Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Terminology: Review

- A function is in Direct Style when it returns its result back to the caller.
- A function is in Continuation Passing Style when it, and every function call in it, passes its result to another function.
- A Tail Call occurs when a function returns the result of another function call without any more computations (eg tail recursion)
- Instead of returning the result to the caller, we pass it forward to another function giving the computation after the call.

CPS Transformation

- Step 1: Add continuation argument to any function definition:
  - let f arg = e $\Rightarrow$ let f arg k = e
  - Idea: Every function takes an extra parameter saying where the result goes
- Step 2: A simple expression in tail position should be passed to a continuation instead of returned:
  - return a $\Rightarrow$ k a
  - Assuming a is a constant or variable.
  - “Simple” = “No available function calls.”

CPS Transformation

- Step 3: Pass the current continuation to every function call in tail position
  - return f arg $\Rightarrow$ f arg k
  - The function "isn't going to return," so we need to tell it where to put the result.

CPS Transformation

- Step 4: Each function call not in tail position needs to be converted to take a new continuation (containing the old continuation as appropriate)
  - return op (f arg) $\Rightarrow$ f arg (fun r -> k(op r))
  - op represents a primitive operation
  - return g(f arg) $\Rightarrow$ f arg (fun r-> g r k)

Example

Before:

```
let rec add_list lst =
match lst with
  [ ] -> 0
| 0 :: xs -> add_list xs
| x :: xs -> (+) x
   (add_list xs);;
```

After:

```
let rec add_listk lst k =
                          (* rule 1 *)
match lst with
| [ ] -> k 0 (* rule 2 *)
| 0 :: xs -> add_listk xs k
                             (* rule 3 *)
| x :: xs -> add_listk xs
            (fun r -> k ((+) x r));;
                       (* rule 4 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                    (* rule 1 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                    (* rule 1 *)

          k false (* rule 2 *)



       k true (* rule 2 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                         (* rule 1 *)

            k false (* rule 2 *)


        k true (* rule 2 *)
          memk (y, xs) k (* rule 3 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                              (* rule 1 *)

               k false (* rule 2 *)

 eqk (x, y)
  (fun b ->   b (* rule 4 *)
          k true (* rule 2 *)
             memk (y, xs) (* rule 3 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                     (* rule 1 *)

          k false (* rule 2 *)

 eqk (x, y)
  (fun b ->if b (* rule 4 *)
then k true (* rule 2 *)
   else memk (y, xs) (* rule 3 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                        (* rule 1 *)
match lst with
| [ ] -> k false (* rule 2 *)
| x :: xs ->
 eqk (x, y)
  (fun b ->if b (* rule 4 *)
then k true (* rule 2 *)
   else memk (y, xs) k (* rule 3 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                        (* rule 1 *)
match lst with
| [ ] -> k false (* rule 2 *)
| x :: xs ->
 eqk (x, y)
  (fun b ->if b (* rule 4 *)
then k true (* rule 2 *)
   else memk (y, xs) k (* rule 3 *)
```

Data type in Ocaml: lists

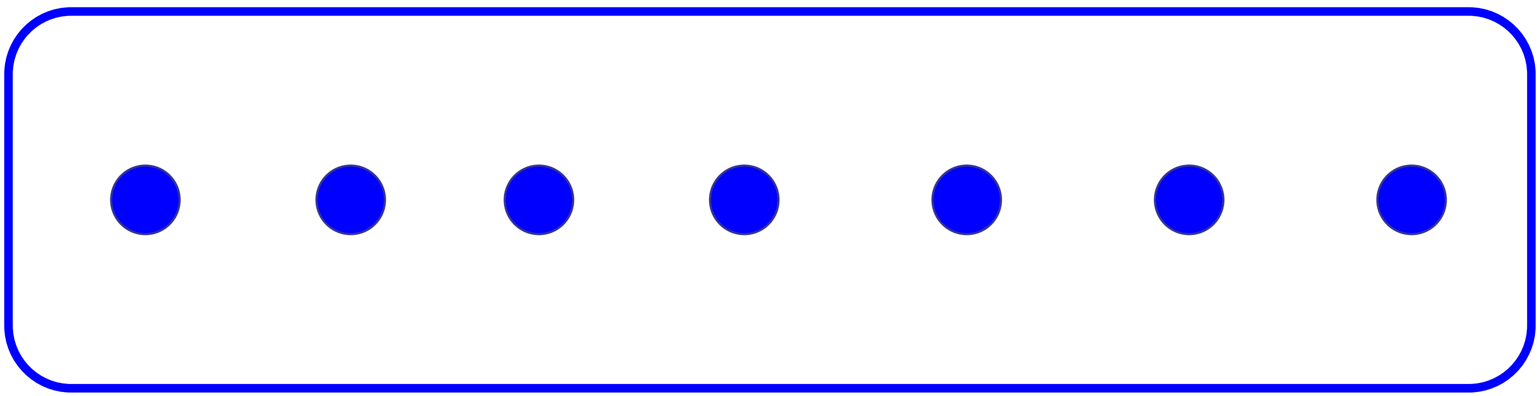
- Frequently used lists in recursive program
- Matched over two structural cases
  - [ ] - the empty list
  - (x :: xs) a non-empty list
- Covers all possible lists
- type ‘a list = [ ] | (::) of ‘a * ‘a list
  - Not quite legitimate declaration because of special syntax

