Programming Languages and Compilers (CS 421)

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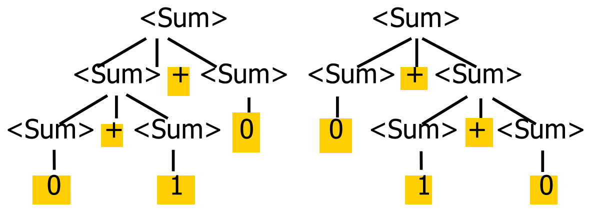
Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF’s for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the value of:

7 – 5 – 2

Example

- What is the value of:

7 – 5 – 2

- Possible answers:
  - In Pascal, C++, SML assoc. left

    7 – 5 – 2 = (7 – 5) – 2 = 0
  - In APL, associate to right

    7 – 5 – 2 = 7 – (5 – 2) = 4

Example

- What is the result for:

3 + 4 * 5 + 6

Example

- What is the result for:

  3 + 4 * 5 + 6

- Possible answers:
  - 41 = ((3 + 4) * 5) + 6
  - 47 = 3 + (4 * (5 + 6))
  - 29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)
  - 77 = (3 + 4) * (5 + 6)

Disambiguating a Grammar

- Given ambiguous grammar G, with start symbol S, find a grammar G′ with same start symbol, such that

  language of G = language of G′

- Not always possible
- No algorithm in general

Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
  - In fact each string of terminals and non-terminals represents all strings having some property (Remember reg exp semantics)
- Identify these properties (often in terms of things that can't happen)
- Use these properties to inductively guarantee every string in language has a unique parse

Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- **Characterize each non-terminal by a language invariant**
- Replace old rules to use new non-terminals
- Rinse and repeat

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoiativity
- Not the only sources of ambiguity
  - Will come back to this

How to Enforce Associativity

- Have at most one recursive call per production
- When two or more recursive calls would be natural leave right-most one for right assoicativity, left-most one for left assoiciativity

Example

- <Sum> ::= 0 | 1 | <Sum> + <Sum>
  | (<Sum>)
- Becomes
  - <Sum> ::= <Num> | <Num> + <Sum>
  - <Num> ::= 0 | 1 | (<Sum>)

<Sum> + <Sum> + <Sum>

Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Precedence Table - Sample

	Fortan	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, /, *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
	+, -		+, -	+, -	::

Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:

<exp> ::= 0 | 1 | <exp> * <exp>
| <exp> + <exp>

- Becomes

<exp> ::= <mult_exp>
| <exp> + <mult_exp>
<mult_exp> ::= <id> | <mult_exp> * <id>
<id> ::= 0 | 1

More Disambiguating Grammars

- M ::= M * M | (M) | M ++ | 6
- Ambiguous because of associativity of *
- because of conflict between * and ++:
- 6 * 6 ++ 6 * 6 ++

M ::= M * M | (M) | M ++ | 6

- How to disambiguate?
- Choose associativity for *
- Choose precedence between * and ++
- Four possibilities
- Four different approaches
- Some easier than others
- Will do --- My choice

M ::= M * M | (M) | M ++ | 6

- Think about 6 * 6 ++ * 6 * 6 ++
- Let's start with observations
- If * binds less tightly than ++, then no * can be the immediate subtree to a ++.
  - We would need a language for things that don't parse as *
- If * binds more tightly than ++, then …
- The right subtree to * can't be a ++
- But the left can!
  - Need different languages of the left and right

M ::= M * M | (M) | M ++ | 6

- * higher prec than ++
  - 6 * 6 ++ * 6 ++ * 6
- M :: = M++ | StarExp | (M) | 6
- What is StarExp
- It is everything that parses as a * and can't parse as a ++
- But what is the associativity of *?
- I'll choose right

M ::= M * M | (M) | M ++ | 6

- * higher prec than ++
  - 6 * 6 ++ * 6 ++ * 6
- * Right assoc but ++ takes all to its left
- M :: = M++ | StarExp | (M) | 6
- StarExp ::= NotStar * PossStarNotPlusPlus
- What is NotStar? It can't b *, but can be 6 or (M) ….
- Can it be ++? YES!
- NotStar ::= M++ | (M) | 6

