

Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Functions

```
# let plus_two n = n + 2;;
```

```
val plus_two : int -> int = <fun>
```

```
# plus_two 17;;
```

```
- : int = 19
```

```
# let plus_two = fun n -> n + 2;;
```

```
val plus_two : int -> int = <fun>
```

```
# plus_two 14;;
```

```
- : int = 16
```

First definition syntactic sugar for second

Using a nameless function

```
# (fun x -> x * 3) 5;; (* An application *)
```

```
- : int = 15
```

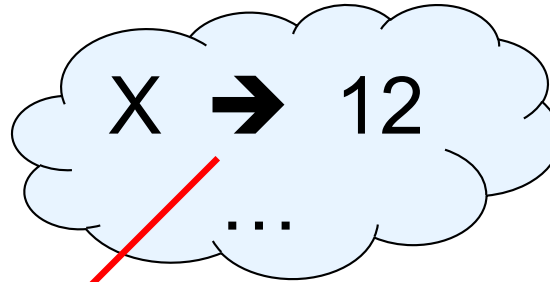
```
# ((fun y -> y +. 2.0), (fun z -> z * 3));;  
(* As data *)
```

```
- : (float -> float) * (int -> int) = (<fun>,  
<fun>)
```

Note: in `fun v -> exp(v)`, scope of variable is only the body `exp(v)`

Values fixed at declaration time

```
# let x = 12;;
```



```
val x : int = 12
```

```
# let plus_x y = y + x;;
```

```
val plus_x : int -> int = <fun>
```

```
# plus_x 3;;
```

What is the result?



Values fixed at declaration time

```
# let x = 12;;
```

```
val x : int = 12
```

```
# let plus_x y = y + x;;
```

```
val plus_x : int -> int = <fun>
```

```
# plus_x 3;;
```

```
- : int = 15
```



Values fixed at declaration time

```
# let x = 7;; (* New declaration, not an  
update *)
```

```
val x : int = 7
```

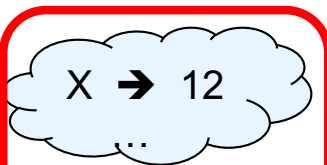
```
# plus_x 3;;
```

What is the result this time?

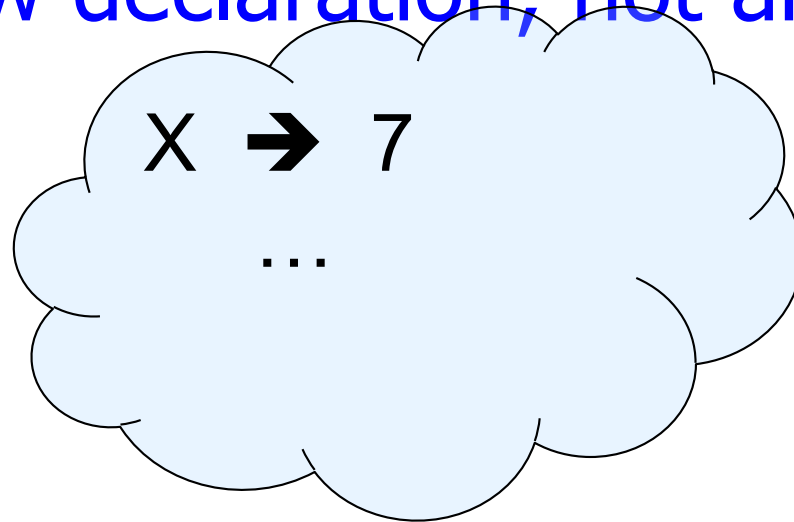
Values fixed at declaration time

```
# let x = 7;; (* New declaration, not an  
update *)
```

```
val x : int = 7
```



```
# plus_x 3;;
```



What is the result this time?



Values fixed at declaration time

```
# let x = 7;; (* New declaration, not an  
update *)
```

```
val x : int = 7
```

```
# plus_x 3;;
```

```
- : int = 15
```




Question

- Observation: Functions are first-class values in this language
- Question: What value does the environment record for a function variable?
- Answer: a closure



Save the Environment!

- A *closure* is a pair of an environment and an association of a formal parameter (the input variables)* with an expression (the function body), written:

$$f \rightarrow \langle (v_1, \dots, v_n) \rightarrow \text{exp}, \rho_f \rangle$$

- Where ρ_f is the environment in effect when f is defined (if f is a simple function)
- * Will come back to the “formal parameter”

Closure for plus_x

- When plus_x was defined, had environment:

$$\rho_{\text{plus_x}} = \{\dots, x \rightarrow 12, \dots\}$$

- Recall: `let plus_x y = y + x`

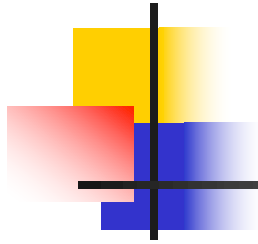
is really `let plus_x = fun y -> y + x`

- Closure for `fun y -> y + x`:

$$\langle y \rightarrow y + x, \rho_{\text{plus_x}} \rangle$$

- Environment just after plus_x defined:

$$\{\text{plus_x} \rightarrow \langle y \rightarrow y + x, \rho_{\text{plus_x}} \rangle\} + \rho_{\text{plus_x}}$$