Variants - Syntax (slightly simplified)

- type *name* = C_1 [of ty_1] | . . . | C_n [of ty_n]
- Introduce a type called *name*
- (fun x -> C_i x) : ty_1 -> *name*
- C_i is called a *constructor*; if the optional type argument is omitted, it is called a *constant*
- Constructors are the basis of almost all pattern matching

Enumeration Types as Variants

An enumeration type is a collection of distinct values



In C and Ocaml they have an order structure; order by order of input

Enumeration Types as Variants

```
# type weekday = Monday | Tuesday | Wednesday
   | Thursday | Friday | Saturday | Sunday;;
type weekday =
    Monday
  | Tuesday
  | Wednesday
  | Thursday
  | Friday
  | Saturday
  | Sunday
```

Functions over Enumerations

```
# let day_after day = match day with
     Monday -> Tuesday
  | Tuesday -> Wednesday
  | Wednesday -> Thursday
  | Thursday -> Friday
  | Friday -> Saturday
  | Saturday -> Sunday
  | Sunday -> Monday;;
 val day_after : weekday -> weekday = <fun>
```

Functions over Enumerations

```
# let rec days_later n day =
    match n with 0 -> day
    | _ -> if n > 0
          then day_after (days_later (n - 1) day)
         else days_later (n + 7) day;;
val days_later : int -> weekday -> weekday
  = <fun>
```

Functions over Enumerations

```
# days_later 2 Tuesday;;
- : weekday = Thursday
# days_later (-1) Wednesday;;
- : weekday = Tuesday
# days_later (-4) Monday;;
- : weekday = Thursday
```

Problem:

```
# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```

- Write function is_weekend : weekday -> bool

```
let is_weekend day =
```

Problem:

type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;

- Write function is_weekend : weekday -> bool

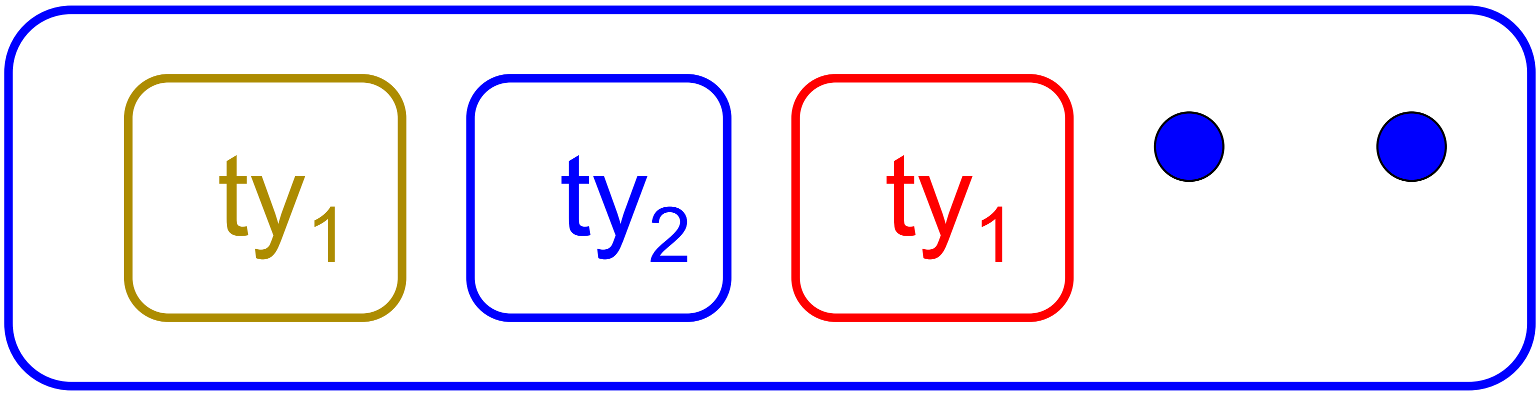
```
let is_weekend day =
    match day with Saturday -> true
     | Sunday -> true
     | _ -> false
```

Example Enumeration Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp

# type mon_op = HdOp | TlOp | FstOp
        | SndOp
```

