From previous class :

- Let's do this mathematically now. We want $\underline{P(s_k \mid m_{1:n})}$ ← offline
- Let's start with a basic result for $\underline{P(s_{1:n} \mid m_{1:n})}$ → trajectory given all measurements

$$P(s_{1:n} \mid m_{1:n}) = \frac{P(s_{1:n}, m_{1:n})}{P(m_{1:n})} \propto \underline{P(s_{1:n}, m_{1:n})}$$

$$\underbrace{P(s_{1:n}, m_{1:n})}_{\mathbb{R}^{2n}} \overset{\text{chain rule}}{=} P(m_n \mid m_{1:n-1}, s_{1:n})\, P(m_{n-1} \mid m_{1:n-2}, s_{1:n}) \cdots$$

$$\cdots P(m_1 \mid s_{1:n})\, P(s_n \mid s_{1:n-1}) \cdots P(s_2 \mid s_1) P(s_1)$$

$P(ABC) = P(A \mid BC) P(B \mid C) P(C)$

Markovian

$$= P(m_n \mid s_n)\, P(m_{n-1} \mid s_{n-1}) \cdots P(m_1 \mid s_1) P(s_n \mid s_{n-1}) \cdots P(s_2 \mid s_1) P(s_1)$$

$$\boxed{\underbrace{P(s_{1:n}, m_{1:n})}_{\text{which trajectory \& measurements are in agreement.}} = P(m_1 \mid s_1) P(s_1) \prod_{i=2}^{n} \underbrace{P(m_i \mid s_i)}_{\text{sensor error}} \underbrace{P(s_i \mid s_{i-1})}_{\text{Transition prob. matrix}}}$$

- Now we want $P(s_k \mid m_{1:n})$

$$P(s_k \mid m_{1:n}) \propto P(s_k, m_{1:n}) = P(s_k, \qquad)$$

$$= P(m_{k+1:n} \mid \qquad) P(\qquad)$$

$$= P(m_{k+1:n} \mid \quad) P(s_k \mid m_{1:k}) P(\quad)$$

$$= P(\qquad) . P(\qquad)$$

↓ ↓

↓ Probability that is at S_k = green st. given surveillance Camera measurements of main street → wright street → 6^{th} street

↓ Probability that 4^{th} to 8^{th} measurement are Neil St. → Kirby road → Lincoln drive → university avenue, given

S_k = green street.


walking
L1
L2
L3
Lr
t1
t2
t3
t4
t5

→ Let's look at the component $P(s_k \mid m_{1:k})$

$$P(s_k \mid m_{1:k}) = \frac{P(s_k, m_{1:k})}{P(m_{1:k})} \propto P(s_k, m_{1:k})$$

By marginalizing RHS $= \sum_{s_{k-1}} P(\qquad\qquad\qquad)$

$$= \sum_{s_{k-1}} P(m_k \mid \qquad\qquad\qquad) \, P(\qquad\qquad\qquad)$$

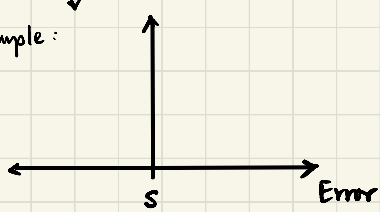
$$= \sum_{s_{k-1}} P(m_k \mid \quad) \, P(s_k \mid \qquad\qquad) \, P(\qquad\qquad)$$

$$P(s_k, m_{1:k}) = \sum_{s_{k-1}} P(m_k \mid s_k) \, P(s_k \mid \qquad) \, P(s_{k-1}, m_{1:k-1})$$

$$\therefore \quad = \sum_{s_{k-1}} P(m_k \mid s_k) \, P(s_k \mid s_{k-1})$$

→ Initial condition $P(s_1, m_1) = \qquad\qquad\qquad$ needs to be known.