M ::= M * M | (M) | M ++ | 6

- * higher prec than ++, * right assoc
- StarExp ::= NotStar * PossStarNoPlusPlus
- What is PossStarNoPlusPlus?
- PossStarNoPlusPlus ::=
  (M) | 6
- | NotStarNoPlusPlus * PossStarNoPlusPlus
- But what is NotStarNotPlusPlus?
- Well, the other two original rules: (M) | 6

M ::= M * M | (M) | M ++ | 6

- * higher prec than ++, * right assoc
- M :: = StarExp |M++ | (M) | 6
- NotStar ::= M++ | (M) | 6
- StarExp ::= NotStar * PossStarNoPlusPlus
- PossStarNoPlusPlus ::=
  (M) | 6
  | NotStarNoPlusPlus * PossStarNoPlusPlus
- NotStarNotPlusPlus ::=(M) | 6

M ::= M * M | (M) | M ++ | 6

- * higher prec than ++, * right assoc
- M :: = StarExp | NotStar
  NotStar ::= M++ | NotStarNotPlusPlus
  StarExp ::= NotStar * PossStarNoPlusPlus
  PossStarNoPlusPlus ::=
  NotStarNotPlusPlus
  | NotStarNoPlusPlus * PossStarNoPlusPlus
  NotStarNotPlusPlus ::=(M) | 6

Parser Code

- <*grammar*>.ml defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point

Ocamlyacc Input

- File format:

%{
    <*header*>
%}
    <*declarations*>
%%
    <*rules*>
%%
    <*trailer*>

Ocamlyacc <*header*>

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- <*footer*> similar. Possibly used to call parser

Ocamlyacc <declarations>

- %token *symbol … symbol*
- Declare given symbols as tokens
- %token <*type*> *symbol … symbol*
- Declare given symbols as token constructors, taking an argument of type <*type*>
- %start *symbol … symbol*
- Declare given symbols as entry points; functions of same names in <*grammar*>.ml

Ocamlyacc <*declarations*>

- %type <*type*> *symbol … symbol*

  Specify type of attributes for given symbols. Mandatory for start symbols
- %left *symbol … symbol*
- %right *symbol … symbol*
- %nonassoc *symbol … symbol*

  Associate precedence and associativity to given symbols. Same line,same precedence; earlier line, lower precedence (broadest scope)

Ocamlyacc <*rules*>

- *nonterminal* :
  symbol ... symbol { *semantic_action* }
  | ...
  | *symbol ... symbol* { *semantic_action* }
  ;
- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: $1 for first symbol, $2 to second …

Example - Base types

```
(* File: expr.ml *)
type expr =
   Term_as_Expr of term
 | Plus_Expr of (term * expr)
 | Minus_Expr of (term * expr)
and term =
   Factor_as_Term of factor
 | Mult_Term of (factor * term)
 | Div_Term of (factor * term)
and factor =
   Id_as_Factor of string
 | Parenthesized_Expr_as_Factor of expr
```


Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter =['a' - 'z' 'A' - 'Z']
rule token = parse
  | "+" {Plus_token}
  | "-"  {Minus_token}
  | "*"  {Times_token}
  | "/"  {Divide_token}
  | "("  {Left_parenthesis}
  | ")"  {Right_parenthesis}
  | letter (letter|numeric|"_")* as id  {Id_token id}
  | [' ' '\t' '\n'] {token lexbuf}
  | eof {EOL}
```


Example - Parser (exprparse.mly)

```
%{ open Expr
%}
%token <string> Id_token
%token Left_parenthesis Right_parenthesis
%token Times_token Divide_token
%token Plus_token Minus_token
%token EOL
%start main
%type <expr> main
%%
```


Example - Parser (exprparse.mly)

```
expr:
  term
     { Term_as_Expr $1 }
 | term Plus_token expr
     { Plus_Expr ($1, $3) }
 | term Minus_token expr
     { Minus_Expr ($1, $3) }
```


Example - Parser (exprparse.mly)

```
term:
  factor
     { Factor_as_Term $1 }
 | factor Times_token term
     { Mult_Term ($1, $3) }
 | factor Divide_token term
     { Div_Term ($1, $3) }
```


Example - Parser (exprparse.mly)

```
factor:
  Id_token
     { Id_as_Factor $1 }
 | Left_parenthesis expr Right_parenthesis
     {Parenthesized_Expr_as_Factor $2 }
main:
 | expr EOL
     { $1 }
```

Example - Using Parser