Disjoint Union Types

- Disjoint union of types, with some possibly occurring more than once



- We can also add in some new singleton elements

Disjoint Union Types

```
# type id = DriversLicense of int
  | SocialSecurity of int | Name of string;;
type id = DriversLicense of int | SocialSecurity
  of int | Name of string
# let check_id id = match id with
     DriversLicense num ->
      not (List.mem num [13570; 99999])
   | SocialSecurity num -> num < 900000000
   | Name str -> not (str = "John Doe");;
 val check_id : id -> bool = <fun>
```

Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

```
type currency =
   Dollar of int
 | Pound of int
 | Euro of int
 | Yen of int
```

Example Disjoint Union Type

```
# type const =
    BoolConst of bool
  | IntConst of int
  | FloatConst of float
  | StringConst of string
  | NilConst
  | UnitConst
```

Example Disjoint Union Type

```
# type const = BoolConst of bool
    | IntConst of int | FloatConst of float
    | StringConst of string  | NilConst
    | UnitConst
```

- How to represent 7 as a const?
- Answer: IntConst 7

Polymorphism in Variants

- The type 'a option is gives us something to represent non-existence or failure

```
# type 'a option = Some of 'a | None;;
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception

Functions producing option

```
# let rec first p list =
    match list with [ ] -> None
    | (x::xs) -> if p x then Some x else first p xs;;
val first : ('a -> bool) -> 'a list -> 'a option = <fun>
# first (fun x -> x > 3) [1;3;4;2;5];;
- : int option = Some 4
# first (fun x -> x > 5) [1;3;4;2;5];;
- : int option = None
```

Functions over option

```
# let result_ok r =
  match r with None -> false
  | Some _ -> true;;
val result_ok : 'a option -> bool = <fun>
# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;
- : bool = true
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;
- : bool = false
```

Problem

- Write a hd and tl on lists that doesn’t raise an exception and works at all types of lists.

Problem

- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.

- let hd list =
    match list with [] -> None
    | (x::xs) -> Some x
- let tl list =
    match list with [] -> None
    | (x::xs) -> Some xs

Mapping over Variants

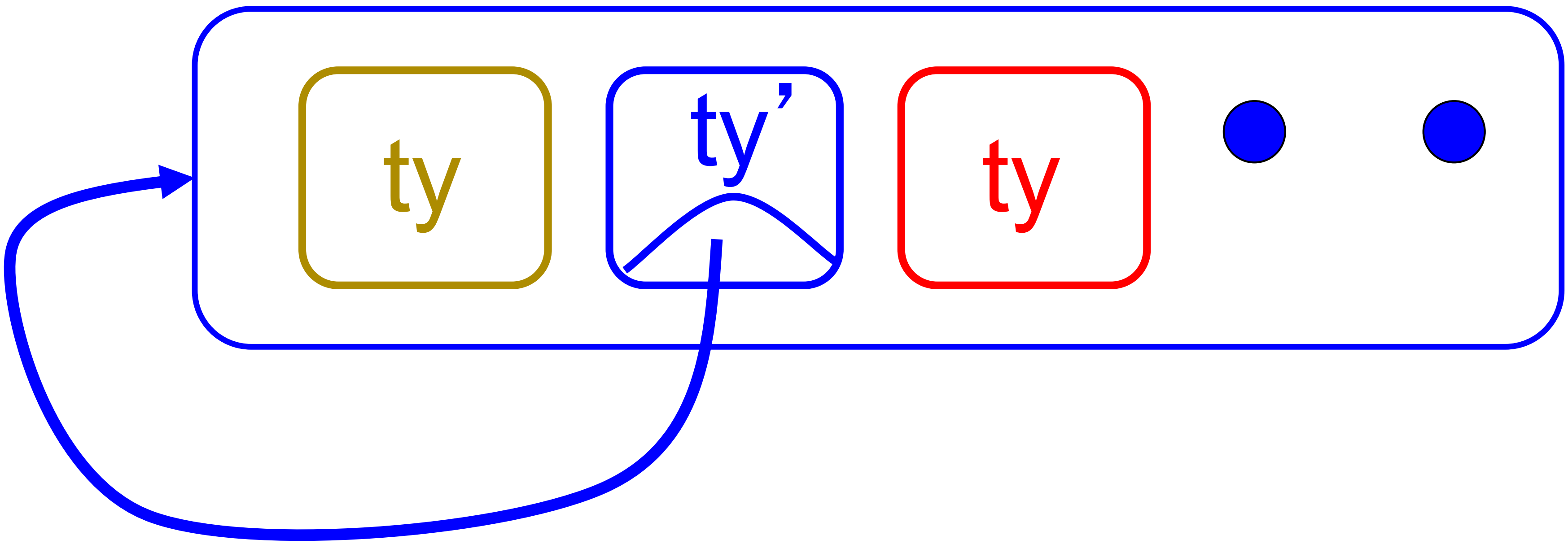
```
# let optionMap f opt =
    match opt with None -> None
    | Some x -> Some (f x);;
val optionMap : ('a -> 'b) -> 'a option -> 'b
  option = <fun>
# optionMap
  (fun x -> x - 2)
  (first (fun x -> x > 3) [1;3;4;2;5]);;
- : int option = Some 2
```

Folding over Variants

