

Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC



<https://courses.engr.illinois.edu/cs421/sp2023>

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Recursive Data Types

```
# type exp =  
  VarExp of string  
| ConstExp of const  
| MonOpAppExp of mon_op * exp  
| BinOpAppExp of bin_op * exp * exp  
| IfExp of exp * exp * exp  
| AppExp of exp * exp  
| FunExp of string * exp
```



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
                | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent 6 as an exp?



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
                | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent (6, 3) as an exp?



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6),
ConstExp (IntConst 3))



Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
           | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp, BinOpAppExp (CommaOp, ConstExp (IntConst 6), ConstExp (IntConst 3)), ConstExp NilConst))));;



Problem

```
type int_Bin_Tree = Leaf of int  
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write `sum_tree : int_Bin_Tree -> int`
- Adds all ints in tree

```
let rec sum_tree t =
```




Problem

```
type int_Bin_Tree = Leaf of int
```

```
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write `sum_tree : int_Bin_Tree -> int`
- Adds all ints in tree

```
let rec sum_tree t =
```

```
  match t with Leaf n -> n
```

```
  | Node(t1,t2) -> sum_tree t1 + sum_tree t2
```



Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?



Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =  
  match exp with VarExp x ->  
    | ConstExp c ->  
    | BinOpAppExp (b, e1, e2) ->  
    | FunExp (x,e) ->  
    | AppExp (e1, e2) ->
```



Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =  
  match exp with VarExp x -> 1  
  | ConstExp c -> 0  
  | BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2  
  | FunExp (x,e) -> 1 + varCnt e  
  | AppExp (e1, e2) -> varCnt e1 + varCnt e2
```



Mapping over Recursive Types

```
# let rec ibtreeMap f tree =  
  match tree with (Leaf n) -> Leaf (f n)  
  | Node (left_tree, right_tree) ->  
    Node (ibtreeMap f left_tree,  
          ibtreeMap f right_tree);;  
val ibtreeMap : (int -> int) -> int_Bin_Tree ->  
  int_Bin_Tree = <fun>
```



Mapping over Recursive Types

```
# ibtreeMap ((+) 2) bin_tree;;
```

```
- : int_Bin_Tree = Node (Node (Leaf 5, Leaf  
8), Leaf (-5))
```



Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =  
  match tree with Leaf n -> leafFun n  
  | Node (left_tree, right_tree) ->  
    nodeFun  
      (ibtreeFoldRight leafFun nodeFun left_tree)  
      (ibtreeFoldRight leafFun nodeFun right_tree);;  
val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->  
  int_Bin_Tree -> 'a = <fun>
```



Folding over Recursive Types

```
# let tree_sum =  
    ibtreeFoldRight (fun x -> x) (+);;  
val tree_sum : int_Bin_Tree -> int = <fun>  
# tree_sum bin_tree;;  
- : int = 2
```




Mutually Recursive Types

