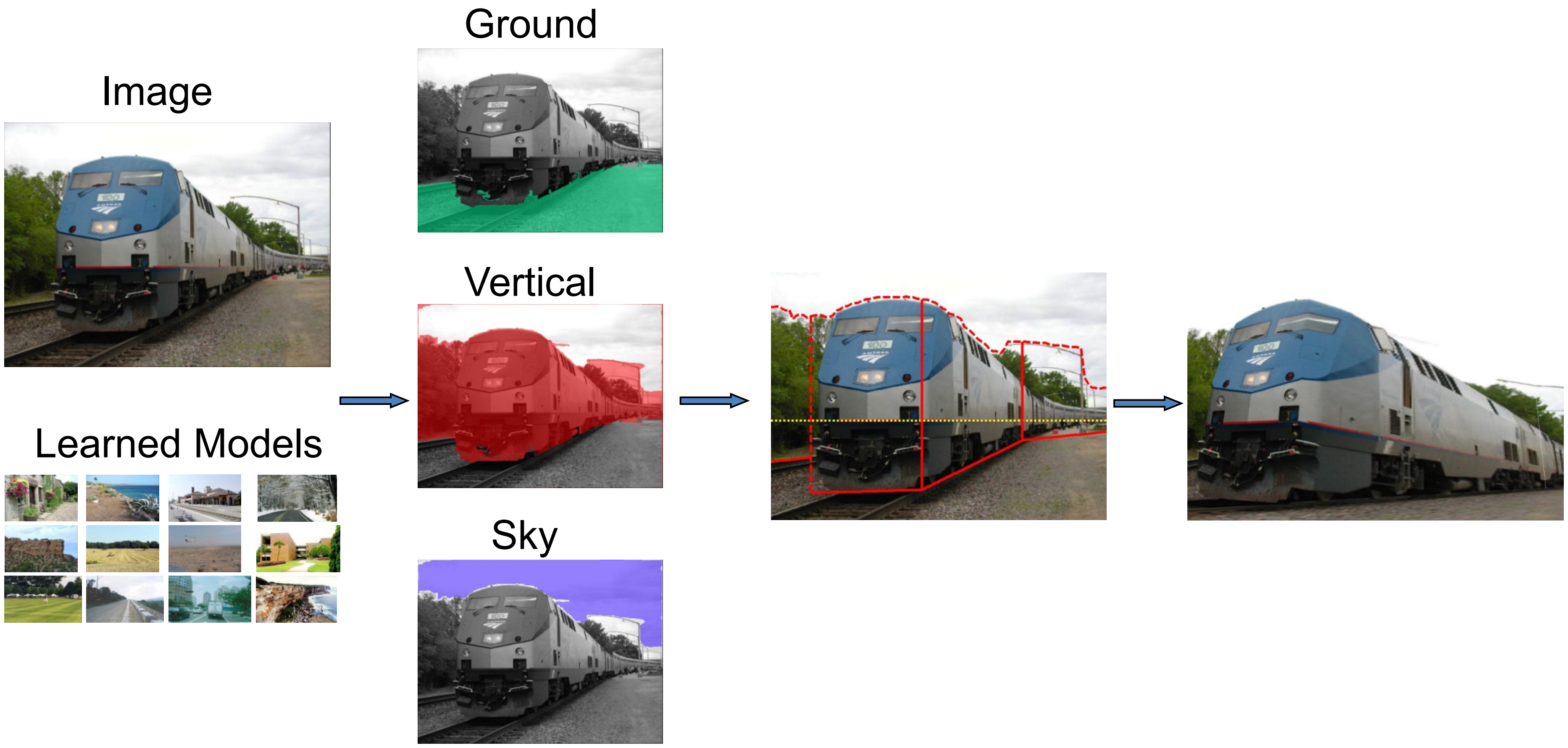




Video from CMU class: http://www.youtube.com/watch?v=dUAtdmGwcuM

# Automatic Photo Pop-up

Input Geometric Labels Cut’n’Fold 3D Model



Image

Learned Models

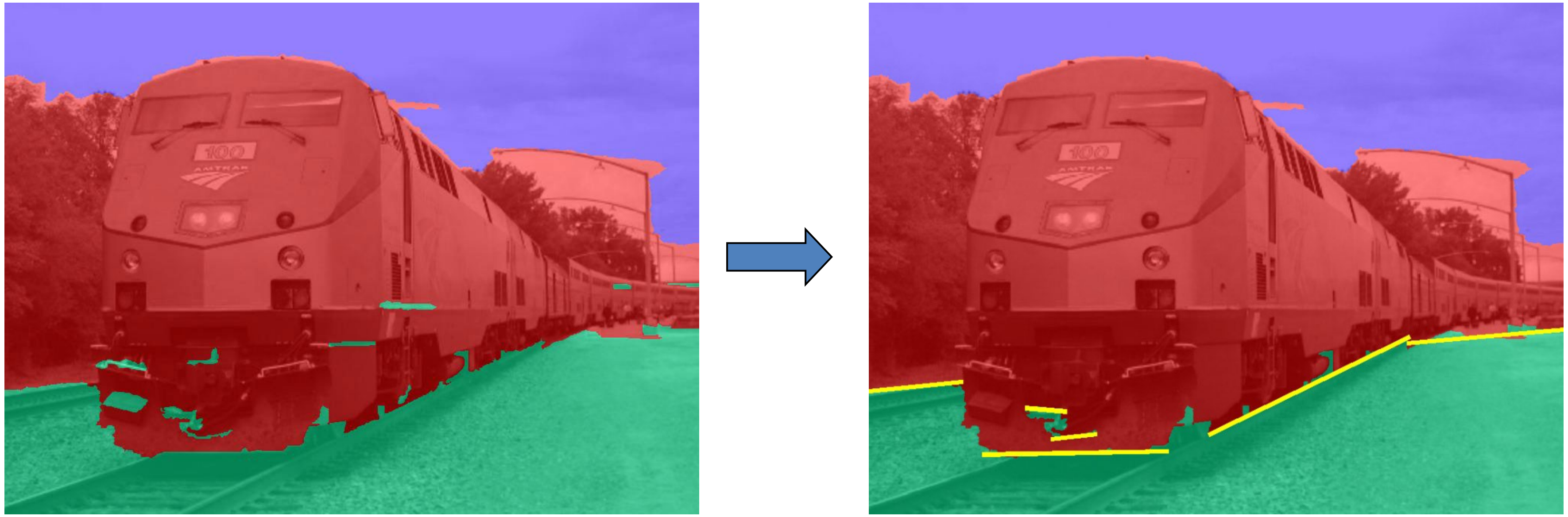
