Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC
http://courses.engr.illinois.edu/cs421

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Unification Problem

Given a set of pairs of terms ("equations")

$\{(s_1, t_1), (s_2, t_2), \ldots, (s_n, t_n)\}$

(the *unification problem*) does there exist a substitution σ (the *unification solution*) of terms for variables such that

$\sigma(s_i) = \sigma(t_i),$

for all $i = 1, \ldots, n$?

Uses for Unification

- Type Inference and type checking
- Pattern matching as in OCaml
  - Can use a simplified version of algorithm
- Logic Programming - Prolog
- Simple parsing

Unification Algorithm

- Let $S = \{(s_1 = t_1), (s_2 = t_2), \ldots, (s_n = t_n)\}$ be a unification problem.
- Case $S = \{\ \}$: $Unif(S)$ = Identity function (i.e., no substitution)
- Case $S = \{(s, t)\} \cup S'$: Four main steps

Unification Algorithm

- Delete: if $s = t$ (they are the same term) then $Unif(S) = Unif(S')$
- Decompose: if $s = f(q_1, \ldots, q_m)$ and $t = f(r_1, \ldots, r_m)$ (same f, same m!), then $Unif(S) = Unif(\{(q_1, r_1), \ldots, (q_m, r_m)\} \cup S')$
- Orient: if $t = x$ is a variable, and s is not a variable, $Unif(S) = Unif(\{(x = s)\} \cup S')$

Unification Algorithm

- Eliminate: if $s = x$ is a variable, and x does not occur in t (the occurs check), then
  - Let $\varphi = \{x \rightarrow t\}$
    - $Unif(S) = Unif(\varphi(S'))\ o\ \{x \rightarrow t\}$
  - Let $\psi = Unif(\varphi(S'))$
  - $Unif(S) = \{x \rightarrow \psi(t)\}\ o\ \psi$
    - Note: $\{x \rightarrow a\}\ o\ \{y \rightarrow b\} = \{y \rightarrow (\{x \rightarrow a\}(b))\}\ o\ \{x \rightarrow a\}$ if y not in a

Tricks for Efficient Unification

- Don't return substitution, rather do it incrementally
- Make substitution be constant time
  - Requires implementation of terms to use mutable structures (or possibly lazy structures)
  - We won't discuss these

Example

- x,y,z variables, f,g constructors
- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = ?

Example

- x,y,z variables, f,g constructors
- S = {(f(x) = f(g(f(z),y))), (g(y,y) = x)} is nonempty
- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,y) = x)
- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,y)) = x)
- Orient: (x = g(y,y))
- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} =
  Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))}

by Orient

Example

- x,y,z variables, f,g constructors
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- {(f(x) = f(g(f(z),y))), (x = g(y,y))} is non-empty
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x = g(y,y))
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x = g(y,y))
- Eliminate x with substitution {x→ g(y,y)}
  - Check: x not in g(y,y)
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (x = g(y,y))
- Eliminate x with substitution {x→ g(y,y)}
- Unify {(f(x) = f(g(f(z),y))), (x = g(y,y))} =
  Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)}

Example

- x,y,z variables, f,g constructors
- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- {(f(g(y,y)) = f(g(f(z),y)))} is non-empty
- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(g(y,y)) = f(g(f(z),y)))
- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(g(y,y)) = f(g(f(z),y)))
- Decompose:(f(g(y,y)) = f(g(f(z),y)))
  becomes {(g(y,y) = g(f(z),y))}
- Unify {(f(g(y,y)) = f(g(f(z),y)))}
  o {x→ g(y,y)} =
  Unify {(g(y,y) = g(f(z),y))} o {x→ g(y,y)}