```
# let optionFold someFun noneVal opt =
    match opt with None -> noneVal
    | Some x -> someFun x;;
val optionFold : ('a -> 'b) -> 'b -> 'a option ->
  'b = <fun>
# let optionMap f opt =
    optionFold (fun x -> Some (f x)) None opt;;
val optionMap : ('a -> 'b) -> 'a option -> 'b
  option = <fun>
```

Recursive Types

- The type being defined may be a component of itself



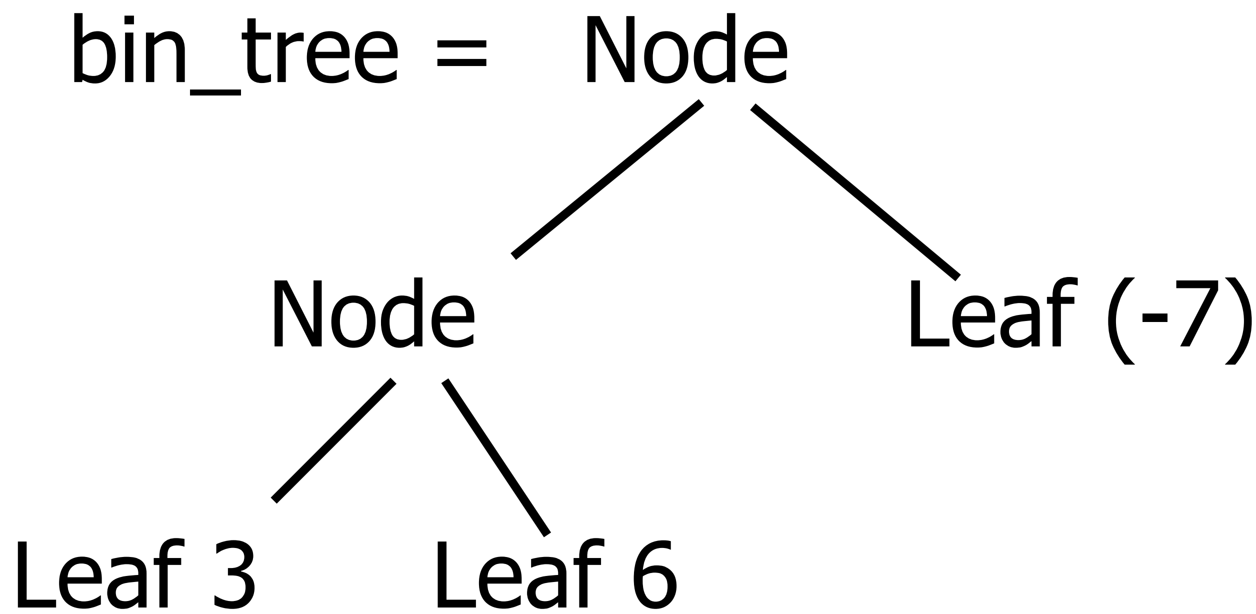
Recursive Data Types