```
# type 'a tree = TreeLeaf of 'a
```

```
  | TreeNode of 'a treeList
```

```
and 'a treeList = Last of 'a tree
```

```
  | More of ('a tree * 'a treeList);;
```

```
type 'a tree = TreeLeaf of 'a | TreeNode of 'a  
treeList
```

```
and 'a treeList = Last of 'a tree | More of ('a  
tree * 'a treeList)
```



Mutually Recursive Types - Values

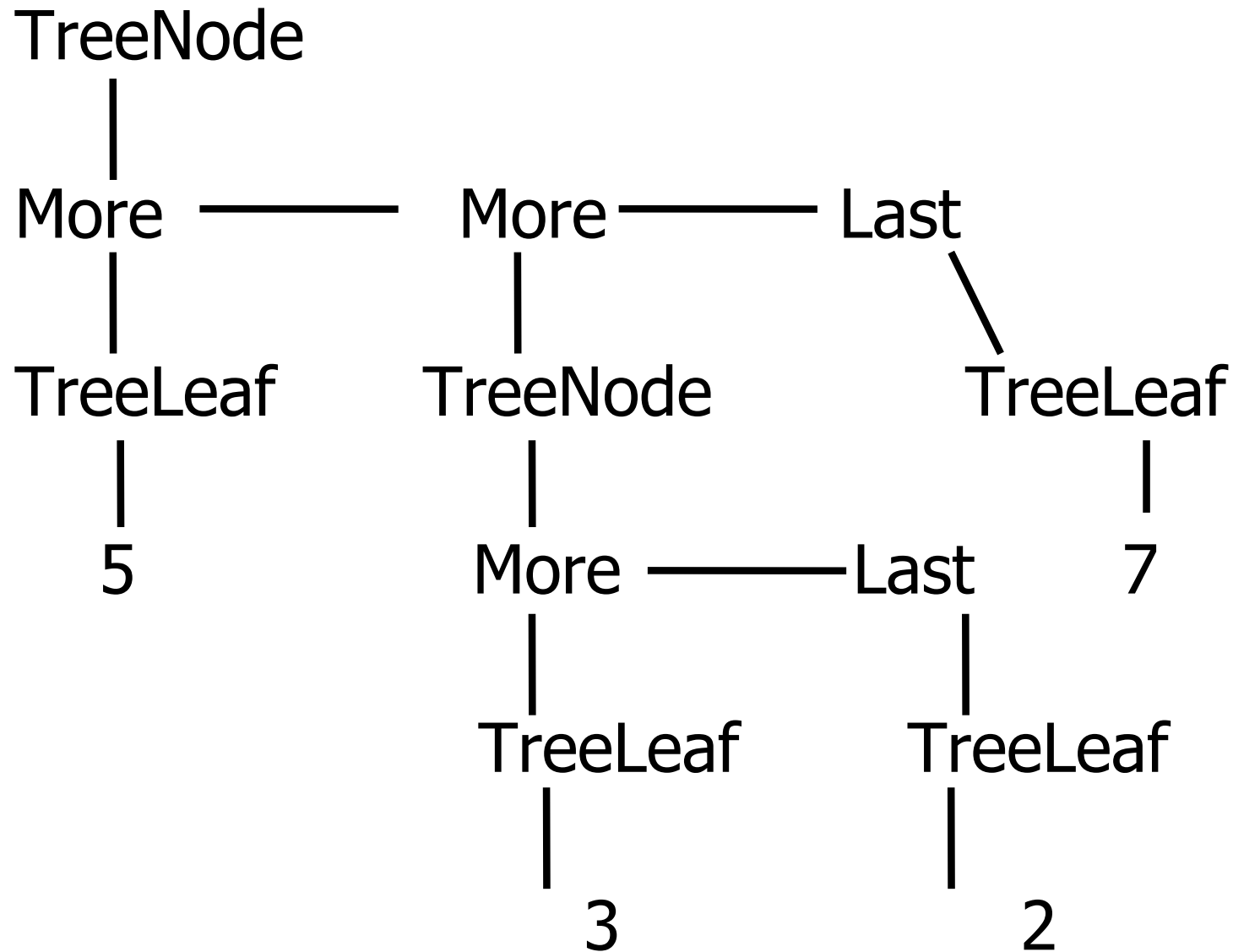
```
# let tree =  
  TreeNode  
    (More (TreeLeaf 5,  
          (More (TreeNode  
                (More (TreeLeaf 3,  
