

Solutions for Sample Questions for Midterm 2 (CS 421 Fall 2025)

Some of these questions may be reused for the exam.

1. Put the following function in full continuation passing style:

```
let rec sum_odd n = if n <= 0 then 0 else ((2 * n) - 1) + sum_odd (n - 1);;
```

Use **addk**, **subk**, **mulk**, **leqk**, for the CPS forms of the primitive operations (+, -, *, <=). All other procedure calls and constructs must be put in CPS.

Solution:

```
let add_k a b k = k(a + b)
```

```
let minus_k a b k = k(a - b)
```

```
let times_k a b k = k(a * b)
```

```
let leq_k a b k = k(a <= b)
```

```
let rec sum_odd_k n k =  
  leq_k n 0 (fun b -> if b then k 0  
    else minus_k n 1  
    (fun d -> sum_odd_k d  
      (fun r -> times_k 2 n  
        (fun t -> minus_k t 1  
          (fun m -> add_k m r k )))))
```

2. Given the following OCAML datatype:

```
type int_seq = Null | Snoc of (int_seq * int)
```

write a tail-recursive function in OCAML **all_pos : int_seq -> bool** that returns **true** if every integer in the input **int_seq** to which **all_pos** is applied is strictly greater than 0 and **false** otherwise. Thus **all_pos (Snoc(Snoc(Snoc(Null, 3), 5), 7))** should return **true**, but **all_pos (Snoc(Null, -1))** and **all_pos (Snoc(Snoc(Null, 3), 0))** should both return **false**.

Solution:

```
let rec all_pos s =  
  (match s with Null -> true  
   | Snoc(seq, x) -> if x <= 0 then false else all_pos seq);;
```

3. Write the definition of an OCAML variant type **reg_exp** to express abstract syntax trees for regular expressions over a base character set of booleans. Thus, a boolean is a **reg_exp**, epsilon is a **reg_exp**, a parenthesized **reg_exp** is a **reg_exp**, the concatenation of two **reg_exp**'s is a **reg_exp**, the "choice" of two **reg_exp**'s is a **reg_exp**, and the Kleene star of a **reg_exp** is a **reg_exp**.

Solution:

```
type reg_exp =  
  Char of bool  
  | Epsilon  
  | Paren of reg_exp
```

- | Concat of (reg_exp * reg_exp)
- | Choice of (reg_exp * reg_exp)
- | Kleene_star of reg_exp

4. Given the following rules for CPS transformation:

$[[x]] K \Rightarrow K x$

$[[c]] K \Rightarrow K c$

$[[\text{let } x = e_1 \text{ in } e_2]] K \Rightarrow [[e_1]] (\text{FN } x \rightarrow [[e_2]] K)$

$[[e_1 \oplus e_2]] K \Rightarrow [[e_2]] (\text{FN } a \rightarrow [[e_1]] (\text{FN } b \rightarrow K (b \oplus a)))$

where e_1 and e_2 are OCaml expressions, K is any continuation, x is a variable and c is a constant, give the step-by-step transformation of

$[[\text{let } x = 2 + 3 \text{ in } x - 4]] \text{REPORTk}$

Solution:

$[[\text{let } x = 2 + 3 \text{ in } x - 4]] \text{REPORTk} \Rightarrow$

$[[2 + 3]] (\text{FN } x \rightarrow [[x - 4]] \text{REPORTk}) \Rightarrow$

$[[2 + 3]] (\text{FN } x \rightarrow [[4]] (\text{FN } n \rightarrow [[x]] (\text{FN } m \rightarrow \text{REPORTk } (m - n)))) \Rightarrow$

$[[2 + 3]] (\text{FN } x \rightarrow [[4]] (\text{FN } n \rightarrow (\text{FN } m \rightarrow \text{REPORTk } (m - n)) x)) \Rightarrow$

$[[2 + 3]] (\text{FN } x \rightarrow (\text{FN } n \rightarrow (\text{FN } m \rightarrow \text{REPORTk } (m - n)) x) 4) \Rightarrow$

$[[3]] (\text{FN } u \rightarrow [[2]] (\text{FN } v \rightarrow (\text{FN } x \rightarrow (\text{FN } n \rightarrow (\text{FN } m \rightarrow \text{REPORTk } (m - n)) x) 4) (v + u))) \Rightarrow$

$[[3]] (\text{FN } u \rightarrow (\text{FN } v \rightarrow (\text{FN } x \rightarrow (\text{FN } n \rightarrow (\text{FN } m \rightarrow \text{REPORTk } (m - n)) x) 4) (v + u)) 2) \Rightarrow$

$(\text{FN } u \rightarrow (\text{FN } v \rightarrow (\text{FN } x \rightarrow (\text{FN } n \rightarrow (\text{FN } m \rightarrow \text{REPORTk } (m - n)) x) 4) (v + u)) 2) 3$

5. Review and be able to write any give clause of **cps_exp** from MP5. On the exam, you would be given all the information you were given in MP5.