Example

- x,y,z variables, f,g constructors
- {(g(y,y) = g(f(z),y))} is non-empty
- Unify {(g(y,y) = g(f(z),y))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (g(y,y) = g(f(z),y))
- Unify {(g(y,y) = g(f(z),y))}
  o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(g(y,y)) = f(g(f(z),y)))
- Decompose: (g(y,y)) = g(f(z),y)) becomes
  {(y = f(z)); (y = y)}
- Unify {(g(y,y) = g(f(z),y))} o {x→ g(y,y)} =
  Unify {(y = f(z)); (y = y)} o {x→ g(y,y)}

Example

- x,y,z variables, f,g constructors
- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- {(y = f(z)); (y = y)} o {x→ g(y,y)} is non-empty
- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (y = f(z))
- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (y = f(z))
- Eliminate y with {y → f(z)}
- Unify {(y = f(z)); (y = y)} o {x→ g(y,y)} =
  Unify {(f(z) = f(z))}
  o ({y → f(z)} o {x→ g(y,y)})=
  Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))}

Example

- x,y,z variables, f,g constructors
- Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))} = ?

Example

- x,y,z variables, f,g constructors
- {(f(z) = f(z))} is non-empty
- Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(z) = f(z))
- Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))} = ?

Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(z) = f(z))
- Delete
- Unify {(f(z) = f(z))}
  o {y → f(z); x→ g(f(z), f(z))} =
  Unify {} o {y → f(z); x→ g(f(z), f(z))}

Example

- x,y,z variables, f,g constructors
- Unify {} o {y → f(z); x→ g(f(z), f(z))} = ?

Example

- x,y,z variables, f,g constructors
- {} is empty
- Unify {} = identity function
- Unify {} o {y → f(z); x→ g(f(z), f(z))} =
  {y → f(z); x→ g(f(z), f(z))}

Example

- Unify {(f(x) = f(g(f(z),y))), (g(y,y) = x)} =
  {y → f(z); x→ g(f(z), f(z))}

f(x) = f(g(f(z), y))
→ f(g(f(z), f(z))) = f(g(f(z), f(z)))

g(y , y) = x
→ g(f(z),f(z)) = g(f(z), f(z))

Example of Failure: Decompose

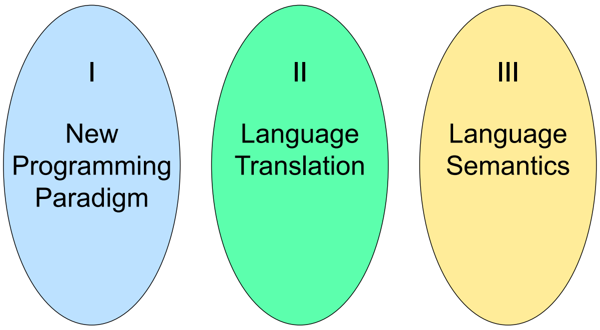
- Unify{(f(x,g(y)) = f(h(y),x))}
- Decompose: (f(x,g(y)) = f(h(y),x))
- = Unify {(x = h(y)), (g(y) = x)}
- Orient: (g(y) = x)
- = Unify {(x = h(y)), (x = g(y))}
- Eliminate: (x = h(y))
- Unify {(h(y) = g(y))} o {x → h(y)}
- No rule to apply! Decompose fails!

Example of Failure: Occurs Check

- Unify{(f(x,g(x)) = f(h(x),x))}
- Decompose: (f(x,g(x)) = f(h(x),x))
- = Unify {(x = h(x)), (g(x) = x)}
- Orient: (g(x) = x)
- = Unify {(x = h(x)), (x = g(x))}
- No rules apply.