                      Last (TreeLeaf 2))),  
                      Last (TreeLeaf 7))))));;
```



Mutually Recursive Types - Values

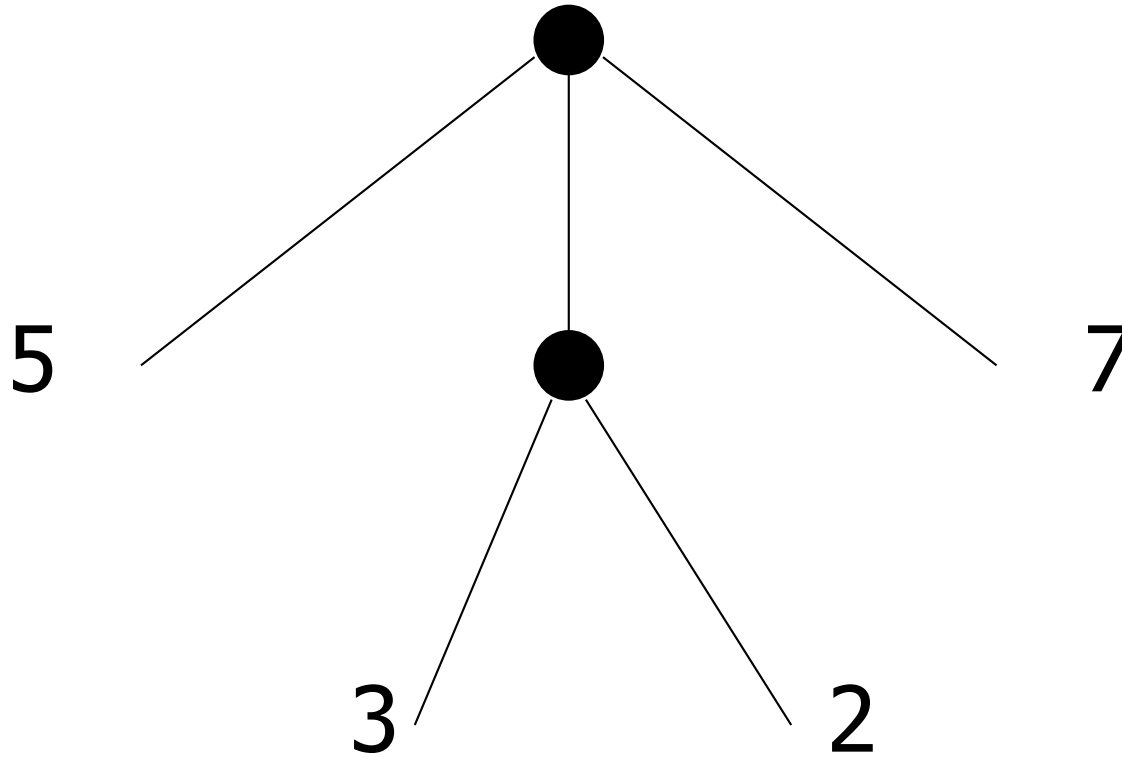
```
val tree : int tree =  
  TreeNode  
    (More  
      (TreeLeaf 5,  
        More  
          (TreeNode (More (TreeLeaf 3, Last  
            (TreeLeaf 2))), Last (TreeLeaf 7))))))
```

Mutually Recursive Types - Values



Mutually Recursive Types - Values

A more conventional picture





Mutually Recursive Functions

