











- Expresses the meaning of syntax
- Static semantics
 - Meaning based only on the form of the expression without executing it
 - Usually restricted to type checking / type inference



Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
 - Operational Semantics
 - Axiomatic Semantics
 - Denotational Semantics

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Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

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Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

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Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from axioms and inference rules
- Mainly suited to simple imperative programming languages

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Axiomatic Semantics

- Used to formally prove a property (post-condition) of the state (the values of the program variables) after the execution of program, assuming another property (pre-condition) of the state before execution
- Written : {Precondition} Program {Postcondition}
- Source of idea of loop invariant



Denotational Semantics

- Construct a function M assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

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Natural Semantics

- Aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

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Simple Imperative Programming Language

- $I \in Identifiers$
- N ∈ Numerals
- B ::= true | false | B & B | B or B | not B | E < E | E = E
- E::= N / I / E + E / E * E / E E / E
- C::= skip | C,C | I::= E | if B then C else C fi | while B do C od

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Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \downarrow m(I)$
- Numerals are values: (*N,m*) ↓ *N*
- Booleans: (true, m) ↓ true (false, m) ↓ false

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Booleans:

$$\frac{(B, m) \Downarrow \text{ false}}{(B \& B', m) \Downarrow \text{ false}} \frac{(B, m) \Downarrow \text{ true } (B', m) \Downarrow b}{(B \& B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{ true}}{(B \text{ or } B', m) \Downarrow \text{ true}} \quad \frac{(B, m) \Downarrow \text{ false } (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

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Relations

$$(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b$$
$$(E \sim E', m) \Downarrow b$$

- By U ~ V = b, we mean does (the meaning of) the relation ~ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and

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Arithmetic Expressions

$$(\underline{E, m}) \Downarrow U \quad (\underline{E', m}) \Downarrow V \quad U \text{ op } V = N$$
$$(\underline{E \text{ op } E', m}) \Downarrow N$$

where N is the specified value for U op V



Commands

Skip:

(skip, m) $\downarrow m$

Assignment:

 $\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I < -- V]}$

Sequencing: $(C,m) \Downarrow m' (C',m') \Downarrow m''$ $(C,C',m) \Downarrow m''$

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If Then Else Command

(B,m) ↓ true (C,m) ↓ m'(if B then C else C' fi, m) ↓ m'

(B,m) ↓ false (C',m) ↓ m' (if B then C else C' fi, m) ↓ m'

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While Command

(B,m) ↓ false(while B do C od, m) ↓ m

 $\frac{(B,m) \forall \text{true } (C,m) \forall m' \text{ (while } B \text{ do } C \text{ od, } m') \forall m''}{\text{(while } B \text{ do } C \text{ od, } m) \ \forall m''}$

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Example: If Then Else Rule

(if x > 5 then y := 2 + 3 else y := 3 + 4 fi, $\{x -> 7\}) \downarrow ?$

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Example: If Then Else Rule

$$(x > 5, \{x -> 7\})$$
 ?

(if x > 5 then y := 2 + 3 else y := 3 + 4 fi, $\{x -> 7\}) \downarrow ?$

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Example: Arith Relation

 $(x,{x->7})$ \Downarrow ? $(5,{x->7})$ \Downarrow ?

 $(x > 5, \{x \to 7\})$?

(if x > 5 then y := 2 + 3 else y := 3 + 4 fi, $\{x -> 7\}) \downarrow ?$

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Example: Identifier(s)

7 > 5 = true

$$(x,(x->7)) \forall 7 \quad (5,(x->7)) \forall 5$$

$$(x > 5, (x -> 7)) \forall ?$$

$$(if x > 5 \text{ then } y:= 2 + 3 \text{ else } y:= 3 + 4 \text{ fi},$$

$$(x -> 7)) \forall ?$$

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Example: Arith Relation

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Example: If Then Else Rule

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Example: Assignment

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Example: Arith Op

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Example: Numerals

$$2 + 3 = 5$$

$$(2,{x->7}) \lor 2 \quad (3,{x->7}) \lor 3$$

$$7 > 5 = \text{true} \qquad (2+3, {x->7}) \lor ?$$

$$(x,{x->7}) \lor 7 \quad (5,{x->7}) \lor 5$$

$$(x > 5, {x -> 7}) \lor \text{true} \qquad \lor ?$$

$$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,}$$

$$\{x -> 7\}) \lor ?$$



Example: Arith Op

$$2 + 3 = 5$$

$$(2,\{x->7\}) \downarrow 2 \quad (3,\{x->7\}) \downarrow 3$$

$$7 > 5 = \text{true} \qquad (2+3,\{x->7\}) \downarrow 5$$

$$(x,\{x->7\}) \downarrow 7 \quad (5,\{x->7\}) \downarrow 5 \qquad (y:= 2 + 3,\{x->7\})$$

$$(x > 5, \{x -> 7\}) \downarrow \text{true} \qquad \qquad \downarrow ?$$

$$(if x > 5 \text{ then } y:= 2 + 3 \text{ else } y:= 3 + 4 \text{ fi,}$$

$$\{x -> 7\}) \downarrow ?$$

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Example: Assignment

$$2 + 3 = 5$$

$$(2,\{x->7\}) \lor 2 \quad (3,\{x->7\}) \lor 3$$

$$7 > 5 = \text{true} \qquad (2+3, \{x->7\}) \lor 5$$

$$(x,\{x->7\}) \lor 7 \quad (5,\{x->7\}) \lor 5$$

$$(x > 5, \{x -> 7\}) \lor \text{true} \qquad \lor \{x->7, y->5\}$$

$$(if x > 5 \text{ then } y:= 2 + 3 \text{ else } y:= 3 + 4 \text{ fi,}$$

$$\{x -> 7\}\} \lor ?$$

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Example: If Then Else Rule

$$2 + 3 = 5$$

$$(2,\{x->7\}) \lor 2 \quad (3,\{x->7\}) \lor 3$$

$$7 > 5 = \text{true} \qquad (2+3,\{x->7\}) \lor 5$$

$$(x,\{x->7\}) \lor 7 \quad (5,\{x->7\}) \lor 5 \qquad (y:=2+3,\{x->7\})$$

$$(x > 5, \{x -> 7\}) \lor \text{true} \qquad \forall \{x->7, y->5\}$$

$$(if x > 5 \text{ then } y:=2+3 \text{ else } y:=3+4 \text{ fi},$$

$$\{x -> 7\}) \lor \{x->7, y->5\}$$

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Let in Command

$$\frac{(E,m) \ \forall v \ (C,m[I < -v]) \ \psi \ m'}{(\text{let } I = E \text{ in } C, m) \ \psi \ m'}$$

Where m''(y) = m'(y) for $y \neq I$ and m''(I) = m(I) if m(I) is defined, and m''(I) is undefined otherwise

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Example

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Example



Comment

- Simple Imperative Programming Language introduces variables implicitly through assignment
- The let-in command introduces scoped variables explictly
- Clash of constructs apparent in awkward semantics

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Interpretation Versus Compilation

- A compiler from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An interpreter of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

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Interpreter

- An Interpreter represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
 - Start with literals
 - Variables
 - Primitive operations
 - Evaluation of expressions
 - Evaluation of commands/declarations