Ground

Vertical

Sky

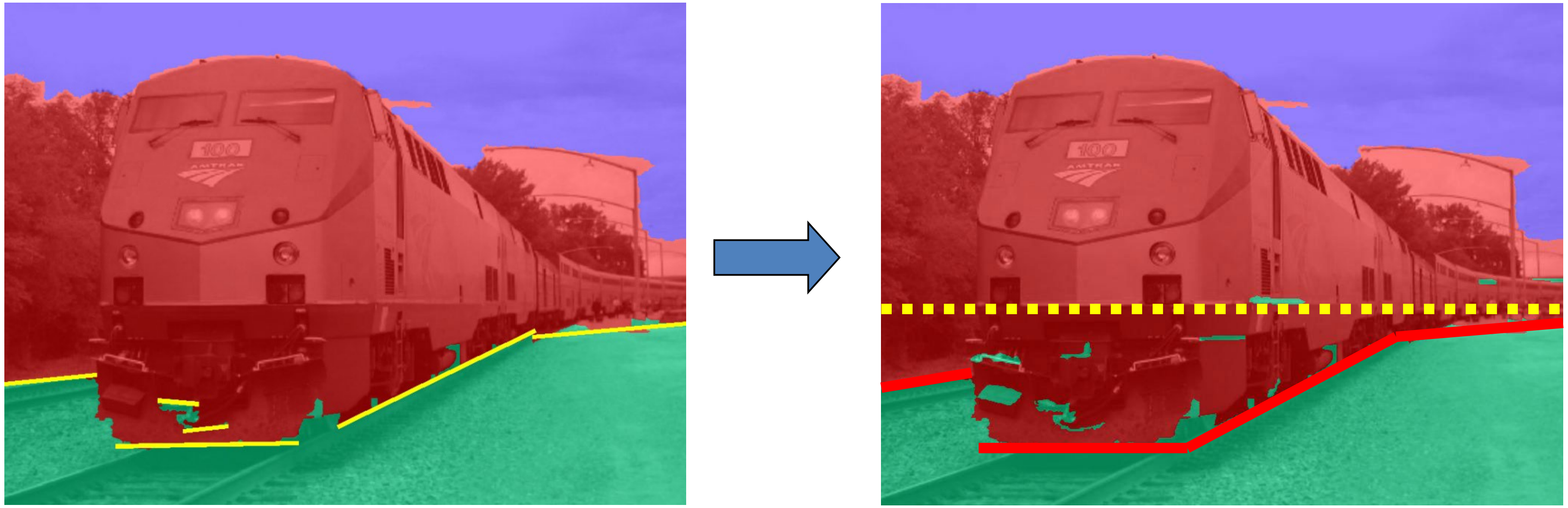
Hoiem et al. 2005

# Cutting and Folding



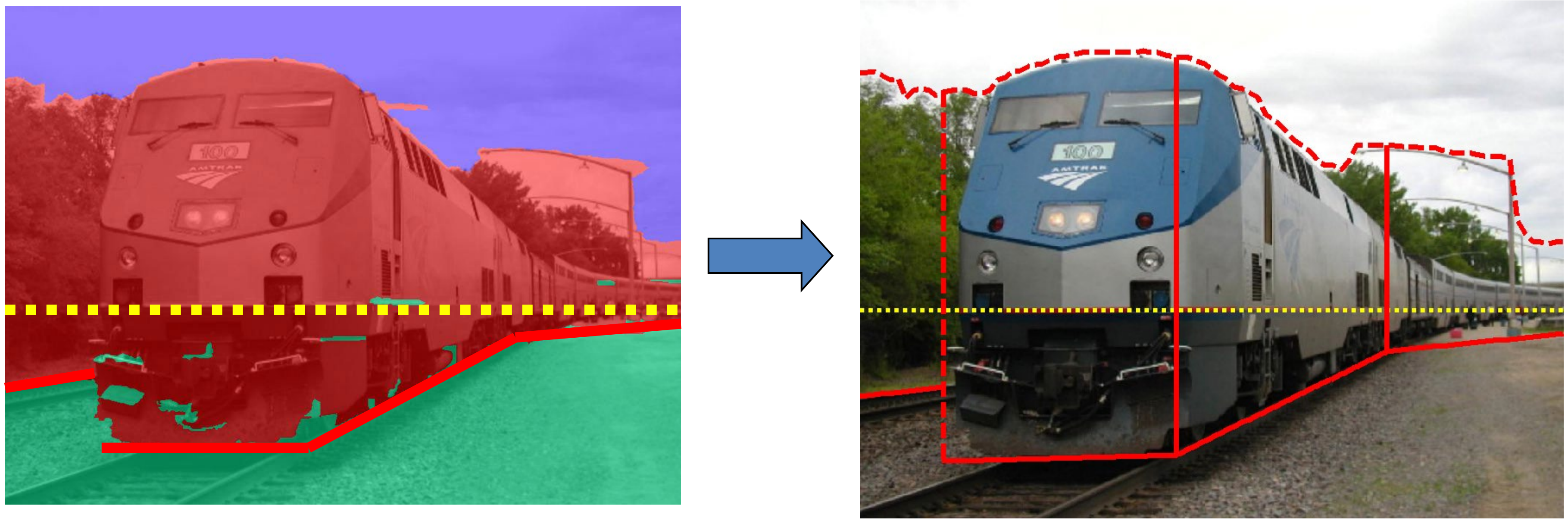
- Fit ground-vertical boundary
  - Iterative Hough transform

