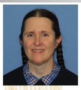
Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC



https://courses.engr.illinois.edu/cs421/sp2023

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Your turn: num_neg – tail recursive

```
# let num_neg list =
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =



 in num_neg_aux ? ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
   match list with [] ->
    | (x :: xs) ->


 in num_neg_aux ? ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
   match list with [] -> curr_neg
    | (x :: xs) ->


 in num_neg_aux ? ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
 let rec num_neg_aux list curr_neg =
   match list with [] -> curr_neg
    | (x :: xs) ->
      num_neg_aux xs ?

 in num_neg_aux ? ?
```

Your turn: num_neg – tail recursive

```
# let num_neg list =
  let rec num_neg_aux list curr_neg =
     match list with [] -> curr_neg
       | (x :: xs) ->
         num_neg_aux xs
           (if x < 0 then 1 + curr_neg
            else curr_neg)
   in num_neg_aux  ?  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
  let rec num_neg_aux list curr_neg =
     match list with [] -> curr_neg
       | (x :: xs) ->
         num_neg_aux xs
           (if x < 0 then 1 + curr_neg
            else curr_neg)
   in num_neg_aux list  ?
```


Your turn: num_neg – tail recursive

```
# let num_neg list =
  let rec num_neg_aux list curr_neg =
     match list with [] -> curr_neg
       | (x :: xs) ->
         num_neg_aux xs
           (if x < 0 then 1 + curr_neg
            else curr_neg)
   in num_neg_aux list 0
```


Tail Recursion - length

- How can we write length with tail recursion?

```
let length list =
    let rec length_aux list acc_length =
           match list
            with [ ] -> acc_length
              | (x::xs) ->
                length_aux xs (1 + acc_length)
     in length_aux list 0
```

accumulated value

initial acc value

combing operation

length, fold_left

```
let length list =
   fold_left
    (fun acc -> fun x -> 1 + acc) // comb op
    0  // initial accumulator cell value
    list
```


Your turn: num_neg, fold_left

```
let num_neg list =
   fold_left
    ?  // comb op



    ?  // initial accumulator cell value
    ?
```

Your turn: num_neg, fold_left

```
let num_neg list =
  fold_left
   ?  // comb op


   0  // initial accumulator cell value
   ?
```


Your turn: num_neg, fold_left

```
let num_neg list =
  fold_left
   (fun curr_neg -> fun x ->
     if x < 0 then 1 + curr_neg else curr_neg)
       // comb op
   0  // initial accumulator cell value
   ?
```


Your turn: num_neg, fold_left

```
let num_neg list =
  fold_left
   (fun curr_neg -> fun x ->
     if x < 0 then 1 + curr_neg else curr_neg)
       // comb op
   0  // initial accumulator cell value
   list
```


Folding

```
# let rec fold_left f a list = match list
  with [] -> a | (x :: xs) -> fold_left f (f a x) xs;;
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a =
  <fun>
```

fold_left f a [x_1; x_2;…;x_n] = f(…(f (f a x_1) x_2)…)x_n

```
# let rec fold_right f list b = match list
  with [ ] -> b | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =
  <fun>
```

fold_right f [x_1; x_2;…;x_n] b = f x_1(f x_2 (…(f x_n b)…))

Folding

- Can replace recursion by fold_right in any forward primitive recursive definition
  - Primitive recursive means here it only recurses on immediate subcomponents of recursive data structure
- Can replace recursion by fold_left in any tail primitive recursive definition

Continuations

- A programming technique for all forms of “non-local” control flow:
  - non-local jumps
  - exceptions
  - general conversion of non-tail calls to tail calls
- Essentially it’s a higher-order function version of GOTO

Continuations

- Idea: Use functions to represent the control flow of a program
- Method: Each procedure takes a function as an extra argument to which to pass its result; outer procedure "returns" no result
- Function receiving the result called a continuation
- Continuation acts as "accumulator" for work still to be done

Continuation Passing Style

- Writing procedures such that all procedure calls take a continuation to which to give (pass) the result, and return no result, is called continuation passing style (CPS)

Continuation Passing Style

- A compilation technique to implement non-local control flow, especially useful in interpreters.
- A formalization of non-local control flow in denotational semantics
- Possible intermediate state in compiling functional code

Why CPS?