Now it's your turn

You should be able to
complete ACT1



Functions with more than one argument

```
# let add_three x y z = x + y + z;;
```

```
val add_three : int -> int -> int -> int = <fun>
```

```
# let t = add_three 6 3 2;;
```

```
val t : int = 11
```

```
# let add_three =
```

```
  fun x -> (fun y -> (fun z -> x + y + z));;
```

```
val add_three : int -> int -> int -> int = <fun>
```

Again, first syntactic sugar for second



Functions with more than one argument

```
# let add_three x y z = x + y + z;;
```

```
val add_three : int -> int -> int -> int = <fun>
```

- What is the value of add_three?
- Let $\rho_{\text{add_three}}$ be the environment before the declaration
- Remember:

```
let add_three =
```

```
  fun x -> (fun y -> (fun z -> x + y + z));;
```

```
Value: <x ->fun y -> (fun z -> x + y + z),  $\rho_{\text{add\_three}}$  >
```



Partial application of functions

```
let add_three x y z = x + y + z;;
```

```
# let h = add_three 5 4;;
```

```
val h : int -> int = <fun>
```

```
# h 3;;
```

```
- : int = 12
```

```
# h 7;;
```

```
- : int = 16
```



Partial application of functions

```
let add_three x y z = x + y + z;;
```

```
# let h = add_three 5 4;;
```

```
val h : int -> int = <fun>
```

```
# h 3;;
```

```
- : int = 12
```

```
# h 7;;
```

```
- : int = 16
```

```
- Partial application also called sectioning
```




Functions as arguments

```
# let thrice f x = f (f (f x));;
```

```
val thrice : ('a -> 'a) -> 'a -> 'a = <fun>
```

```
# let g = thrice plus_two;;
```

```
val g : int -> int = <fun>
```

```
# g 4;;
```

```
- : int = 10
```

```
# thrice (fun s -> "Hi! " ^ s) "Good-bye!";;
```

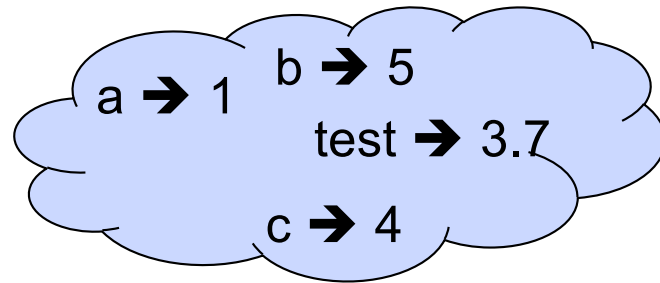
```
- : string = "Hi! Hi! Hi! Good-bye!"
```

Tuples as Values

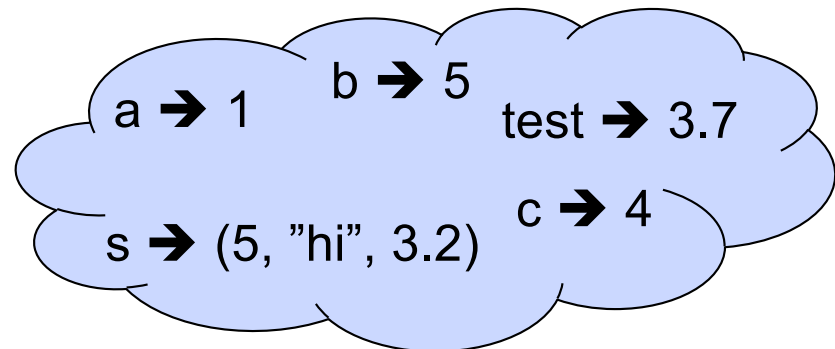
```
// ρ7 = {c → 4, test → 3.7,  
          a → 1, b → 5}
```

```
# let s = (5, "hi", 3.2);;
```

```
val s : int * string * float = (5, "hi", 3.2)
```

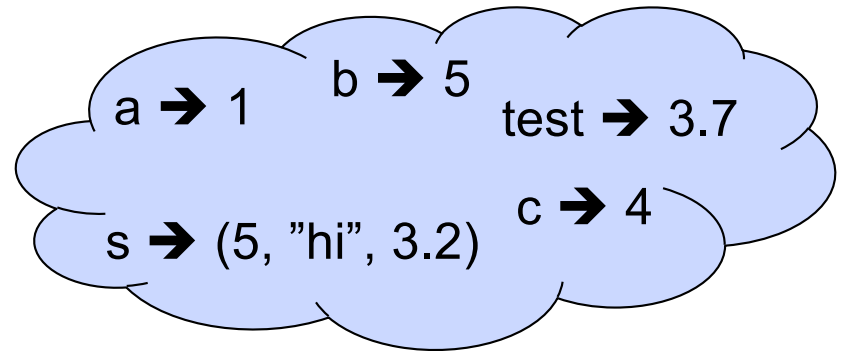


```
// ρ8 = {s → (5, "hi", 3.2),  
          c → 4, test → 3.7,  
          a → 1, b → 5}
```