```
# type int_Bin_Tree =
 Leaf of int | Node of (int_Bin_Tree *
  int_Bin_Tree);;

type int_Bin_Tree = Leaf of int | Node of
  (int_Bin_Tree * int_Bin_Tree)
```

Recursive Data Type Values

```
# let bin_tree =
 Node(Node(Leaf 3, Leaf 6),Leaf (-7));;

val bin_tree : int_Bin_Tree = Node (Node
  (Leaf 3, Leaf 6), Leaf (-7))
```

Recursive Data Type Values



Recursive Functions

```
# let rec first_leaf_value tree =
   match tree with (Leaf n) -> n
   | Node (left_tree, right_tree) ->
    first_leaf_value left_tree;;
val first_leaf_value : int_Bin_Tree -> int =
  <fun>
# let left = first_leaf_value bin_tree;;
val left : int = 3
```

Recursive Data Types

```
# type exp =
    VarExp of string
  | ConstExp of const
  | MonOpAppExp of mon_op * exp
  | BinOpAppExp of bin_op * exp * exp
  | IfExp of exp* exp * exp
  | AppExp of exp * exp
  | FunExp of string * exp
```

Recursive Data Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent 6 as an exp?

Recursive Data Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)

Recursive Data Types

```
# type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent (6, 3) as an exp?

Recursive Data Types

\# type bin_op = IntPlusOp | IntMinusOp
| EqOp | CommaOp | ConsOp | …

\# type const = BoolConst of bool | IntConst of int |
…

\# type exp = VarExp of string | ConstExp of const
| BinOpAppExp of bin_op * exp * exp | …

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6),
ConstExp (IntConst 3))

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …

type const = BoolConst of bool | IntConst of int | …

type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp | …

- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp, BinOpAppExp (CommaOp, ConstExp (IntConst 6), ConstExp (IntConst 3)), ConstExp NilConst))));;

Problem

```
type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

```
let rec sum_tree t =
```

Problem

```
type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

```
let rec sum_tree t =
    match t with Leaf n -> n
    | Node(t1,t2) -> sum_tree t1 + sum_tree t2
```

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp
   | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp
  | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =
  match exp with VarExp x ->
   | ConstExp c ->
   | BinOpAppExp (b, e1, e2) ->
   | FunExp (x,e) ->
   | AppExp (e1, e2) ->
```

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp
   | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =
   match exp with VarExp x -> 1
    | ConstExp c -> 0
    | BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2
    | FunExp (x,e) -> 1 + varCnt e
    | AppExp (e1, e2) -> varCnt e1 + varCnt e2
```

Your turn now

Try Problem 3 on MP5

Mapping over Recursive Types

```
# let rec ibtreeMap f tree =
   match tree with (Leaf n) -> Leaf (f n)
   | Node (left_tree, right_tree) ->
    Node (ibtreeMap f left_tree,
          ibtreeMap f right_tree);;
val ibtreeMap : (int -> int) -> int_Bin_Tree ->
  int_Bin_Tree = <fun>
```

Mapping over Recursive Types

```
# ibtreeMap ((+) 2) bin_tree;;
```

```
- : int_Bin_Tree = Node (Node (Leaf 5, Leaf
  8), Leaf (-5))
```

Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =
    match tree with Leaf n -> leafFun n
    | Node (left_tree, right_tree) ->
      nodeFun
      (ibtreeFoldRight leafFun nodeFun left_tree)
      (ibtreeFoldRight leafFun nodeFun right_tree);;
val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->
  int_Bin_Tree -> 'a = <fun>
```

Folding over Recursive Types

```
# let tree_sum =
    ibtreeFoldRight (fun x -> x) (+);;
val tree_sum : int_Bin_Tree -> int = <fun>
# tree_sum bin_tree;;
- : int = 2
```

600 minutes

Mutually Recursive Types