```
# #use "expr.ml";;
…
# #use "exprparse.ml";;
…
# #use "exprlex.ml";;
…
# let test s =
  let lexbuf = Lexing.from_string (s^"\n") in
      main token lexbuf;;
```


Example - Using Parser

```
# test "a + b";;
- : expr =
Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Term_as_Expr (Factor_as_Term
   (Id_as_Factor "b")))
```


LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=	(<Sum> ●) + 0	shift
=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

=>	(<Sum>) ● + 0	reduce
=	(<Sum> ●) + 0	shift
=>	(<Sum> + <Sum> ●) + 0	reduce
=>	(<Sum> + 1 ●) + 0	reduce
=	(<Sum> + ● 1) + 0	shift
=	(<Sum> ● + 1) + 0	shift
=>	(0 ● + 1) + 0	reduce
=	(● 0 + 1) + 0	shift
=	● (0 + 1) + 0	shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>

= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> =>
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> => <Sum> + <Sum > ● reduce
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
=> (<Sum> + 1 ●) + 0 reduce
= (<Sum> + ● 1) + 0 shift
= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

<Sum> ● => <Sum> + <Sum > ● reduce
=> <Sum> + 0 ● reduce
= <Sum> + ● 0 shift
= <Sum> ● + 0 shift
=> (<Sum>) ● + 0 reduce
= (<Sum> ●) + 0 shift
=> (<Sum> + <Sum> ●) + 0 reduce
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= (<Sum> ● + 1) + 0 shift
=> (0 ● + 1) + 0 reduce
= (● 0 + 1) + 0 shift
= ● (0 + 1) + 0 shift

Example

(0 + 1) + 0

Example

(0 + 1) + 0

Example

(0 + 1) + 0

Example

<Sum>

(0 + 1) + 0

Example

<Sum>

(0 + 1) + 0

Example

<Sum>

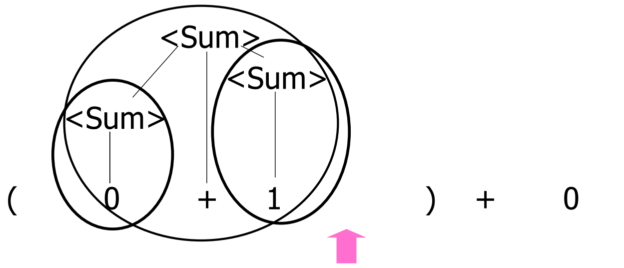
(0 + 1) + 0

Example

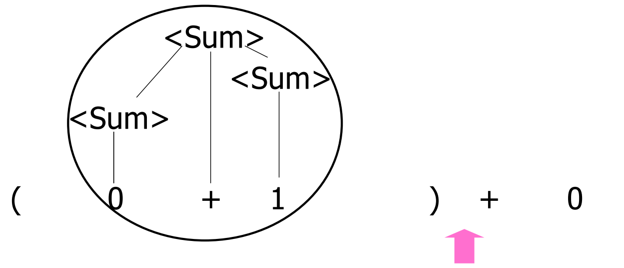
<Sum> <Sum>

(0 + 1) + 0

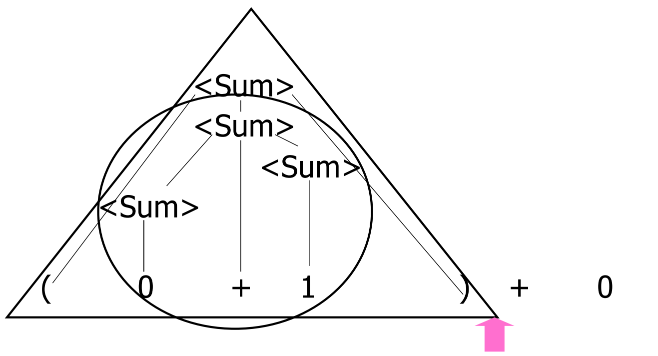
Example



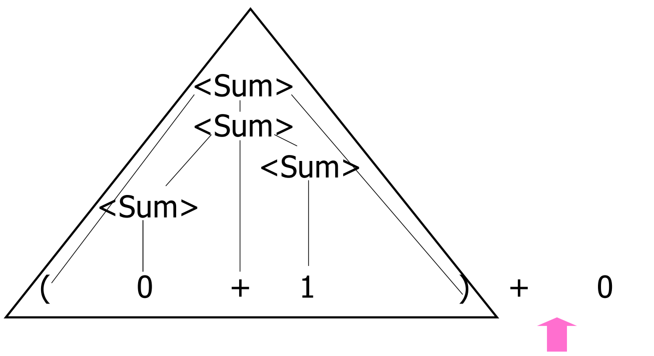
Example



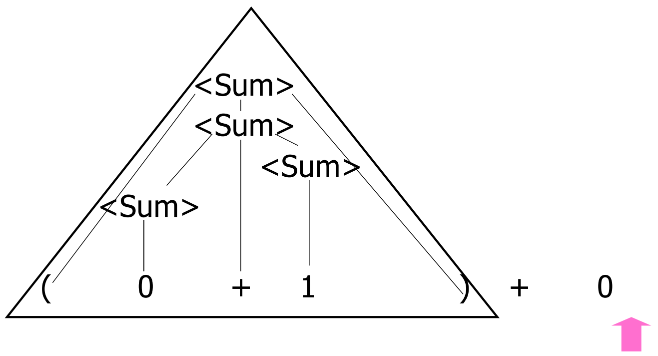
Example



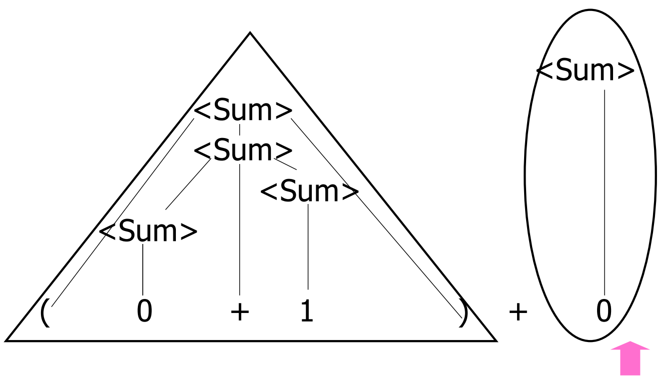
Example



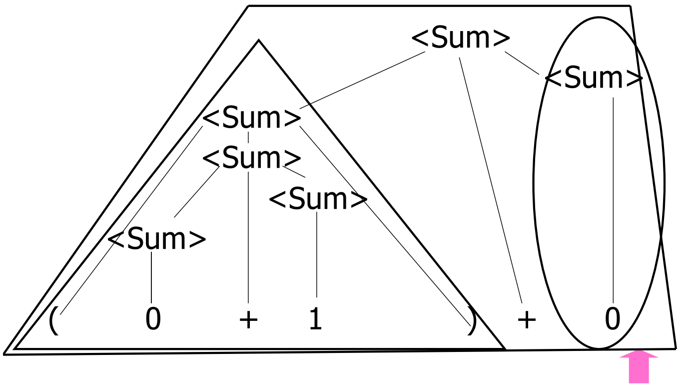
Example



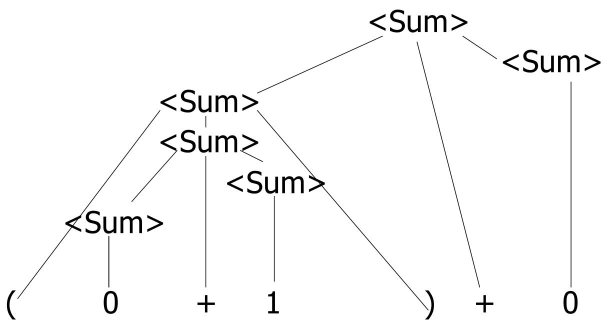
Example



Example



Example



LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and "end-of-tokens" marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals

Action and Goto Tables

- Given a state and the next input, Action table says either
  - **shift** and go to state *n*, or
  - **reduce** by production *k* (explained in a bit)
  - **accept** or **error**
- Given a state and a non-terminal, Goto table says
  - go to state *m*

LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

LR(i) Parsing Algorithm

0. Insure token stream ends in special "end-of-tokens" symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
3. Look at next *i* tokens from token stream (*toks*) (don't remove yet)
4. If top symbol on stack is **state**(*n*), look up action in Action table at (*n*, *toks*)

LR(i) Parsing Algorithm

5. If action = **shift** *m*,
   a) Remove the top token from token stream and push it onto the stack
   b) Push **state**(*m*) onto stack
   c) Go to step 3

LR(i) Parsing Algorithm

6. If action = **reduce** *k* where production *k* is E ::= u
   a) Remove 2 * length(u) symbols from stack (u and all the interleaved states)
   b) If new top symbol on stack is **state**(*m*), look up new state *p* in Goto(*m*,E)
   c) Push E onto the stack, then push **state**(*p*) onto the stack
   d) Go to step 3

LR(i) Parsing Algorithm

7. If action = **accept**
   - Stop parsing, return success
8. If action = **error**,
   - Stop parsing, return failure

Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
  - gather the recorded attributes from each non-terminal popped from stack
  - Compute new attribute for non-terminal pushed onto stack

Shift-Reduce Conflicts

- **Problem**: can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

Example: <Sum> = 0 | 1 | (<Sum>) | <Sum> + <Sum>

● 0 + 1 + 0	shift
-> 0 ● + 1 + 0	reduce
-> <Sum> ● + 1 + 0	shift
-> <Sum> + ● 1 + 0	shift
-> <Sum> + 1 ● + 0	reduce
-> <Sum> + <Sum> ● + 0	

Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

Example

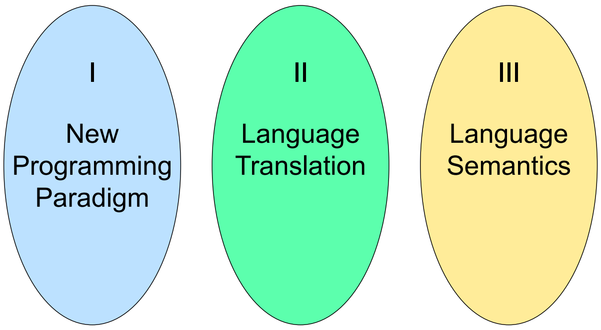
- S ::= A | aB   A ::= abc   B ::= bc

● abc	shift
a ● bc	shift
ab ● c	shift
abc ●	

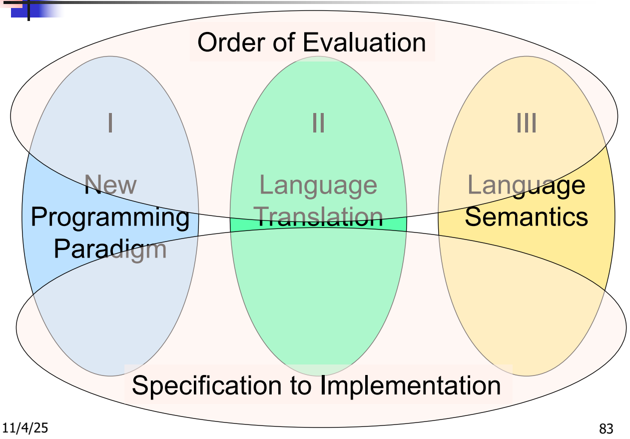
