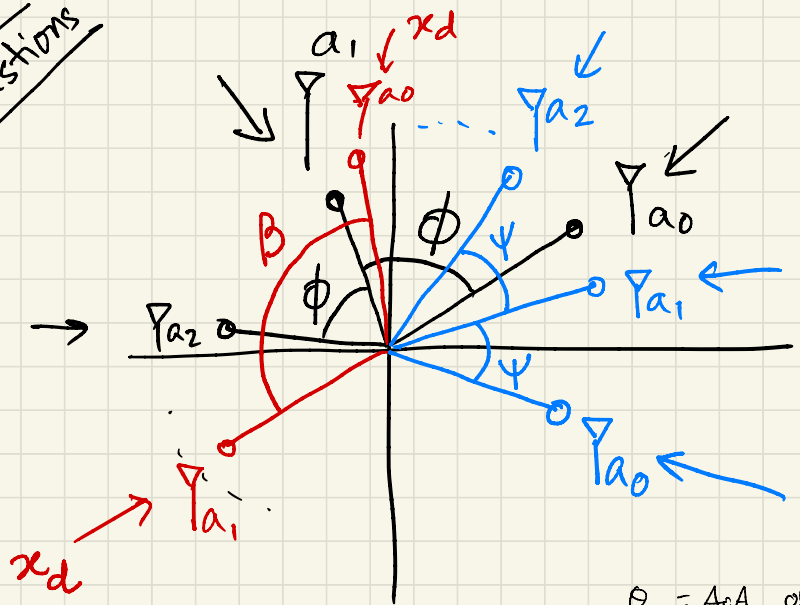
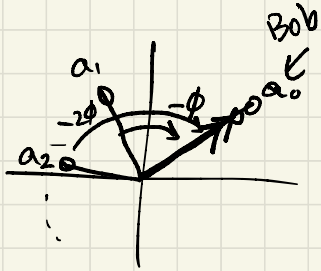
Questions



$$\phi = \frac{2\pi}{\lambda} d\cos\theta_1$$

$$\psi = \frac{2\pi}{\lambda} d\cos\theta_2$$

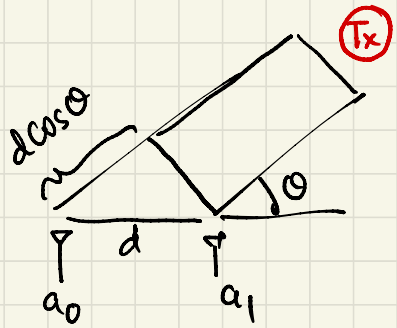


θ_1 = AoA of signal x_1

θ_2 = AoA of signal x_2

θ_d = AoA of signal x_d

① Narrowband and wideband signal.



Phase diff. between a_0 and a_1 is $\phi = \frac{2\pi}{\lambda} d\cos\theta$.

We assumed that signal has only 1 freq, ∴ only one λ.

↳ often called narrowband signals or tones.

② Real-world signals have wider bandwidth ↔ human - 20KHz, WiFi - 20MHz, 5G - 500MHz

↳ $\phi_{(fi)} = \frac{2\pi}{\lambda_{(fi)}} d\cos\theta$

⊕ Sub-space based AoA : MUSIC algorithm

steering matrix

$$Y = A\bar{s} + \bar{n}$$

$$\begin{bmatrix} \\ \\ Y \\ \\ \\ \end{bmatrix} = \begin{bmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix}$$

$$YY^H = (As+n)(As+n)^H$$

$$Y(Y^*)^T$$

$$= (As+n)(s^H A^H + n^H)$$

$$= Ass^H A^H + As.n^H + ns^H A^H + nn^H$$

$$E[YY^H] = E[Ass^H A^H + As.n^H + ns^H A^H + nn^H]$$

$$R_{YY} = AR_{ss}A^H + 0 + 0 + \sigma^2 I$$

- R_{YY}: $m \times m$ matrix, Rank m
- $A_{m \times d}\, R_{dd}\, A^H_{d \times m}$, Rank d
- $I_{m \times m}$, Rank m

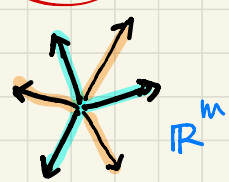
m mic. d signals $m > d$

$$\text{Rank}\{AB\} = \text{at most } \min\{R_A, R_B\}$$

⊕ Intuition :