# Cutting and Folding



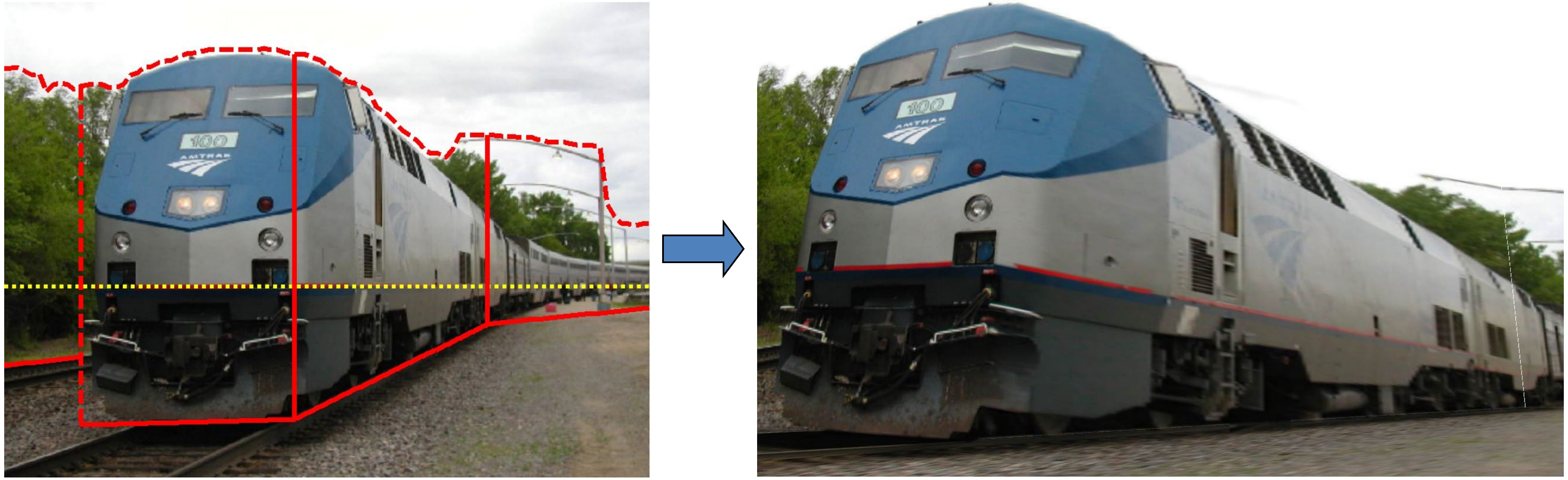
- Form polylines from boundary segments
  - Join segments that intersect at slight angles
  - Remove small overlapping polylines
- Estimate horizon position from perspective cues

# Cutting and Folding



- ``Fold’’ along polylines and at corners
- ``Cut’’ at ends of polylines and along vertical-sky boundary