```
# let rec fringe tree =  
  match tree with (TreeLeaf x) -> [x]  
  | (TreeNode list) -> list_fringe list  
and list_fringe tree_list =  
  match tree_list with (Last tree) -> fringe tree  
  | (More (tree,list)) ->  
    (fringe tree) @ (list_fringe list);;
```

```
val fringe : 'a tree -> 'a list = <fun>
```

```
val list_fringe : 'a treeList -> 'a list = <fun>
```



Mutually Recursive Functions

```
# fringe tree;;
```

```
- : int list = [5; 3; 2; 7]
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;  
Define tree_size
```




Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =  
    match t with TreeLeaf _ ->  
    | TreeNode ts ->
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
```

```
    match t with TreeLeaf _ -> 1
```

```
    | TreeNode ts -> treeList_size ts
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList  
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define `tree_size` and `treeList_size`

```
let rec tree_size t =
```

```
    match t with TreeLeaf _ -> 1
```

```
    | TreeNode ts -> treeList_size ts
```

```
and treeList_size ts =
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size ts
and treeList_size ts =
    match ts with Last t ->
    | More t ts' ->
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size and treeList_size
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size ts
and treeList_size ts =
    match ts with Last t -> tree_size t
    | More t ts' -> tree_size t + treeList_size ts'
```



Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;

Define tree_size and treeList_size

let rec tree_size t =
  match t with TreeLeaf _ -> 1
  | TreeNode ts -> treeList_size ts
and treeList_size ts =
  match ts with Last t -> tree_size t
  | More t ts' -> tree_size t + treeList_size ts'
```



Nested Recursive Types

```
# type 'a labeled_tree =
```

```
  TreeNode of ('a * 'a labeled_tree  
  list);;
```

```
type 'a labeled_tree = TreeNode of ('a  
  * 'a labeled_tree list)
```



Nested Recursive Type Values

```
# let ltree =  
  TreeNode(5,  
    [TreeNode (3, []);  
      TreeNode (2, [TreeNode (1, []);  
                          TreeNode (7, [])]);  
      TreeNode (5, [])]);;
```