- Makes order of evaluation explicitly clear
- Allocates variables (to become registers) for each step of computation
- Essentially converts functional programs into imperative ones
  - Major step for compiling to assembly or byte code
- Tail recursion (and forward recursion) easily identified

Other Uses for Continuations

- CPS designed to preserve order of evaluation
- Continuations used to express order of evaluation
- Can be used to change order of evaluation
- Implements:
  - Exceptions and exception handling
  - Co-routines
  - (pseudo, aka green) threads

Example

- Simple reporting continuation:

```
# let report x = (print_int x; print_newline( ) );;
val report : int -> unit = <fun>
```

- Simple function using a continuation:

```
# let addk (a, b) k = k (a + b);;
val addk : int * int -> (int -> ’a) -> ’a = <fun>
# addk (22, 20) report;;
42
- : unit = ()
```

Simple Functions Taking Continuations

- Given a primitive operation, can convert it to pass its result forward to a continuation
- Examples:

```
# let subk (x, y) k = k(x - y);;
val subk : int * int -> (int -> 'a) -> 'a = <fun>
# let eqk (x, y) k = k(x = y);;
val eqk : 'a * 'a -> (bool -> 'b) -> 'b = <fun>
# let timesk (x, y) k = k(x * y);;
val timesk : int * int -> (int -> 'a) -> 'a = <fun>
```


Nesting Continuations

```
# let add_triple (x, y, z) = (x + y) + z;;
val add_triple : int * int * int -> int = <fun>
# let add_triple (x,y,z)=let p = x + y in p + z;;
val add_triple : int * int * int -> int = <fun>
# let add_triple_k (x, y, z) k =
  addk (x, y) (fun p -> addk (p, z) k );;
val add_triple_k: int * int * int -> (int -> 'a) ->
  'a = <fun>
```


add_three: a different order

- # let add_triple (x, y, z) = x + (y + z);;
- How do we write add_triple_k to use a different order?
- let add_triple_k (x, y, z) k =

add_three: a different order

- # let add_triple (x, y, z) = x + (y + z);;
- How do we write add_triple_k to use a different order?
- let add_triple_k (x, y, z) k =
  addk (y,z) (fun r -> addk(x,r) k)

Recursive Functions

- Recall:

```
# let rec factorial n =
   if n = 0 then 1 else n * factorial (n - 1);;
  val factorial : int -> int = <fun>
# factorial 5;;
- : int = 120
```


Terms

- A function is in Direct Style when it returns its result back to the caller.
- A function is in Continuation Passing Style when it, and every function call in it, passes its result to another function.
- Instead of returning the result to the caller, we pass it forward to another function giving the computation after the call.

Recursive Functions

```
# let rec factorial n =
    let b = (n = 0) in (* First computation *)
    if b then 1 (* Returned value *)
    else let s = n – 1 in (* Second computation *)
         let r = factorial s in  (* Third computation *)
         n * r (* Returned value *) ;;
val factorial : int -> int = <fun>
# factorial 5;;
- : int = 120
```


Recursive Functions

```
# let rec factorialk n k =
   eqk (n, 0)
   (fun b ->  (* First computation *)
    if b then k 1 (* Passed value *)
    else subk (n, 1)  (* Second computation *)
    (fun s -> factorialk s  (* Third computation *)
      (fun r -> timesk (n, r) k))) (* Passed value *)
val factorialk : int -> (int -> 'a) -> 'a = <fun>
# factorialk 5 report;;
120
- : unit = ()
```


Recursive Functions

- To make recursive call, must build intermediate continuation to
  - take recursive value: r
  - build it to final result: n * r
  - And pass it to final continuation:
  - times (n, r) k = k (n * r)

Recursive Functions


```
# let rec factorialk n k =
   eqk (n, 0)
   (fun b ->  (* First computation *)
    if b then k 1 (* Passed value *)
    else subk (n, 1)  (* Second computation *)
    (fun s -> factorialk s  (* Third computation *)
      (fun r -> timesk (n, r) k))) (* Passed value *)
val factorialk : int -> (int -> 'a) -> 'a = <fun>
# factorialk 5 report;;
120
- : unit = ()
```


Example: CPS for length

```
let rec length list = match list with [] -> 0
    | (a :: bs) -> 1 + length bs
```

What is the let-expanded version of this?