- Problem: reduce by B ::= bc then by S ::= aB, or by A::= abc then S::A?

Programming Languages & Compilers

Three Main Topics of the Course

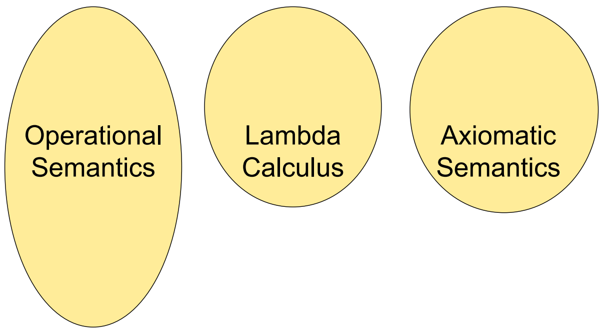


Programming Languages & Compilers

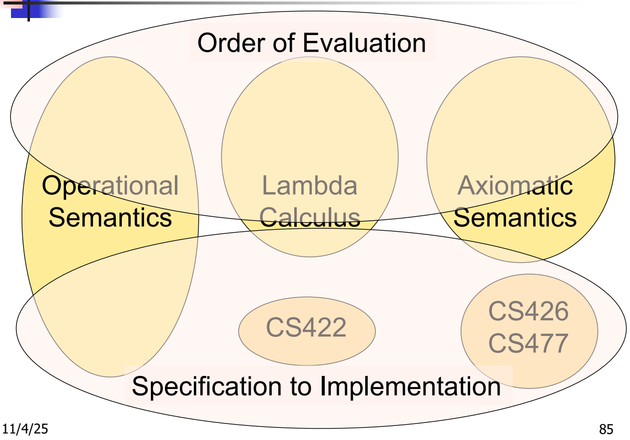


Programming Languages & Compilers

III : Language Semantics



Programming Languages & Compilers



Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :

  {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | C;C | I ::= E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$

  $(\text{false},m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation ~ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \; op \; V = N}{(E \; op \; E', m) \Downarrow N}$$

where N is the specified value for U op V

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment: $\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I \leftarrow V]}$

Sequencing: $\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \; (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: If Then Else Rule

$$\frac{\frac{}{(x > 5, \{x \to 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow ?}$$

Example: Arith Relation

$$\frac{\frac{? > ? = ? \quad (x,\{x\to 7\})\Downarrow ? \quad (5,\{x\to 7\})\Downarrow ?}{(x > 5, \{x \to 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow ?}$$

Example: Identifier(s)

$$\frac{\frac{7 > 5 = \text{true} \quad (x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{(x > 5, \{x \to 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow ?}$$

Example: Arith Relation

$$\frac{\frac{7 > 5 = \text{true} \quad (x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{(x > 5, \{x \to 7\}) \Downarrow \text{true}}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow ?}$$

Example: If Then Else Rule

$$\frac{\frac{7 > 5 = \text{true} \quad (x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{(x > 5, \{x \to 7\}) \Downarrow \text{true}} \quad \frac{}{(y{:=}\ 2 + 3,\{x\to 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow ?}$$

Example: Assignment