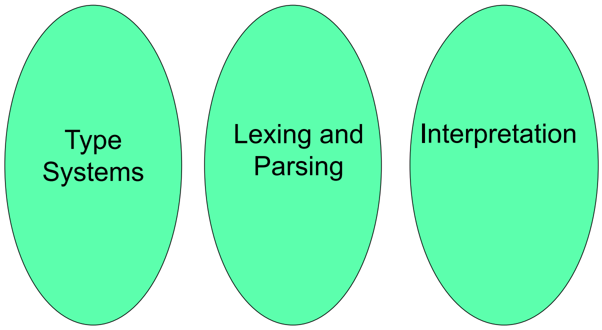
Programming Languages & Compilers

Three Main Topics of the Course

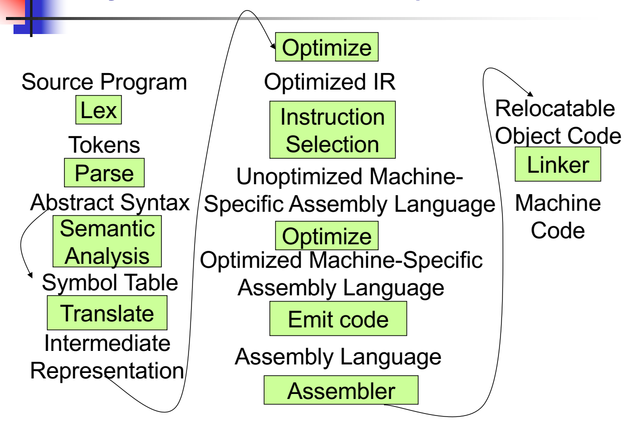


Programming Languages & Compilers

II : Language Translation



Major Phases of a Compiler



Modified from "Modern Compiler Implementation in ML", by Andrew Appel

Where We Are Going Next?

- We want to turn strings (code) into computer instructions
- Done in phases
- Turn strings into abstract syntax trees (parse)
- Translate abstract syntax trees into executable instructions (interpret or compile)

Meta-discourse

- Language Syntax and Semantics
- Syntax
  - Regular Expressions, DFSAs and NDFSAs
  - Grammars
- Semantics
  - Natural Semantics
  - Transition Semantics

Language Syntax

- Syntax is the description of which strings of symbols are meaningful expressions in a language
- It takes more than syntax to understand a language; need meaning (semantics) too
- Syntax is the entry point

Syntax of English Language

- Pattern 1

Subject	Verb
David	*sings*
The dog	*barked*
Susan	*yawned*

- Pattern 2

Subject	Verb	Direct Object
David	*sings*	*ballads*
The professor	*wants*	*to retire*
The jury	*found*	*the defendant guilty*

Elements of Syntax

- Character set – previously always ASCII, now often 64 character sets
- Keywords – usually reserved
- Special constants – cannot be assigned to
- Identifiers – can be assigned to
- Operator symbols
- Delimiters (parenthesis, braces, brackets)
- Blanks (aka white space)

Elements of Syntax

- Expressions
  if ... then begin ... ; ... end else begin ... ; ... end
- Type expressions
  $typexpr_1$ -> $typexpr_2$
- Declarations (in functional languages)
  let *pattern* = *expr*
- Statements (in imperative languages)
  a = b + c
- Subprograms
  let $pattern_1$ = $expr_1$ in *expr*

Elements of Syntax

- Modules
- Interfaces
- Classes (for object-oriented languages)

Lexing and Parsing

- Converting strings to abstract syntax trees done in two phases
  - **Lexing:** Converting string (or streams of characters) into lists (or streams) of tokens (the "words" of the language)
    - Specification Technique: Regular Expressions
  - **Parsing:** Convert a list of tokens into an abstract syntax tree
    - Specification Technique: BNF Grammars

Formal Language Descriptions

- Regular expressions, regular grammars, finite state automata
- Context-free grammars, BNF grammars, syntax diagrams
- Whole family more of grammars and automata – covered in automata theory

Grammars

- Grammars are formal descriptions of which strings over a given character set are in a particular language
- Language designers write grammar
- Language implementers use grammar to know what programs to accept
- Language users use grammar to know how to write legitimate programs

Regular Expressions - Review

- Start with a given character set – **a, b, c…**
- *L*(**ε**) = {“” }
- Each character is a regular expression
  - It represents the set of one string containing just character
  - *L*(**a**) = {a}