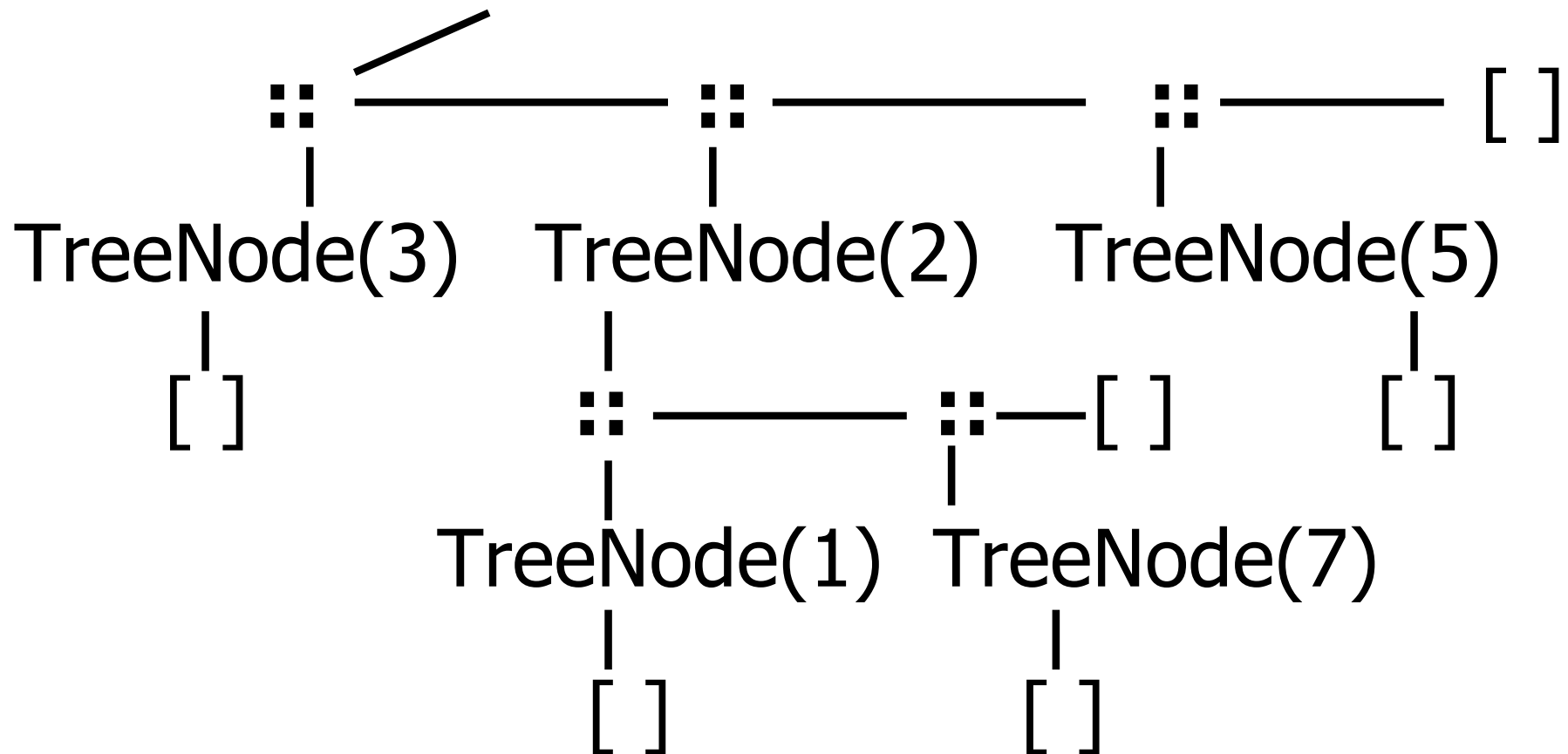



Nested Recursive Type Values

```
val ltree : int labeled_tree =  
  TreeNode  
    (5,  
     [TreeNode (3, []); TreeNode (2,  
      [TreeNode (1, []); TreeNode (7, [])]);  
     TreeNode (5, [])])
```

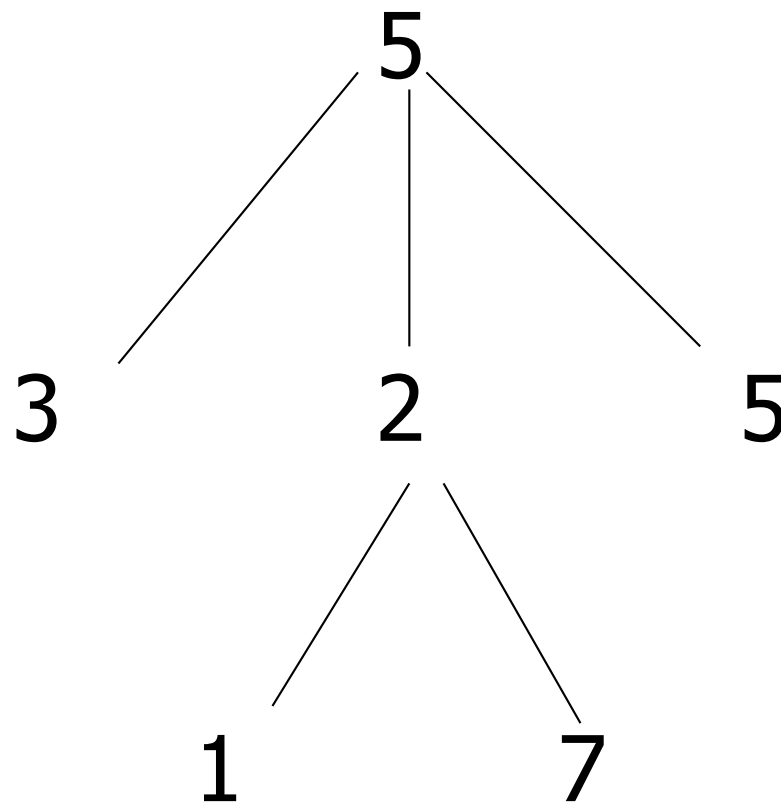
Nested Recursive Type Values

Ltree = TreeNode(5)





Nested Recursive Type Values





Mutually Recursive Functions

```
# let rec flatten_tree labtree =  
  match labtree with TreeNode (x,treelist)  
    -> x::flatten_tree_list treelist  
and flatten_tree_list treelist =  
  match treelist with [] -> []  
  | labtree::labtrees  
    -> flatten_tree labtree  
      @ flatten_tree_list labtrees;;
```