# Cutting and Folding

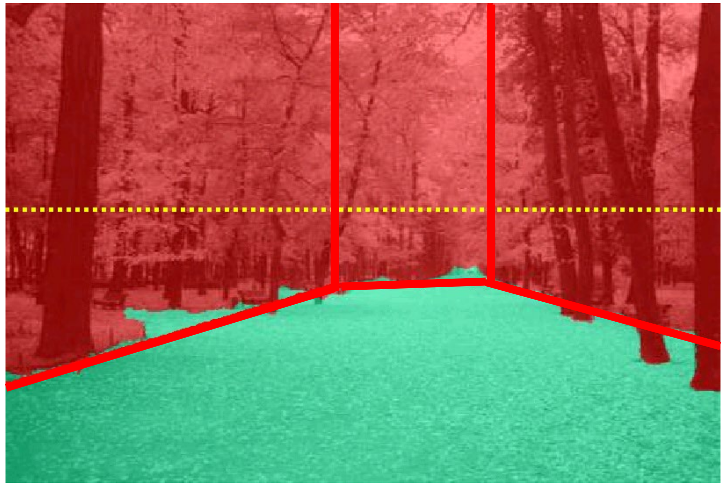


- Construct 3D model
- Texture map

# Results

http://www.cs.illinois.edu/homes/dhoiem/projects/popup/

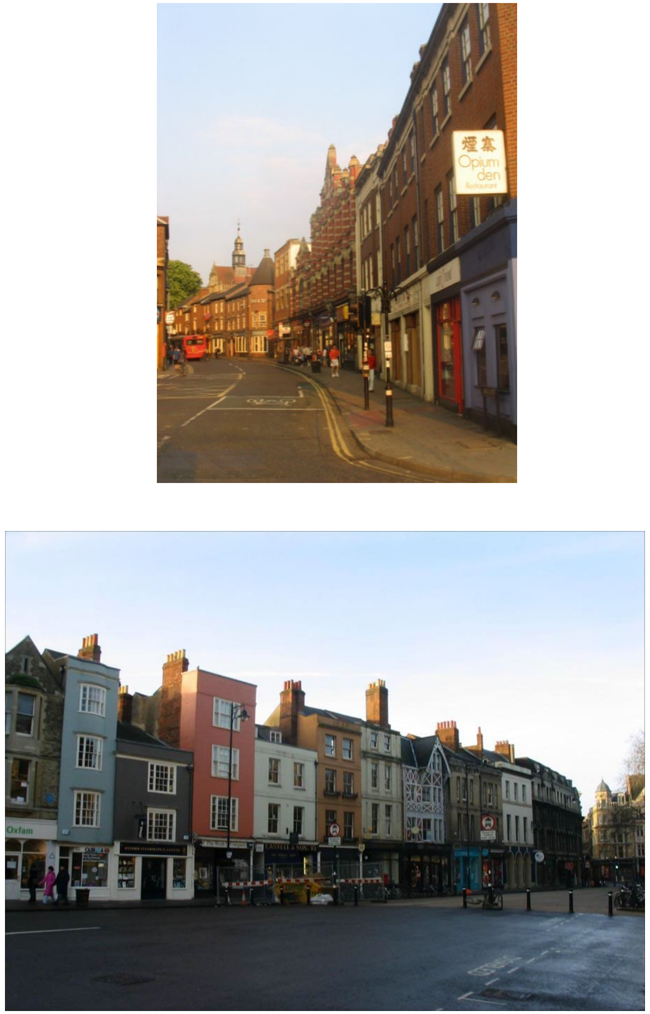
Input Image



Cut and Fold

Automatic Photo Pop-up

# Results

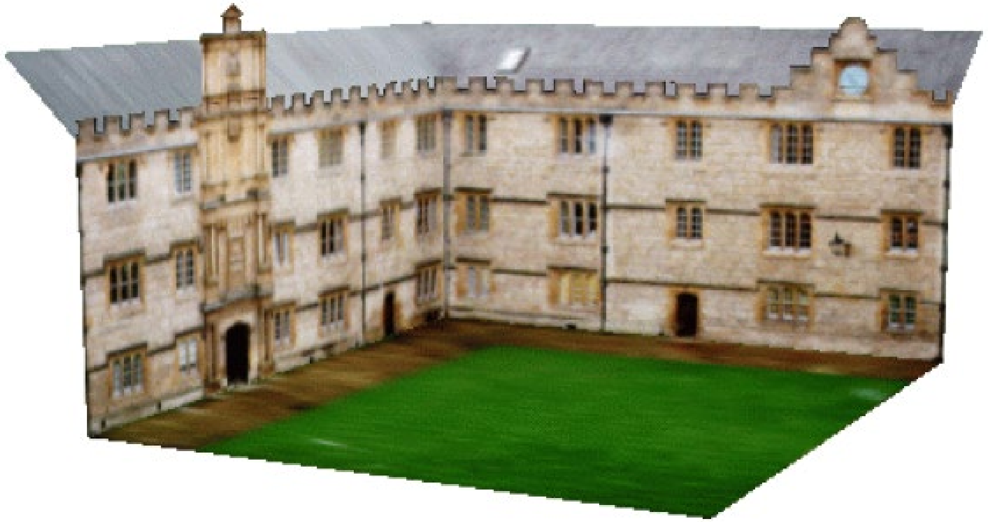


Input Image

Automatic Photo Pop-up

# Comparison with Manual Method

Input Image

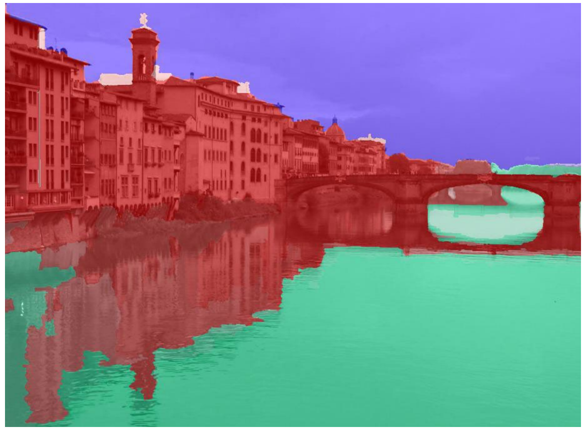


[Liebowitz et al. 1999] (manual)