Regular Expressions

- If **x** and **y** are regular expressions, then **xy** is a regular expression
  - It represents the set of all strings made from first a string described by **x** then a string described by **y**

If *L*(x)={a,ab} and *L*(y)={c,d}
then *L*(xy) ={ac,ad,abc,abd}

Regular Expressions

- If **x** and **y** are regular expressions, then **x**∨**y** is a regular expression
  - It represents the set of strings described by either **x** or **y**

If *L*(x)={a,ab} and *L*(y)={c,d}
then *L*(x ∨ y)={a,ab,c,d}

Regular Expressions

- If **x** is a regular expression, then so is (**x**)
  - It represents the same thing as **x**
- If **x** is a regular expression, then so is **x***
  - It represents strings made from concatenating zero or more strings from **x**

If *L*(x) = {a,ab} then *L*(x*) ={“”,a,ab,aa,aab,abab,…}

- ε
  - It represents {“”}, set containing the empty string
- *Φ*
  - It represents { }, the empty set

Example Regular Expressions

- **(0∨1)*1**
  - The set of all strings of **0**’s and **1**’s ending in 1, **{1, 01, 11,…}**
- **a*b(a*)**
  - The set of all strings of a’s and b’s with exactly one b
- **((01) ∨(10))***
  - You tell me
- Regular expressions (equivalently, regular grammars) important for lexing, breaking strings into recognized words

Right Regular Grammars

- Subclass of BNF (covered in detail sool)
- Only rules of form
  <nonterminal>::=<terminal><nonterminal> or
  <nonterminal>::=<terminal> or
  <nonterminal>::=ε
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)
- Close connection to nondeterministic finite state automata – nonterminals ≅ states; rule ≅ edge

Example

- Right regular grammar:
  <Balanced> ::= ε
  <Balanced> ::= 0<OneAndMore>
  <Balanced> ::= 1<ZeroAndMore>
  <OneAndMore> ::= 1<Balanced>
  <ZeroAndMore> ::= 0<Balanced>
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

Implementing Regular Expressions

- Regular expressions reasonable way to generate strings in language
- Not so good for recognizing when a string is in language
- Problems with Regular Expressions
  - which option to choose,
  - how many repetitions to make
- Answer: finite state automata
- Should have seen in CS374

Example: Lexing

- Regular expressions good for describing lexemes (words) in a programming language
  - Identifier = (a ∨ b ∨ … ∨ z ∨ A ∨ B ∨ … ∨ Z) (a ∨ b ∨ … ∨ z ∨ A ∨ B ∨ … ∨ Z ∨ 0 ∨ 1 ∨ … ∨ 9)*
  - Digit = (0 ∨ 1 ∨ … ∨ 9)
  - Number = 0 ∨ (1 ∨ … ∨ 9)(0 ∨ … ∨ 9)* ∨ ~ (1 ∨ … ∨ 9)(0 ∨ … ∨ 9)*
  - Keywords: if = if, while = while,…

Lexing

- Different syntactic categories of "words": tokens

Example:

- Convert sequence of characters into sequence of strings, integers, and floating point numbers.
- "asd 123 jkl 3.14" will become:
  [String "asd"; Int 123; String "jkl"; Float 3.14]

Lex, ocamllex

- Could write the reg exp, then translate to DFA by hand
  - A lot of work
- Better: Write program to take reg exp as input and automatically generates automata
- Lex is such a program
- ocamllex version for ocaml

How to do it

- To use regular expressions to parse our input we need:
  - Some way to identify the input string — call it a lexing buffer
  - Set of regular expressions,
  - Corresponding set of actions to take when they are matched.

How to do it

- The lexer will take the regular expressions and generate a state machine.
- The state machine will take our lexing buffer and apply the transitions...
- If we reach an accepting state from which we can go no further, the machine will perform the appropriate action.

