

Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Example: If Then Else Rule

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$) \Downarrow ?



Example: If Then Else Rule

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}) \Downarrow ?$



Example: Arith Relation

? > ? = ?

$(x, \{x \rightarrow 7\}) \Downarrow ? \quad (5, \{x \rightarrow 7\}) \Downarrow ?$

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



Example: Identifier(s)

$7 > 5 = \text{true}$

$(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5$

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



Example: Arith Relation

$7 > 5 = \text{true}$

$(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5$

$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$

Example: If Then Else Rule

$7 > 5 = \text{true}$

$\frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}}$

$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$

$\frac{(y := 2 + 3, \{x \rightarrow 7\})}{\Downarrow ?}$

$\Downarrow ?$

$\frac{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \quad (y := 2 + 3, \{x \rightarrow 7\}) \Downarrow ?}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}$

Example: Assignment

$$\begin{array}{c}
 7 > 5 = \text{true} \\
 \hline
 \frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}} \quad \frac{(2+3, \{x \rightarrow 7\}) \Downarrow ?}{(y := 2 + 3, \{x \rightarrow 7\}) \Downarrow ?} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?
 \end{array}$$



Example: Arith Op

$$\begin{array}{r} \text{? + ? = ?} \\ \hline (2, \{x \rightarrow 7\}) \Downarrow? \quad (3, \{x \rightarrow 7\}) \Downarrow? \\ \hline (2+3, \{x \rightarrow 7\}) \Downarrow? \\ \hline (y := 2 + 3, \{x \rightarrow 7\}) \\ \Downarrow? \end{array}$$
$$\begin{array}{r} 7 > 5 = \text{true} \\ \hline (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\ \hline (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\ \hline (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ \quad \{x \rightarrow 7\}) \Downarrow ? \end{array}$$

Example: Numerals

$$\begin{array}{r}
 2 + 3 = 5 \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow ? \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow ? \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow ?
 \end{array}$$

Example: Arith Op

$$\begin{array}{r}
 2 + 3 = 5 \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow ? \\
 \\
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow ?
 \end{array}$$

Example: Assignment

$$\begin{array}{c}
 2 + 3 = 5 \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow \{x \rightarrow 7, y \rightarrow 5\} \\
 \hline
 \begin{array}{c}
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow ?
 \end{array}
 \end{array}$$

Example: If Then Else Rule

$$\begin{array}{c}
 2 + 3 = 5 \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow \{x \rightarrow 7, y \rightarrow 5\} \\
 \hline
 \begin{array}{c}
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow \{x \rightarrow 7, y \rightarrow 5\}
 \end{array}
 \end{array}$$



Let in Command

$$\frac{(E, m) \Downarrow v \quad (C, m[I \leftarrow v]) \Downarrow m'}{(\text{let } I = E \text{ in } C, m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and
 $m''(I) = m(I)$ if $m(I)$ is defined,
and $m''(I)$ is undefined otherwise



Example

$$\frac{\frac{(x, \{x \rightarrow 5\}) \Downarrow 5 \quad (3, \{x \rightarrow 5\}) \Downarrow 3}{(x+3, \{x \rightarrow 5\}) \Downarrow 8}}{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}}{(\text{let } x = 5 \text{ in } (x := x+3), \{x \rightarrow 17\}) \Downarrow ?}$$



Example

$$\frac{\frac{(x, \{x \rightarrow 5\}) \Downarrow 5 \quad (3, \{x \rightarrow 5\}) \Downarrow 3}{(x+3, \{x \rightarrow 5\}) \Downarrow 8}}{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}{\text{(let } x = 5 \text{ in } (x := x+3), \{x \rightarrow 17\}) \Downarrow \{x \rightarrow 17\}}$$



Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explicitly*
- Clash of constructs apparent in awkward semantics



Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed



Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
 - Start with literals
 - Variables
 - Primitive operations
 - Evaluation of expressions
 - Evaluation of commands/declarations



Interpreter

- Takes abstract syntax trees as input
 - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
 - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next “state”
 - To get final value, put in a loop