Pattern Matching with Tuples

```
/ ρ8 = {s → (5, "hi", 3.2),  
         c → 4, test → 3.7,  
         a → 1, b → 5}
```

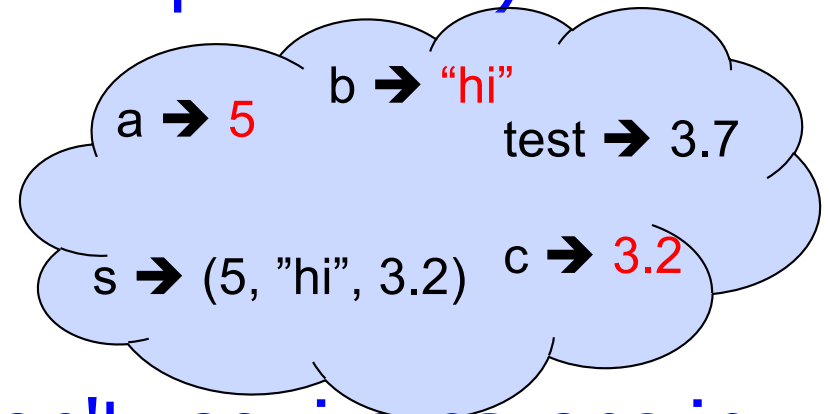


```
# let (a,b,c) = s;; (* (a,b,c) is a pattern *)
```

```
val a : int = 5
```

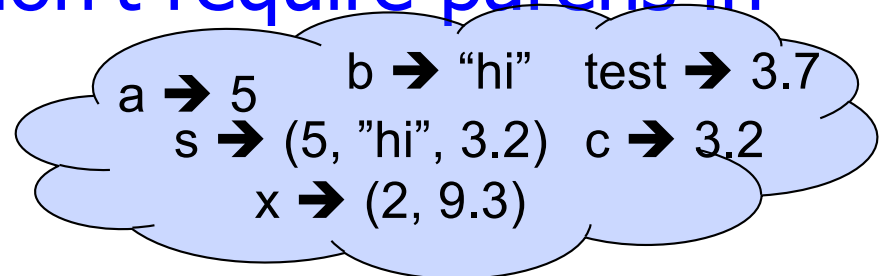
```
val b : string = "hi"
```

```
val c : float = 3.2
```



```
# let x = 2, 9.3;; (* tuples don't require parens in  
                  Ocaml *)
```

```
val x : int * float = (2, 9.3)
```





Nested Tuples

```
# (*Tuples can be nested *)
```

```
let d = ((1,4,62),("bye",15),73.95);;
```

```
val d : (int * int * int) * (string * int) * float =  
  ((1, 4, 62), ("bye", 15), 73.95)
```

```
# (*Patterns can be nested *)
```

```
let (p,(st,_),_) = d;; (* _ matches all, binds nothing  
*)
```

```
val p : int * int * int = (1, 4, 62)
```

```
val st : string = "bye"
```



Functions on tuples

```
# let plus_pair (n,m) = n + m;;
```

```
val plus_pair : int * int -> int = <fun>
```

```
# plus_pair (3,4);;
```

```
- : int = 7
```

```
# let double x = (x,x);;
```

```
val double : 'a -> 'a * 'a = <fun>
```

```
# double 3;;
```

```
- : int * int = (3, 3)
```

```
# double "hi";;
```

```
- : string * string = ("hi", "hi")
```



Match Expressions

```
# let triple_to_pair triple =
```

```
  match triple
```

```
  with (0, x, y) -> (x, y)
```

```
  | (x, 0, y) -> (x, y)
```

```
  | (x, y, _) -> (x, y);;
```

- Each clause: pattern on left, expression on right
- Each x, y has scope of only its clause
- Use first matching clause

```
val triple_to_pair : int * int * int -> int * int =  
  <fun>
```



Closure for plus_pair

- Assume $\rho_{\text{plus_pair}}$ was the environment just before `plus_pair` defined

- Closure for `plus_pair`:

$$\langle (n,m) \rightarrow n + m, \rho_{\text{plus_pair}} \rangle$$

- Environment just after `plus_pair` defined:

$$\{\text{plus_pair} \rightarrow \langle (n,m) \rightarrow n + m, \rho_{\text{plus_pair}} \rangle\}$$

$$+ \rho_{\text{plus_pair}}$$



Save the Environment!

- A *closure* is a pair of an environment and an association of a pattern (e.g. (v_1, \dots, v_n) giving the input variables) with an expression (the function body), written:

$$\langle (v_1, \dots, v_n) \rightarrow \underline{\text{exp}}, \rho \rangle$$

- Where ρ is the environment in effect when the function is defined (for a simple function)