Mechanics

- Put table of reg exp and corresponding actions (written in ocaml) into a file <*filename*>.mll
- Call

  ocamllex <*filename*>.mll
- Produces Ocaml code for a lexical analyzer in file <*filename*>.ml

Sample Input

```
rule main = parse
 ['0'-'9']+ { print_string "Int\n"}
 | ['0'-'9']+'.'['0'-'9']+ { print_string "Float\n"}
 | ['a'-'z']+ { print_string "String\n"}
 | _ { main lexbuf }
 {
 let newlexbuf = (Lexing.from_channel stdin) in
 main newlexbuf
}
```


General Input

{ *header* }
let *ident* = *regexp* ...
rule *entrypoint* [*arg1*... *argn*] = parse
    regexp { *action* }
  | ...
  | *regexp* { *action* }
and *entrypoint* [*arg1*... *argn*] = parse ...and
 ...
{ *trailer* }

Ocamllex Input

- *header* and *trailer* contain arbitrary ocaml code put at top an bottom of <*filename*>.ml
- let *ident* = *regexp* ... Introduces *ident* for use in later regular expressions

Ocamllex Input

- *<filename>*.ml contains one lexing function per *entrypoint*
  - Name of function is name given for *entrypoint*
  - Each entry point becomes an Ocaml function that takes *n*+1 arguments, the extra implicit last argument being of type Lexing.lexbuf
- *arg1*... a*rgn* are for use in *action*

Ocamllex Regular Expression

- Single quoted characters for letters: 'a'
- _: (underscore) matches any letter
- Eof: special "end_of_file" marker
- Concatenation same as usual
- "*string*": concatenation of sequence of characters
- e_1 / e_2: choice - what was $e_1 \vee e_2$

Ocamllex Regular Expression

- [c_1 - c_2]: choice of any character between first and second inclusive, as determined by character codes
- [^c_1 - c_2]: choice of any character NOT in set
- *e**: same as before
- *e*+: same as *e e**
- *e*?: option - was $e \vee \varepsilon$
- (*e*): same as *e*

Ocamllex Regular Expression

- e_1 # e_2: the characters in e_1 but not in e_2; e_1 and e_2 must describe just sets of characters
- *ident*: abbreviation for earlier reg exp in let *ident* = *regexp*
- e_1 as *id*: binds the result of e_1 to *id* to be used in the associated *action*

Ocamllex Manual

- More details can be found at

Version for ocaml 4.07:

https://v2.ocaml.org/releases/4.07/htmlman/lexyacc.html

Current version (ocaml 4.14)

https://v2.ocaml.org/releases/4.14/htmlman/lexyacc.html

(same, except formatting, I think)

End of Lect 18

Example : test.mll

```
{ type result = Int of int | Float of float |
  String of string }
let digit = ['0'-'9']
let digits = digit +
let lower_case = ['a'-'z']
let upper_case = ['A'-'Z']
let letter = upper_case | lower_case
let letters = letter +
```


Example : test.mll

```
rule main = parse
  (digits)'.'digits as f  { Float (float_of_string f) }
 | digits as n             { Int (int_of_string n) }
 | letters as s            { String s}
 | _ { main lexbuf }
 { let newlexbuf = (Lexing.from_channel stdin) in
 print_newline ();
 main newlexbuf  }
```


Example

```
# #use "test.ml";;
...
val main : Lexing.lexbuf -> result = <fun>
val __ocaml_lex_main_rec : Lexing.lexbuf -> int ->
  result = <fun>
hi there 234 5.2
- : result = String "hi"
```

What happened to the rest?!?