Example:



Now let's look at the backward part : $P(\qquad\qquad)$

$$P(m_{k+1:n} \mid s_k) = \frac{P(m_{k+1:n},\ s_k)}{\qquad}$$

$$= \frac{1}{P(s_k)} \sum P$$

$$= \frac{1}{P(s_k)} \sum_{s_{k+1}} P(\qquad\qquad).$$

$$P(\qquad\qquad).$$

$$P(\qquad)P(\ )$$

$$= \sum_{s_{k+1}} P(\qquad\qquad)P(\qquad)P(\qquad)$$

Say LHS $= P(m_{k+1:n} \mid s_k) =$

$\therefore \quad \beta_k = \sum_{s_{k+1}}$

How should we initialize this β_k ?

$$\beta_{n-1} = P(\qquad\qquad) =$$

$$= \frac{1}{\quad} \sum_{s_n} P(m_n \mid s_n, s_{n-1})$$

$$\beta_{n-1} = \sum_{s_n} P(\qquad) P(s_n \mid s_{n-1})$$

$$\beta_{n-2} = \sum$$

⊙ Recall original goal : $P(s_k \mid m_{1:n})$ ⇒ offline version

$$P(s_k \mid m_{1:n}) = P(s_k \mid m_{1:k}) \, P(m_{k+1:n} \mid s_k)$$

$$\alpha_k = \sum_{s_{k-1}} P(m_k \mid s_k) \, P(s_k \mid s_{k-1}) \, \alpha_{k-1}$$

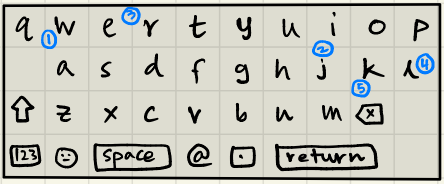
$$\beta_k = \sum_{s_{k+1}} \beta_{k+1} \, P(m_{k+1} \mid s_{k+1}) \, P(s_{k+1} \mid s_k)$$

HMM's ⇒ identifying the most likely value of a from a large space of

⇒ Possible to also compute the full trajectory
↳ called

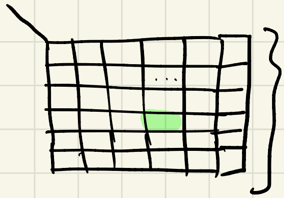
Some other applications of HMM (informal discussion)

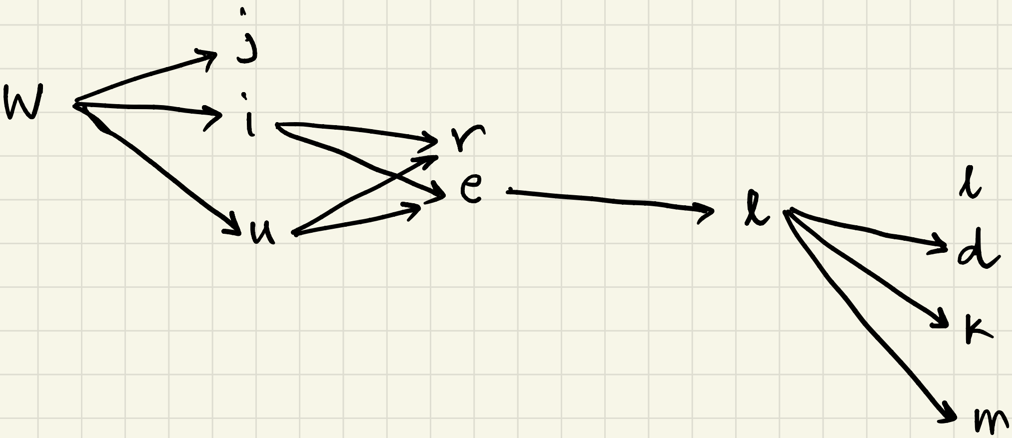
I. in smartphone keyboard.



$P(m_1 | s_1) = P(\qquad\qquad) =$

$P(s_2 | s_1) = P(\qquad\qquad) =$





$\Downarrow$

Decodes to

Similar application in

$P(m_k | s_k) = P(\qquad\qquad)$