Automatic Photo Pop-up

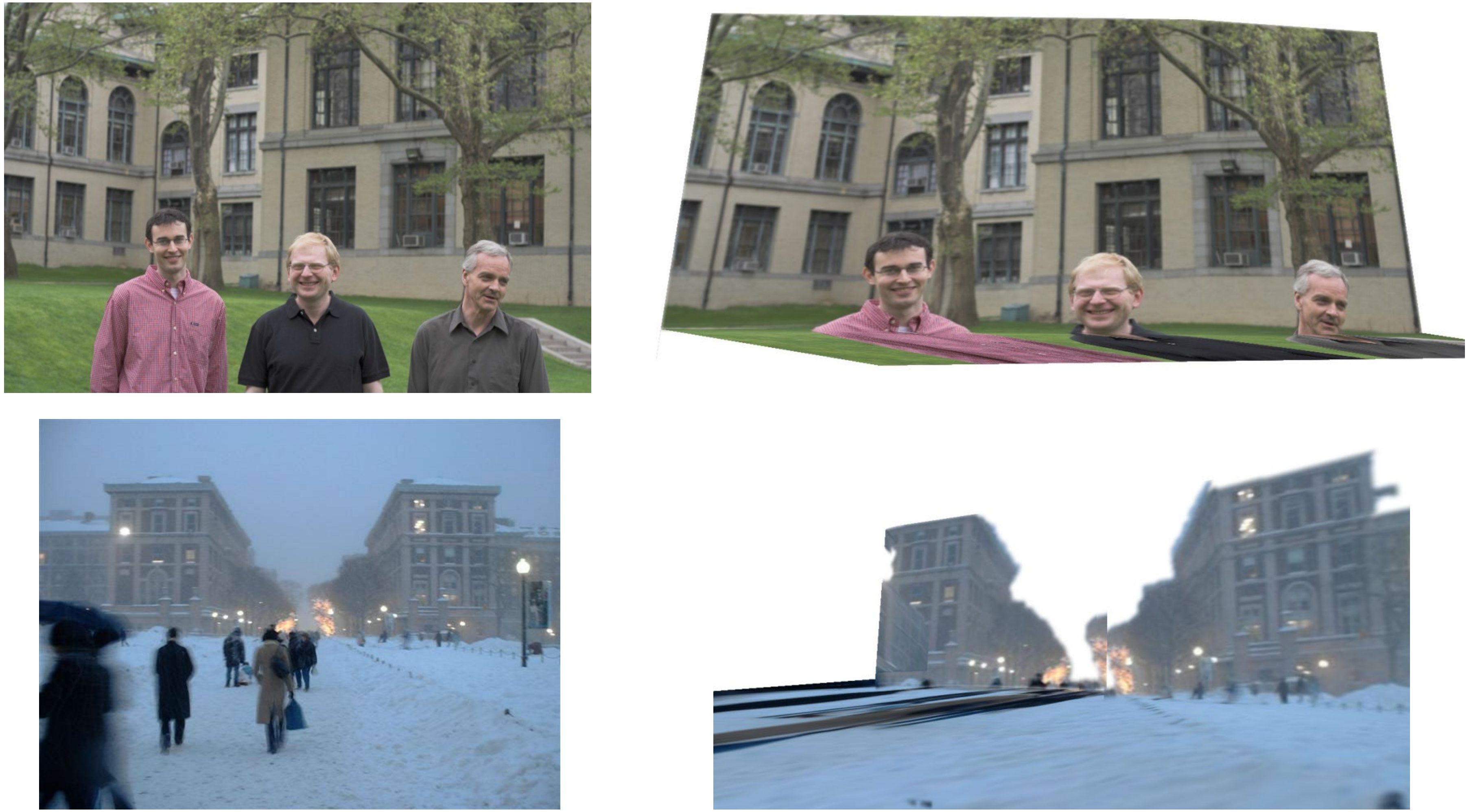
# Failures

## Labeling Errors

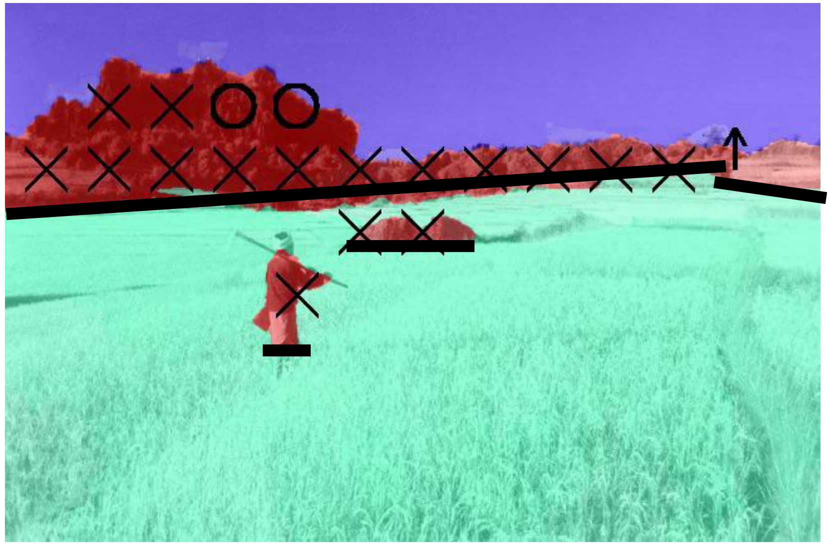


# Failures

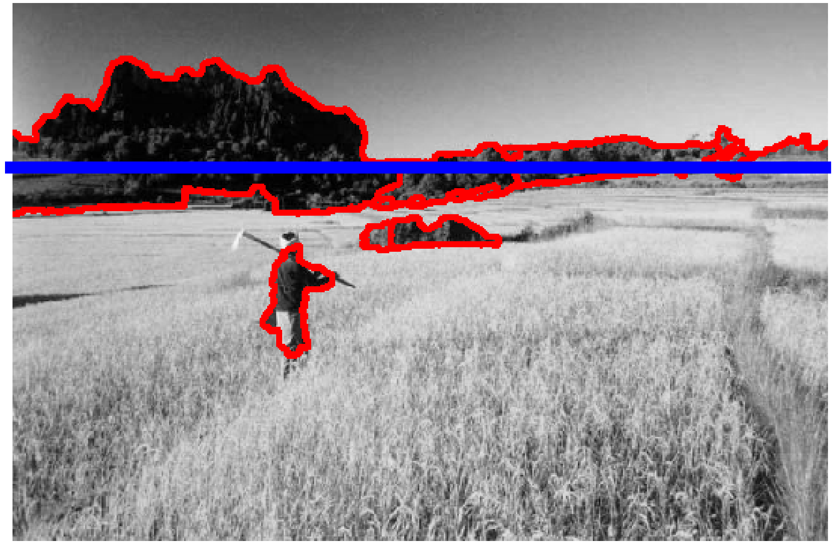
## Foreground Objects



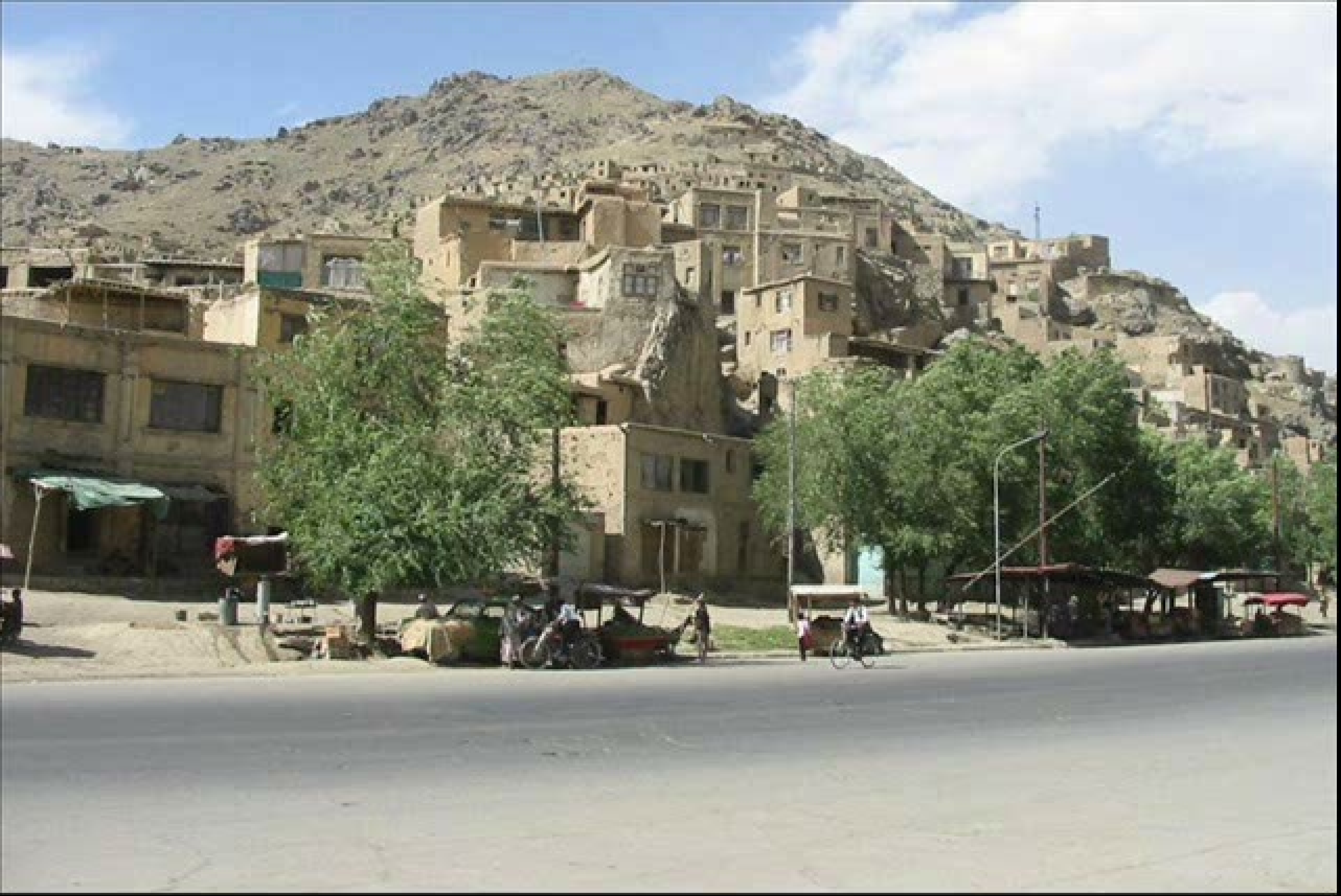
# Adding Foreground Labels



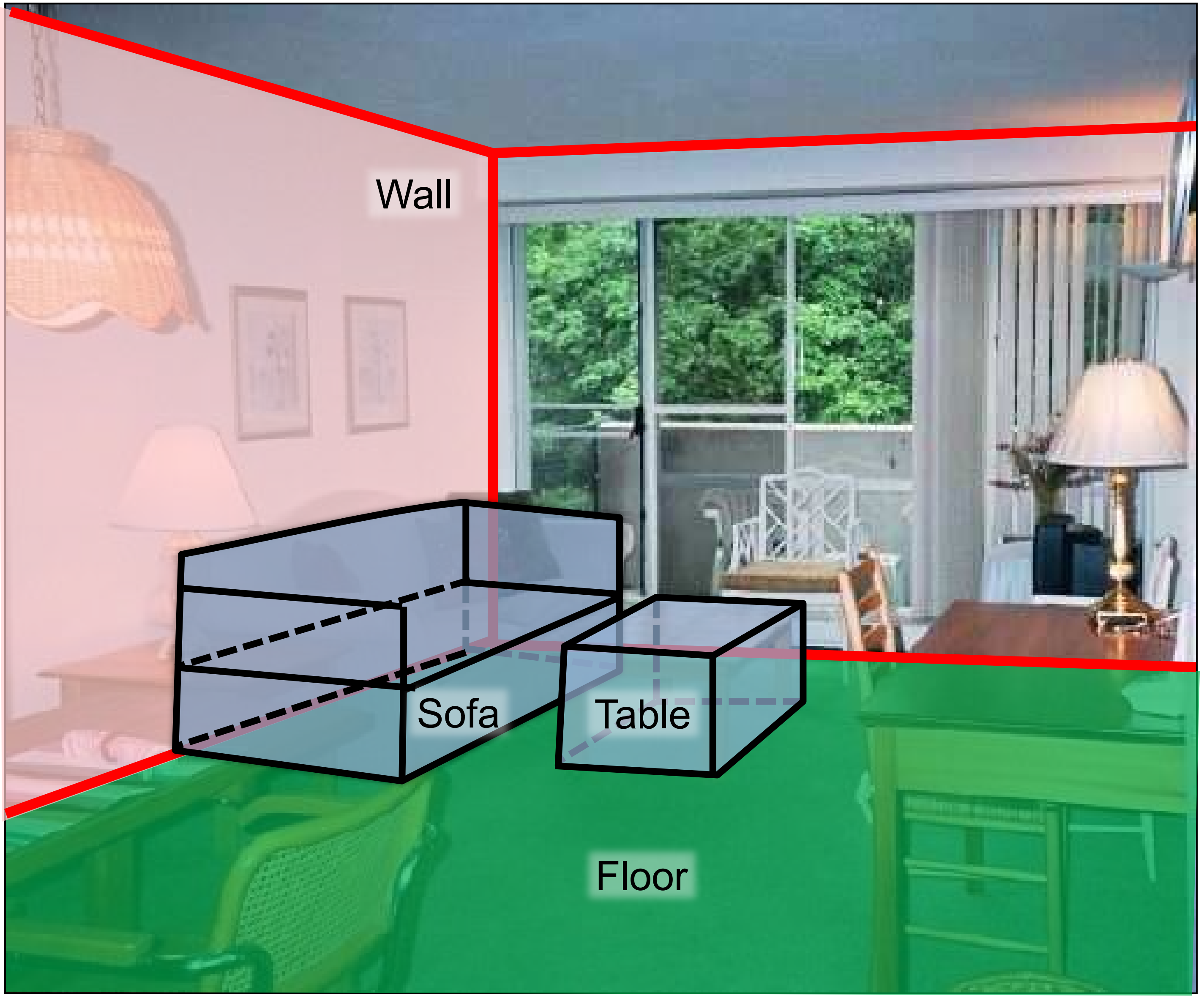
Recovered Surface Labels +
Ground-Vertical Boundary Fit



Object Boundaries + Horizon

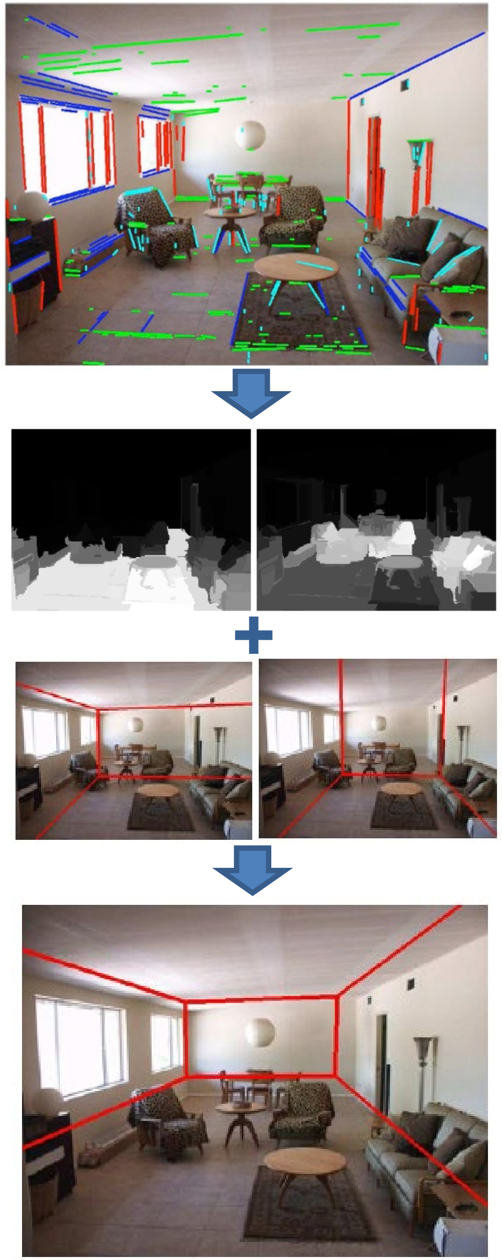


# Fitting boxes to indoor scenes



Hedau et al. 2010

# Box Layout Algorithm



1. Detect edges
2. Estimate 3 orthogonal vanishing points
3. Apply region classifier to label pixels with visible surfaces
    - Boosted decision trees on region based on color, texture, edges, position
4. Generate box candidates by sampling pairs of rays from VPs
5. Score each box based on edges and pixel labels
    - Learn score via structured learning
6. Jointly refine box layout and pixel labels to get final estimate