Mutually Recursive Functions

```
val flatten_tree : 'a labeled_tree -> 'a list =  
  <fun>
```

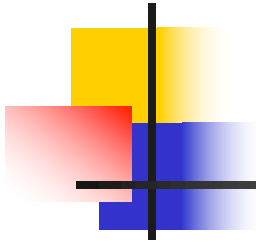
```
val flatten_tree_list : 'a labeled_tree list -> 'a  
  list = <fun>
```

```
# flatten_tree ltree;;
```

```
- : int list = [5; 3; 2; 1; 7; 5]
```

- Nested recursive types lead to mutually recursive functions

625 minutes



Extra Material



Infinite Recursive Values

```
# let rec ones = 1::ones;;
```

```
val ones : int list =
```

```
[1; 1; 1; 1; ...]
```

```
# match ones with x::_ -> x;;
```

Characters 0-25:

Warning: this pattern-matching is not exhaustive.

Here is an example of a value that is not matched:

```
[]
```

```
match ones with x::_ -> x;;
```

```
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
```

```
- : int = 1
```




Infinite Recursive Values

```
# let rec lab_tree = TreeNode(2, tree_list)
  and tree_list = [lab_tree; lab_tree];;

val lab_tree : int labeled_tree =
  TreeNode (2, [TreeNode(...); TreeNode(...)])

val tree_list : int labeled_tree list =
  [TreeNode (2, [TreeNode(...);
  TreeNode(...)]);
  TreeNode (2, [TreeNode(...);
  TreeNode(...)])]
```



Infinite Recursive Values

```
# match lab_tree  
  with TreeNode (x, _) -> x;;  
- : int = 2
```



Records

- Records serve the same programming purpose as tuples
- Provide better documentation, more readable code
- Allow components to be accessed by label instead of position
 - Labels (aka *field names* must be unique)
 - Fields accessed by suffix dot notation



Record Types

- Record types must be declared before they can be used in OCaml

```
# type person = {name : string; ss : (int * int * int); age : int};;
```

```
type person = { name : string; ss : int * int * int; age : int; }
```

- person is the type being introduced
- name, ss and age are the labels, or fields



Record Values

- Records built with labels; order does not matter

```
# let teacher = {name = "Elsa L. Gunter";  
  age = 102; ss = (119,73,6244)};;
```

```
val teacher : person =
```

```
{name = "Elsa L. Gunter"; ss = (119, 73,  
  6244); age = 102}
```



Record Pattern Matching

```
# let {name = elsa; age = age; ss =  
      (_,_,s3)} = teacher;;
```

```
val elsa : string = "Elsa L. Gunter"
```

```
val age : int = 102
```

```
val s3 : int = 6244
```



Record Field Access

```
# let soc_sec = teacher.ss;;
```

```
val soc_sec : int * int * int = (119,  
73, 6244)
```



Record Values

```
# let student = {ss=(325,40,1276);  
  name="Joseph Martins"; age=22};;
```

```
val student : person =
```

```
{name = "Joseph Martins"; ss = (325, 40,  
  1276); age = 22}
```

```
# student = teacher;;
```

```
- : bool = false
```




New Records from Old

```
# let birthday person = {person with age =  
    person.age + 1};;  
val birthday : person -> person = <fun>  
# birthday teacher;;  
- : person = {name = "Elsa L. Gunter"; ss =  
    (119, 73, 6244); age = 103}
```



New Records from Old

```
# let new_id name soc_sec person =  
  {person with name = name; ss = soc_sec};;  
val new_id : string -> int * int * int -> person  
  -> person = <fun>  
# new_id "Giuseppe Martin" (523,04,6712)  
student;;  
- : person = {name = "Giuseppe Martin"; ss  
= (523, 4, 6712); age = 22}
```



End of Extra Material

625 minutes