```
# type 'a tree = TreeLeaf of 'a
   | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree
   | More of ('a tree * 'a treeList);;
type 'a tree = TreeLeaf of 'a | TreeNode of 'a
  treeList
and 'a treeList = Last of 'a tree | More of ('a
  tree * 'a treeList)
```

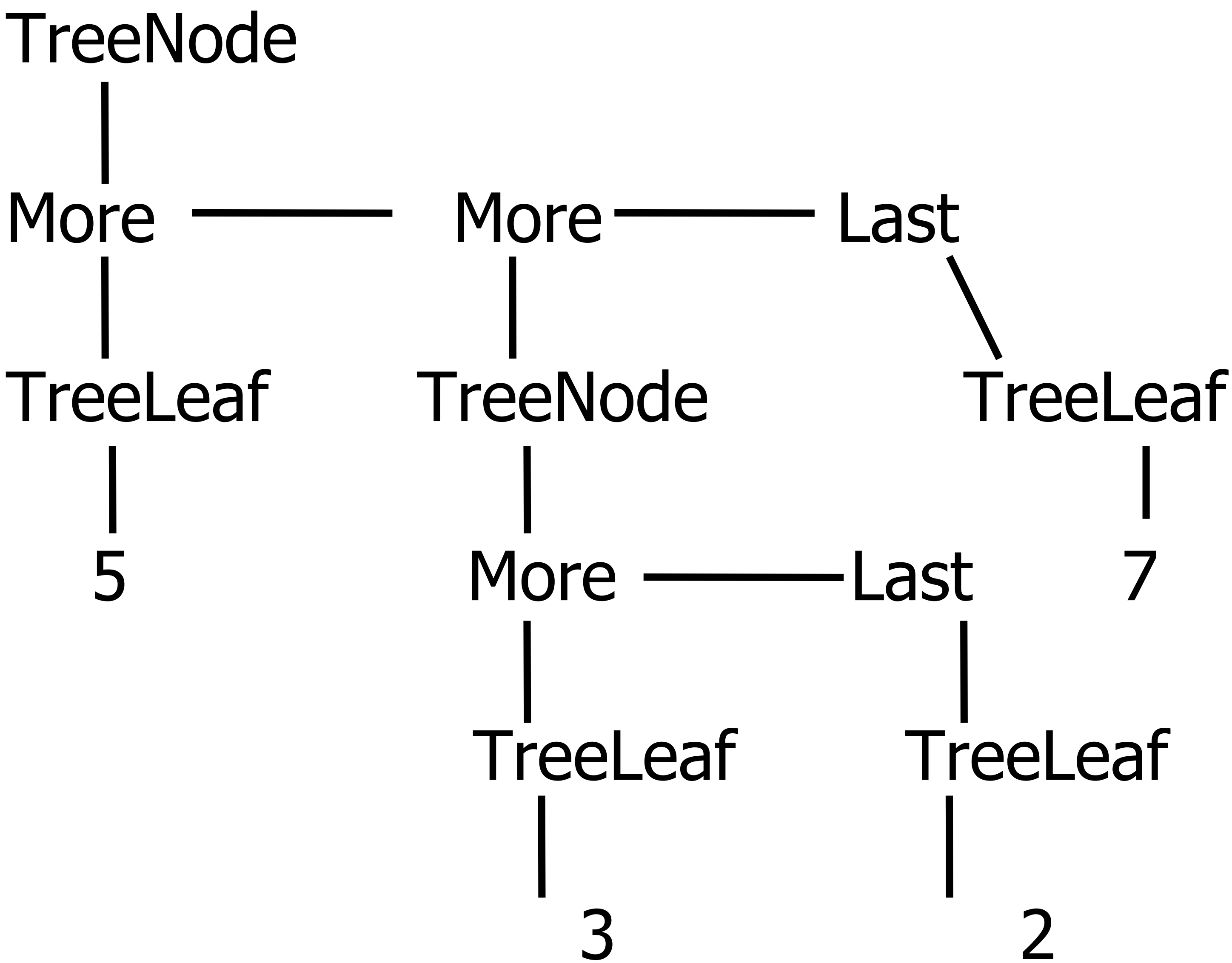
Mutually Recursive Types - Values