# Experimental results

Detected Edges

Surface Labels

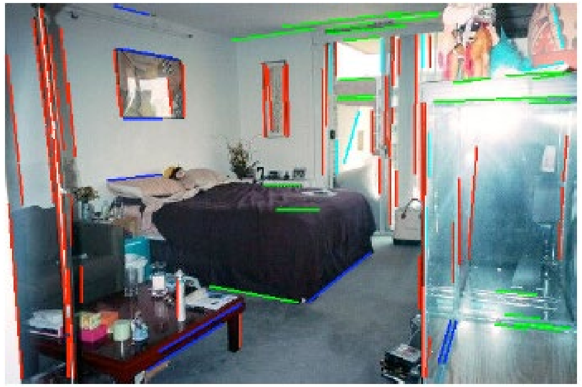
Box Layout

Detected Edges

Surface Labels

Box Layout

# Experimental results



Detected Edges

Surface Labels

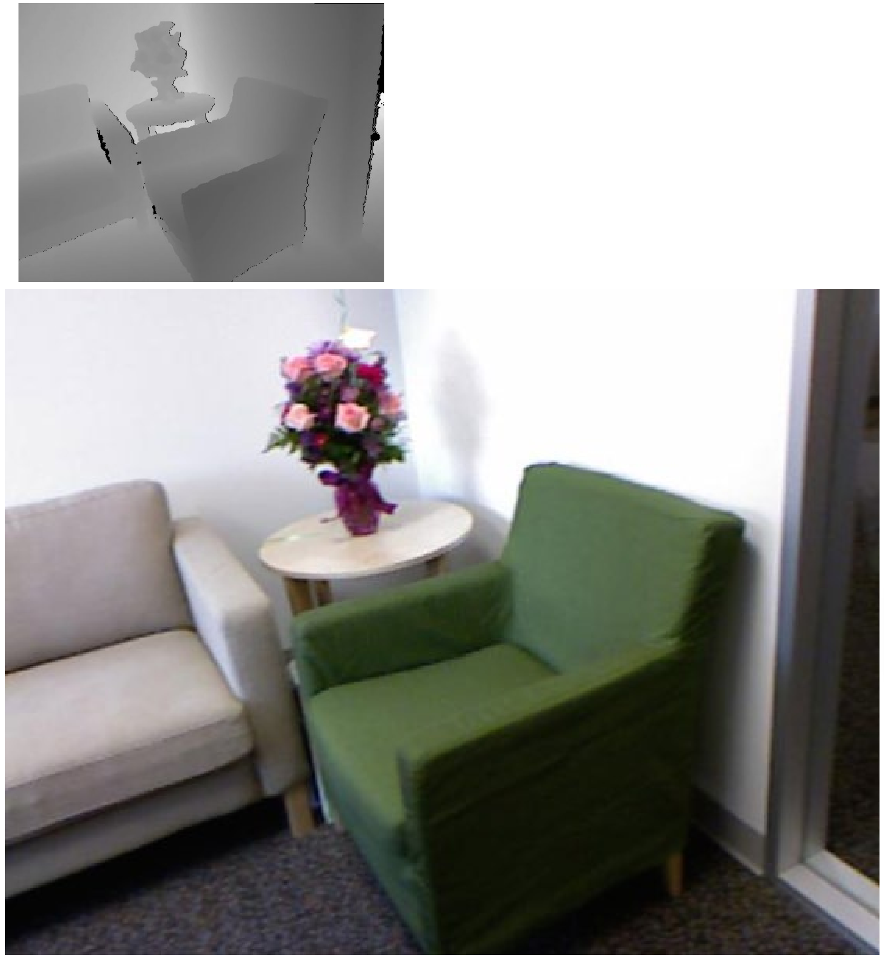
Box Layout

Detected Edges

Surface Labels

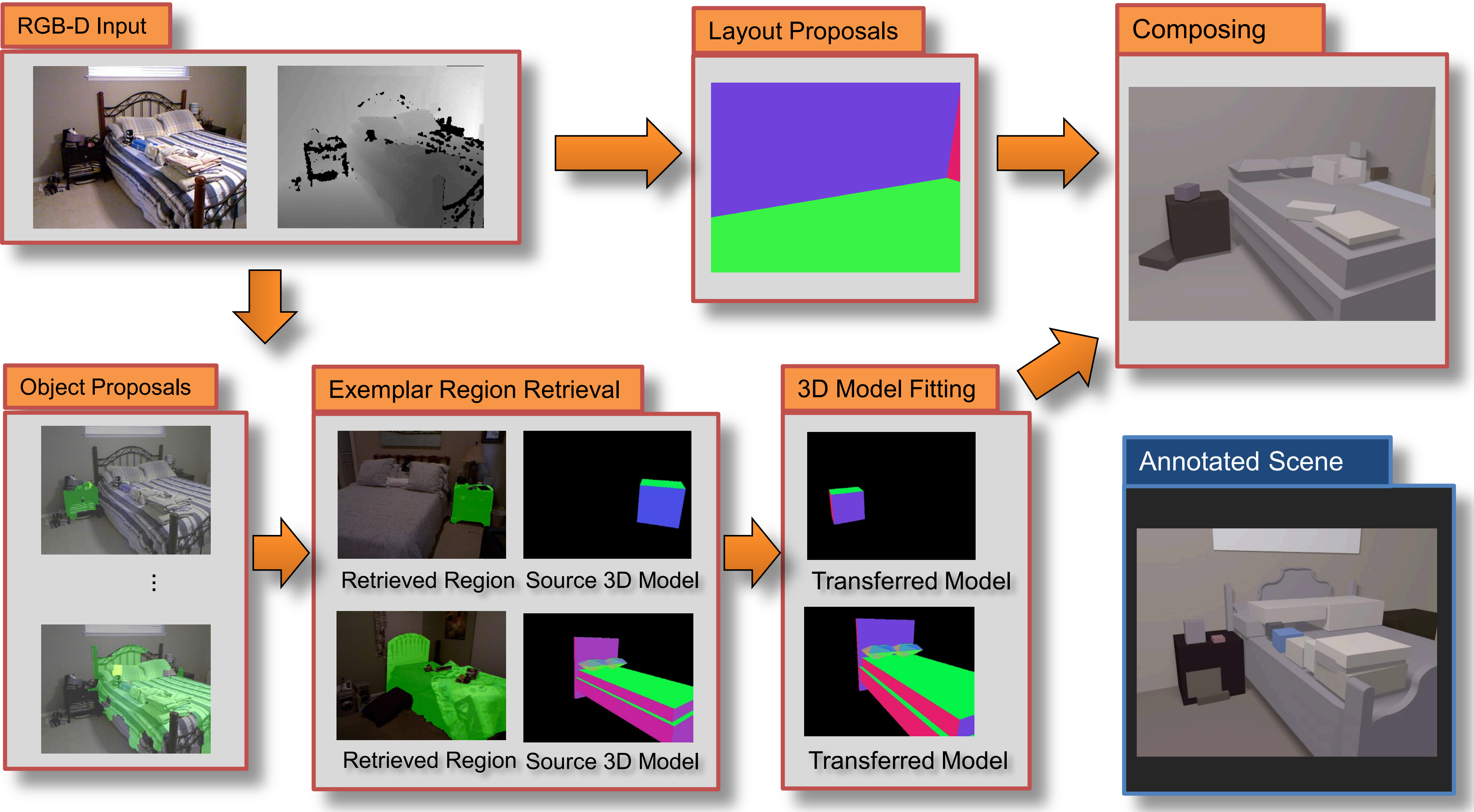
Box Layout

# Complete 3D from RGBD



Zou et al. IJCV 2018

# Complete 3D from RGBD



RGB-D Input

Layout Proposals

Composing

Object Proposals

⋮

Exemplar Region Retrieval

Retrieved Region

Source 3D Model

Retrieved Region

Source 3D Model

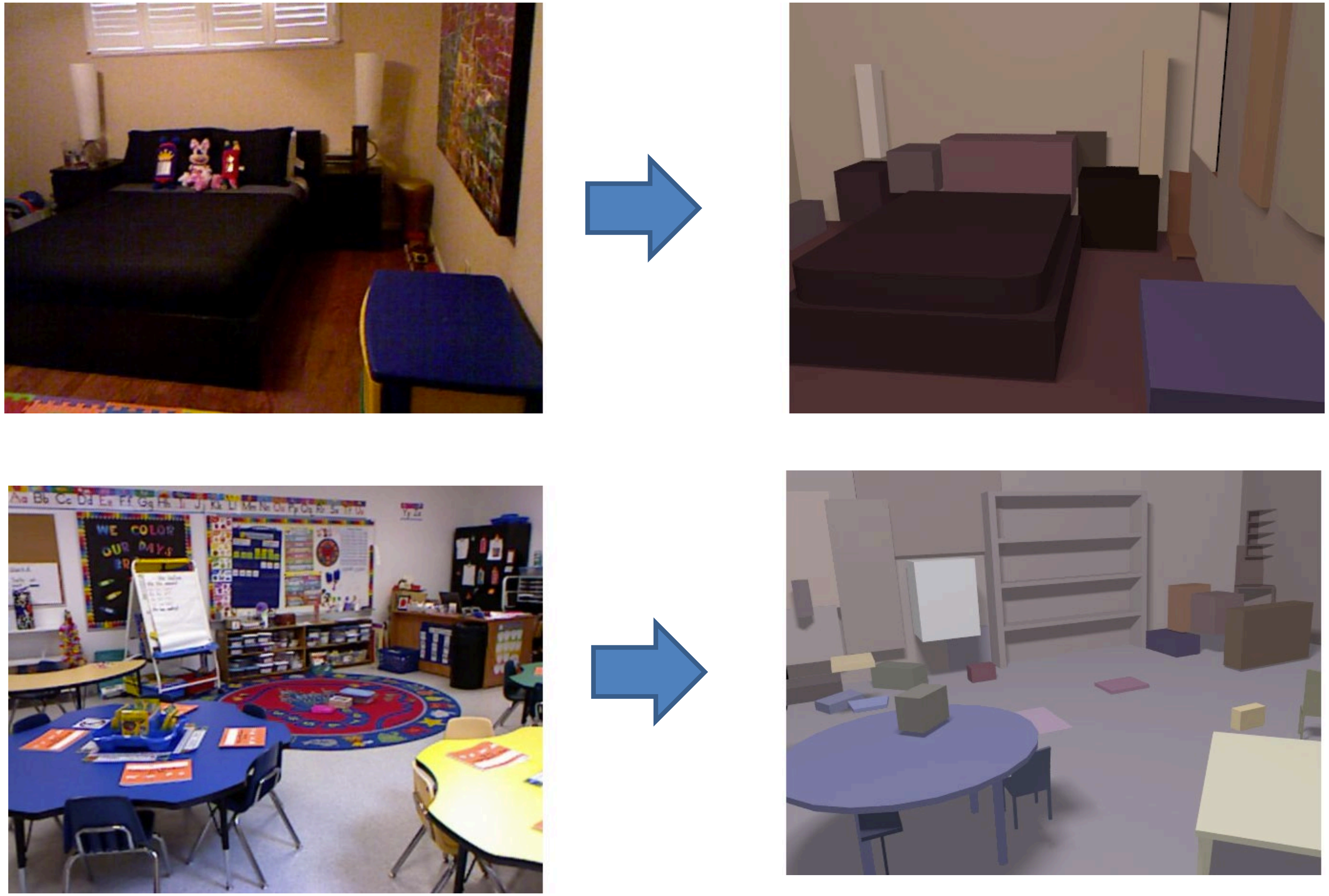