$$\frac{\frac{7 > 5 = \text{true} \quad (x,\{x\to 7\})\Downarrow 7 \quad (5,\{x\to 7\})\Downarrow 5}{(x > 5, \{x \to 7\}) \Downarrow \text{true}} \quad \frac{\frac{}{(2+3, \{x\to 7\})\Downarrow ?}}{(y{:=}\ 2 + 3,\{x\to 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow ?}$$

Example: Arith Op

? + ? = ?
(2,{x->7})⇓? (3,{x->7}) ⇓?
7 > 5 = true (2+3, {x->7})⇓?
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})
(x > 5, {x -> 7})⇓true ⇓? .
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Numerals

2 + 3 = 5
(2,{x->7})⇓2 (3,{x->7}) ⇓3
7 > 5 = true (2+3, {x->7})⇓?
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})
(x > 5, {x -> 7})⇓true ⇓ ?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Arith Op

2 + 3 = 5
(2,{x->7})⇓2 (3,{x->7}) ⇓3
7 > 5 = true (2+3, {x->7})⇓5
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})
(x > 5, {x -> 7})⇓true ⇓?
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Assignment

2 + 3 = 5
(2,{x->7})⇓2 (3,{x->7}) ⇓3
7 > 5 = true (2+3, {x->7})⇓5
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})
(x > 5, {x -> 7})⇓true ⇓ {x->7, y->5}
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓?

Example: If Then Else Rule

2 + 3 = 5
(2,{x->7})⇓2 (3,{x->7}) ⇓3
7 > 5 = true (2+3, {x->7})⇓5
(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})
(x > 5, {x -> 7})⇓true ⇓ {x->7, y->5}
(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ {x->7, y->5}

Let in Command

$$\frac{(E,m) \Downarrow v \quad (C, m[I <- v]) \Downarrow m'}{(\text{let } I = E \text{ in } C,\ m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and $m''(I) = m(I)$ if $m(I)$ is defined, and $m''(I)$ is undefined otherwise

Example

$$\frac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3), \{x \text{ -> } 17\}) \Downarrow \text{?}}$$

Example

$$\frac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3), \{x \text{ -> } 17\}) \Downarrow \{x\text{->}17\}}$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =
  if compute_exp (b,m) = Bool(true)
  then compute_com (c1,m)
  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then