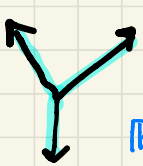
$$R_{xx} = AR_{ss}A^H + \sigma^2 I$$

$R_{xx} - \sigma^2 I$: $m \times m$



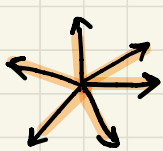
$\mathbb{R}^m$

signal + noise space



$\mathbb{R}^d$

signal sub-space



$\mathbb{R}^m$

$$(R_{xx} - \sigma^2 I)\,\bar{e}_{d+1} = 0 = AR_{ss}A^H \cdot \bar{e}_{d+1}$$

λe_{d+1}

Both full rank

$\therefore A^H \cdot \bar{e}_{d+1} = 0$

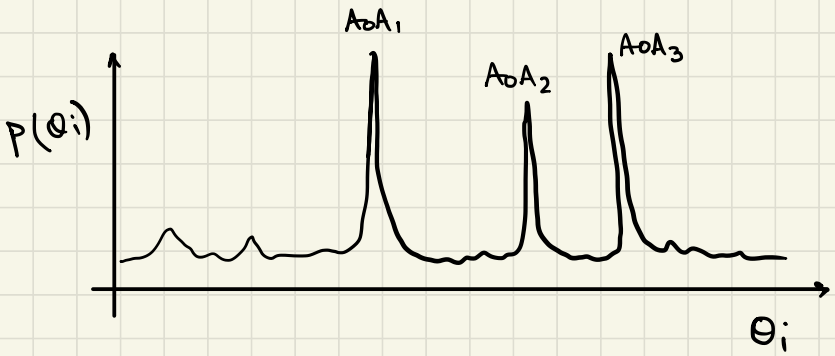
$m = 10$, $d = 4$

10×10, 6 eigenvalues $= 0$

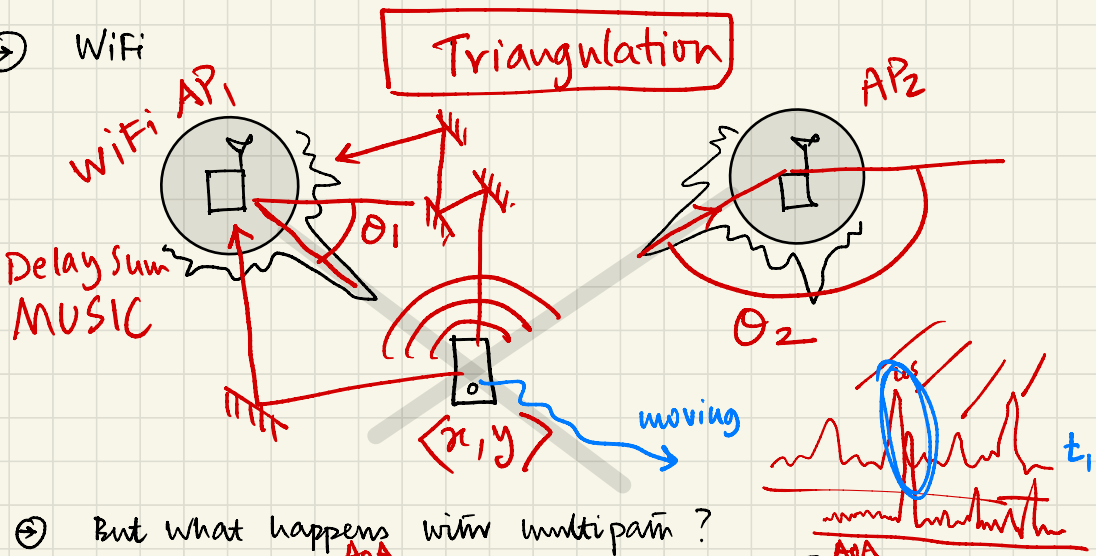
AoA spectrum $P(\theta_i) = \frac{1}{\| A_{\theta_i}^H e_{noise} \|_{L_1}}$

$1 \times m$ $\quad$ $m \times (m-d)$

$$[-A_{\theta_i}-]\begin{bmatrix} | & & | \\ e_{d+1} & \cdots & e_m \\ | & & | \end{bmatrix} = [\cdots\cdots]$$



- WiFi



- But what happens with multipath ?
  - LOS path AoA varies slowly while multipath AoA varies faster.
  - Identify LOS by observing stable AoA spectrum peak.
  - Apply Maximum Likelihood Estimation (MLE) for localization