Solution:

(* Problem 5 *)

let rec cps_exp e k =

 match e with

(* [[x]] k = k x *)

 VarExp x -> VarCPS (k, x)

(* [[c]] k = k c *)

 ConstExp n -> ConstCPS (k, n)

(* [[~ e]] k = [[e]]_ (fun r -> k (~ r)) *)

 MonOpAppExp (m, e) ->

 let r = freshFor (freeVarsInContCPS k)

 in cps_exp e (FnContCPS (r, MonOpAppCPS (k, m, r)))

(* [[(e1 + e2)]] k = [[e2]]_ fun s -> [[e1]]_ fun r -> k (r + s) *)

 BinOpAppExp (b, e1, e2) ->

 let v2 = freshFor (freeVarsInContCPS k @ freeVarsInExp e1) in

 let v1 = freshFor (v2 :: (freeVarsInContCPS k)) in

 let e2CPS =

 cps_exp e1 (FnContCPS (v1, BinOpAppCPS (k, b, v1, v2))) in

 cps_exp e2 (FnContCPS (v2, e2CPS))

```

(*[[if e1 then e2 else e3]]k = [[e1]]_(fun r -> if r then [[e2]]k else [[e3]]k)*)
| IfExp (e1,e2,e3) ->
  let r = freshFor (freeVarsInContCPS k @
    freeVarsInExp e2 @ freeVarsInExp e3) in
  let e2cps = cps_exp e2 k in
  let e3cps = cps_exp e3 k in
  cps_exp e1 (FnContCPS(r, IfCPS(r, e2cps, e3cps)))
(*[[e1 e2]]k = [[e2]]_fun v2 -> [[e1]]_fun v1 -> k (v1 v2)*)
| AppExp (e1,e2) ->
  let v2 = freshFor (freeVarsInContCPS k @ freeVarsInExp e1) in
  let v1 = freshFor (v2 :: freeVarsInContCPS k) in
  let e1cps = cps_exp e1 (FnContCPS (v1, AppCPS(k, v1, v2))) in
  cps_exp e2 (FnContCPS (v2, e1cps))
(*[[fun x -> e]]k = k(fnk x kx -> [[e]]kx) *)
| FunExp (x,e) ->
  let ecps = cps_exp e (ContVarCPS Kvar) in
  FunCPS (k, x, Kvar, ecps)
(*[[let x = e1 in e2]]k = [[e1]]_fun x -> [[e2]]k) *)
| LetInExp (x,e1,e2) ->
  let e2cps = cps_exp e2 k in
  let fx = FnContCPS (x, e2cps) in
  cps_exp e1 fx
(*[[let rec f x = e1 in e2]]k = (FN f -> [[e2]]_k)(FIX f. FUN x -> fn kx => [[e1]]kx) *)
| LetRecInExp(f,x,e1,e2) ->
  let e1cps = cps_exp e1 (ContVarCPS Kvar) in
  let e2cps = cps_exp e2 k in
  FixCPS(FnContCPS (f,e2cps),f,x,Kvar,e1cps)

```

6. Given a polymorphic type derivation for $\{\} \vdash \text{let id} = \text{fun } x \rightarrow x \text{ in id id true} : \text{bool}$

Solution:

Let $\Gamma = \{\text{id} : \forall 'a. 'a \rightarrow 'a\}$

		Instance: $\{ 'a \rightarrow \text{bool} \rightarrow \text{bool} \}$		
Var -----	$\Gamma \vdash \text{id} : (\text{bool} \rightarrow \text{bool}) \rightarrow$	Instance: $\{ 'a \rightarrow \text{bool} \}$	Var -----	
	$\text{bool} \rightarrow \text{bool}$		$\Gamma \vdash \text{id} : \text{bool} \rightarrow \text{bool}$	Const
Var -----	App-----		App-----	Const
$\{ x : 'a \} \vdash x : 'a$	$\Gamma \vdash \text{id id} : \text{bool} \rightarrow \text{bool}$		$\Gamma \vdash \text{true} : \text{bool}$	
Fun -----	App-----			
$\{ \} \vdash \text{fun } x \rightarrow x$	$\{ \text{id} : \forall 'a. 'a \rightarrow 'a \} \vdash \text{id id true}$			
Let -----				
	$\{ \} \vdash \text{let id} = \text{fun } x \rightarrow x \text{ in id id true} : \text{bool}$			