Example

```
# let b = Lexing.from_channel stdin;;
# main b;;
hi 673 there
- : result = String "hi"
# main b;;
- : result = Int 673
# main b;;
- : result = String "there"
```


Your Turn

- Work on MP8
  - Add a few keywords
  - Implement booleans and unit
  - Implement Ints and Floats
  - Implement identifiers

Problem

- How to get lexer to look at more than the first token at one time?
- Answer: *action* has to tell it to -- recursive calls
  - Not what you want to sew this together with ocamlyacc
- Side Benefit: can add “state” into lexing
- Note: already used this with the _ case

Example

```
rule main = parse
  (digits) '.' digits as f { Float
  (float_of_string f) :: main lexbuf}
 | digits as n        { Int (int_of_string n) ::
   main lexbuf }
 | letters as s       { String s :: main
   lexbuf}
 | eof                 { [] }
 | _                   { main lexbuf }
```


Example Results

hi there 234 5.2

- : result list = [String "hi"; String "there"; Int 234; Float 5.2]

#

Used Ctrl-d to send the end-of-file signal

Dealing with comments

First Attempt

```
let open_comment = "(*"
let close_comment = "*)"
rule main = parse
  (digits) '.' digits as f { Float (float_of_string
  f) :: main lexbuf}
 | digits as n        { Int (int_of_string n) ::
   main lexbuf }
 | letters as s       { String s :: main lexbuf}
```


Dealing with comments

```
| open_comment        { comment  lexbuf}
| eof                 { [] }
| _ { main lexbuf }
and comment = parse
  close_comment       { main lexbuf }
| _                   { comment lexbuf }
```


Dealing with nested comments

```
rule main = parse …
 | open_comment        { comment 1 lexbuf}
 | eof                 { [] }
 | _ { main lexbuf }
and comment depth = parse
  open_comment         { comment (depth+1) lexbuf
  }
 | close_comment       { if depth = 1
                          then main lexbuf
                          else comment (depth - 1) lexbuf }
 | _                    { comment depth lexbuf }
```


Dealing with nested comments

```
rule main = parse
  (digits) '.' digits as f { Float (float_of_string f) ::
  main lexbuf}
 | digits as n        { Int (int_of_string n) :: main
   lexbuf }
 | letters as s       { String s :: main lexbuf}
 | open_comment       { (comment 1 lexbuf}
 | eof                { [] }
 | _ { main lexbuf }
```

Dealing with nested comments

```
and comment depth = parse
   open_comment        { comment (depth+1) lexbuf
   }
 | close_comment       { if depth = 1
                        then main lexbuf
                       else comment (depth - 1) lexbuf }
 | _                  { comment depth lexbuf }
```


Types of Formal Language Descriptions

- Regular expressions, regular grammars
- Context-free grammars, BNF grammars, syntax diagrams

- Finite state automata
- Pushdown automata
- Whole family more of grammars and automata – covered in automata theory

Sample Grammar

- Language: Parenthesized sums of 0's and 1's

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)

BNF Grammars

- Start with a set of characters, **a,b,c,…**
  - We call these *terminals*
- Add a set of different characters, **X,Y,Z,…**
  - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*

BNF Grammars

- BNF rules (aka *productions*) have form

  X ::= *y*

  where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

Sample Grammar

- Terminals: 0 1 + ( )
- Nonterminals: <Sum>
- Start symbol = <Sum>

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)
- Can be abbreviated as

```
<Sum> ::= 0 | 1
          | <Sum> + <Sum> | (<Sum>)
```

BNF Derivations

- Given rules

  X::= *y***Z***w* and **Z**::=*v*

we may replace **Z** by *v* to say

  X => *y***Z***w* => *yvw*

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

BNF Derivations

- Start with the start symbol:

<Sum> =>

BNF Derivations

- Pick a non-terminal

<Sum> =>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= (<Sum>)

<Sum> => <Sum> + <Sum >

  => (<Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 1

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a rule and substitute
  - <Sum> ::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) 0
=> (0 + 1) + 0

BNF Derivations

- (0 + 1) + 0 is generated by grammar

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0
=> (0 + 1) + 0

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>