```
# let tree =
   TreeNode
    (More (TreeLeaf 5,
          (More (TreeNode
                (More (TreeLeaf 3,
                      Last (TreeLeaf 2))),
                Last (TreeLeaf 7)))));;
```

Mutually Recursive Types - Values

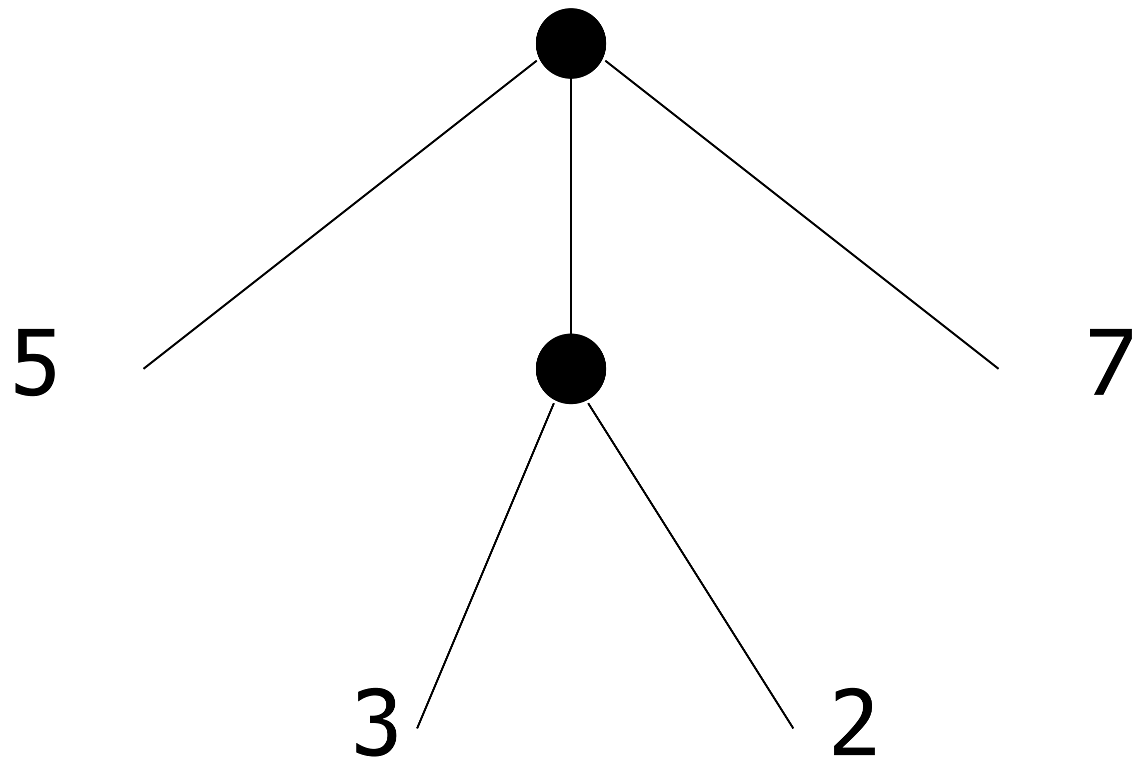
```
val tree : int tree =
 TreeNode
  (More
    (TreeLeaf 5,
     More
      (TreeNode (More (TreeLeaf 3, Last
 (TreeLeaf 2))), Last (TreeLeaf 7))))
```

Mutually Recursive Types - Values



Mutually Recursive Types - Values

A more conventional picture



Mutually Recursive Functions

```
# let rec fringe tree =
    match tree with (TreeLeaf x) -> [x]
  | (TreeNode list) -> list_fringe list
and list_fringe tree_list =
    match tree_list with (Last tree) -> fringe tree
  | (More (tree,list)) ->
    (fringe tree) @ (list_fringe list);;

val fringe : 'a tree -> 'a list = <fun>
val list_fringe : 'a treeList -> 'a list = <fun>
```

Mutually Recursive Functions

```
# fringe tree;;
- : int list = [5; 3; 2; 7]
```

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
      match t with TreeLeaf _ ->
      | TreeNode ts ->
```

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
```

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
      match t with TreeLeaf _ -> 1
      | TreeNode ts -> treeList_size  ts
and treeList_size ts =
```

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
      match t with TreeLeaf _ -> 1
      | TreeNode ts -> treeList_size  ts
and treeList_size ts =
      match ts with Last t ->
      | More t ts’ ->
```

Problem

type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;

Define tree_size and treeList_size