3D Model Fitting

Transferred Model

Transferred Model

Annotated Scene

# Complete 3D from RGBD



# Final project idea

- Interactive program to make 3D model from an image (e.g., output in VRML, or draw path for animation)
  - Add tools for cutting out foreground objects and automatic hole-filling

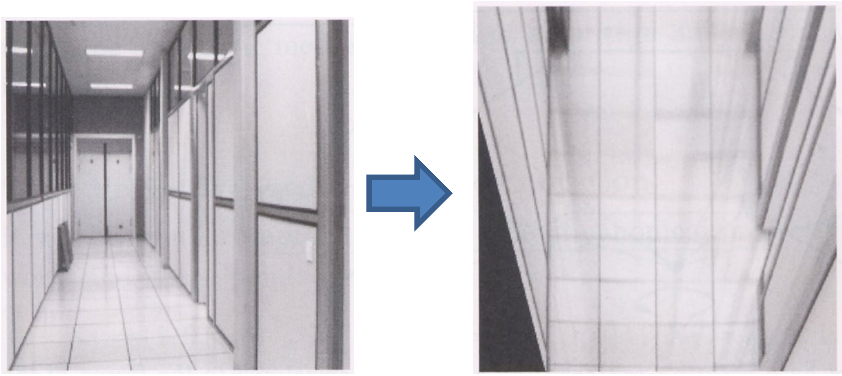
# Summary

- 2D→3D is mathematically impossible

  (but we do it without even thinking)

- Need right assumptions about the world geometry

- Important tools
  - Vanishing points
  - Camera matrix
  - Homography



# Next Week

- Exam review next Tuesday
- Exam next Thursday