Natural Semantics Example

- $\text{compute_exp}(\text{Var}(v), m) = \text{look_up } v \ m$
- $\text{compute_exp}(\text{Int}(n), _) = \text{Num } (n)$
- ...
- $\text{compute_com}(\text{IfExp}(b, c1, c2), m) =$
if $\text{compute_exp}(b, m) = \text{Bool}(\text{true})$
then $\text{compute_com}(c1, m)$
else $\text{compute_com}(c2, m)$



Natural Semantics Example

- $\text{compute_com}(\text{While}(b,c), m) =$
if $\text{compute_exp}(b,m) = \text{Bool}(\text{false})$
then m
else compute_com
 $(\text{While}(b,c), \text{compute_com}(c,m))$
- May fail to terminate - exceed stack limits
- Returns no useful information then



Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
$$(C, m) \dashrightarrow (C', m') \quad \text{or} \quad (C, m) \dashrightarrow m'$$
- C, C' is code remaining to be executed
- m, m' represent the state/store/memory/environment
 - Partial mapping from identifiers to values
 - Sometimes m (or C) not needed
- Indicates exactly one step of computation



Expressions and Values

- C, C' used for commands; E, E' for expressions; U, V for values
- Special class of expressions designated as *values*
 - Eg 2, 3 are values, but $2+3$ is only an expression
- Memory only holds values
 - Other possibilities exist



Evaluation Semantics

- Transitions successfully stops when E/C is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

1525 minutes



Simple Imperative Programming Language

- $I \in \text{Identifiers}$
- $N \in \text{Numerals}$
- $B ::= \text{true} \mid \text{false} \mid B \ \& \ B \mid B \ \text{or} \ B \mid \text{not } B \mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E$
| $\text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$



Transitions for Expressions

- Numerals are values
- Boolean values = {true, false}
- Identifiers: $(I, m) \dashrightarrow (m(I), m)$



Boolean Operations:

- Operators: (short-circuit)

$$\begin{array}{l} (\text{false} \ \& \ B, \ m) \ \rightarrow (\text{false}, m) \\ (\text{true} \ \& \ B, \ m) \ \rightarrow (B, m) \end{array} \quad \frac{(B, \ m) \ \rightarrow (B'', \ m)}{(B \ \& \ B', \ m) \ \rightarrow (B'' \ \& \ B', \ m)}$$
$$\begin{array}{l} (\text{true} \ \text{or} \ B, \ m) \ \rightarrow (\text{true}, m) \\ (\text{false} \ \text{or} \ B, \ m) \ \rightarrow (B, m) \end{array} \quad \frac{(B, \ m) \ \rightarrow (B'', \ m)}{(B \ \text{or} \ B', \ m) \ \rightarrow (B'' \ \text{or} \ B', \ m)}$$
$$\begin{array}{l} (\text{not true}, \ m) \ \rightarrow (\text{false}, m) \\ (\text{not false}, \ m) \ \rightarrow (\text{true}, m) \end{array} \quad \frac{(B, \ m) \ \rightarrow (B', \ m)}{(\text{not } B, \ m) \ \rightarrow (\text{not } B', \ m)}$$



Relations

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \sim E', m) \dashrightarrow (E'' \sim E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \sim E, m) \dashrightarrow (V \sim E', m)}$$

$(U \sim V, m) \dashrightarrow (\text{true}, m)$ or (false, m)
depending on whether $U \sim V$ holds or not



Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \text{ op } E', m) \dashrightarrow (E'' \text{ op } E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \text{ op } E, m) \dashrightarrow (V \text{ op } E', m)}$$

$(U \text{ op } V, m) \dashrightarrow (N, m)$ where N is the specified value for $U \text{ op } V$



Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory



Commands

$$(\text{skip}, m) \dashrightarrow m$$

$$\frac{(E, m) \dashrightarrow (E', m)}{}$$

$$(\text{I} ::= E, m) \dashrightarrow (\text{I} ::= E', m)$$

$$(\text{I} ::= V, m) \dashrightarrow m[I \leftarrow V]$$

$$\frac{(C, m) \dashrightarrow (C'', m')}{}$$

$$(\text{C}; \text{C}', m) \dashrightarrow (\text{C}''; \text{C}', m')$$

$$\frac{(C, m) \dashrightarrow m'}{}$$

$$(\text{C}; \text{C}', m) \dashrightarrow (\text{C}', m')$$