```
let rec tree_size t =
      match t with TreeLeaf _ -> 1
      | TreeNode ts -> treeList_size  ts
and treeList_size ts =
      match ts with Last t -> tree_size t
      | More t ts’ -> tree_size t + treeList_size ts’
```

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
and treeList_size ts =
    match ts with Last t -> tree_size t
    | More t ts’ -> tree_size t + treeList_size ts’
```

Nested Recursive Types

- # type 'a labeled_tree =
  TreeNode of ('a * 'a labeled_tree
  list);;

type 'a labeled_tree = TreeNode of ('a * 'a labeled_tree list)

Nested Recursive Type Values

```
# let ltree =
 TreeNode(5,
  [TreeNode (3, []);
   TreeNode (2, [TreeNode (1, []);
                 TreeNode (7, [])]);
   TreeNode (5, [])]);;
```

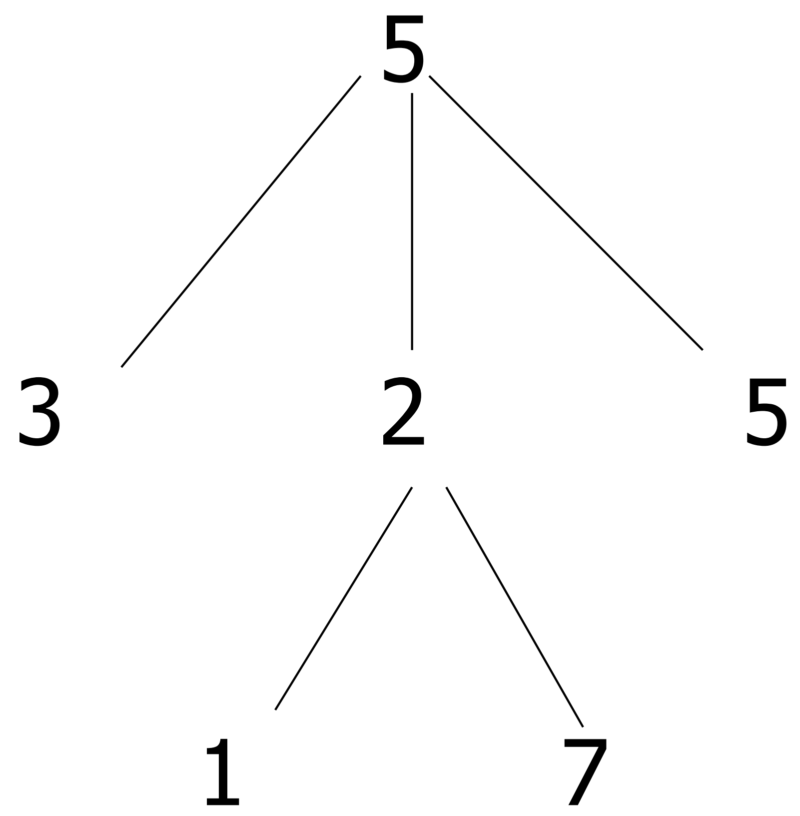
Nested Recursive Type Values

```
val ltree : int labeled_tree =
  TreeNode
   (5,
    [TreeNode (3, []); TreeNode (2,
  [TreeNode (1, []); TreeNode (7, [])]);
     TreeNode (5, [])])
```

Nested Recursive Type Values

```
Ltree =  TreeNode(5)
                /
        ::  ————————  ::  ————————  ::  ——————  [ ]
        |             |             |
TreeNode(3)    TreeNode(2)     TreeNode(5)
   |              |                   |
  [ ]             ::  ——————  :: —— [ ]      [ ]
                  |           |
           TreeNode(1)  TreeNode(7)
                  |           |
                 [ ]         [ ]
```

Nested Recursive Type Values



Mutually Recursive Functions

```
# let rec flatten_tree labtree =
  match labtree with TreeNode (x,treelist)
    -> x::flatten_tree_list treelist
  and flatten_tree_list treelist =
  match treelist with [] -> []
  | labtree::labtrees
    -> flatten_tree labtree
      @ flatten_tree_list labtrees;;
```

Mutually Recursive Functions

val flatten_tree : 'a labeled_tree -> 'a list = <fun>

val flatten_tree_list : 'a labeled_tree list -> 'a list = <fun>

flatten_tree ltree;;

- : int list = [5; 3; 2; 1; 7; 5]

- Nested recursive types lead to mutually recursive functions