



Programming Languages and Compilers (CS 421)

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4218 SC, UIUC



<https://courses.grainger.illinois.edu/cs421/fa2023/>

Based heavily on slides by Elsa Gunter, which were based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Objectives for Today

- **Reminder:** We want to turn strings (code) into computer instructions
- Done in **phases**
 - Turn strings into abstract syntax trees (**parse**)
 - Translate abstract syntax trees into executable instructions (**interpret** or **compile**)
- Tuesday we finished **lexing** those strings into tokens, and started the rest of **parsing**
- Today we will continue **parsing**



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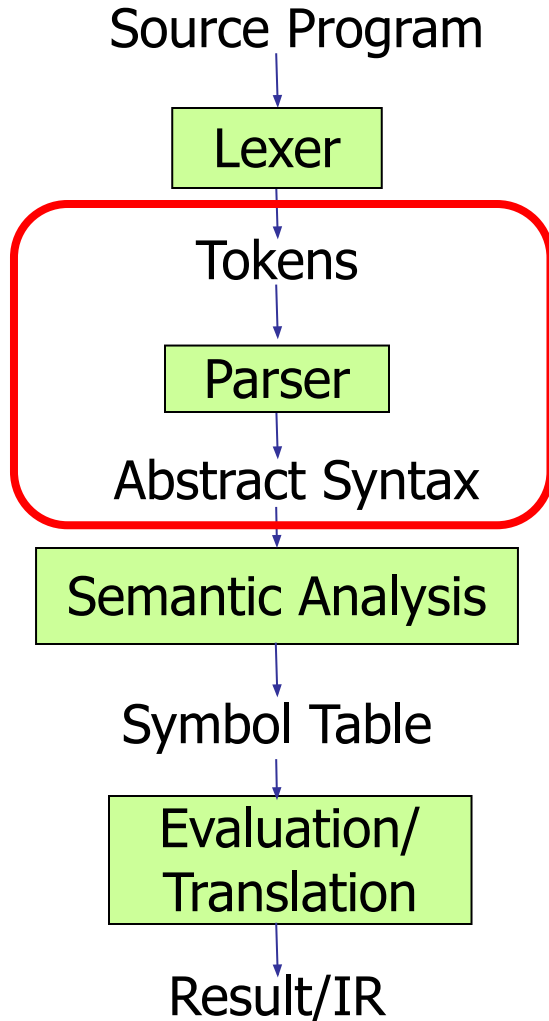


Questions from Tuesday?



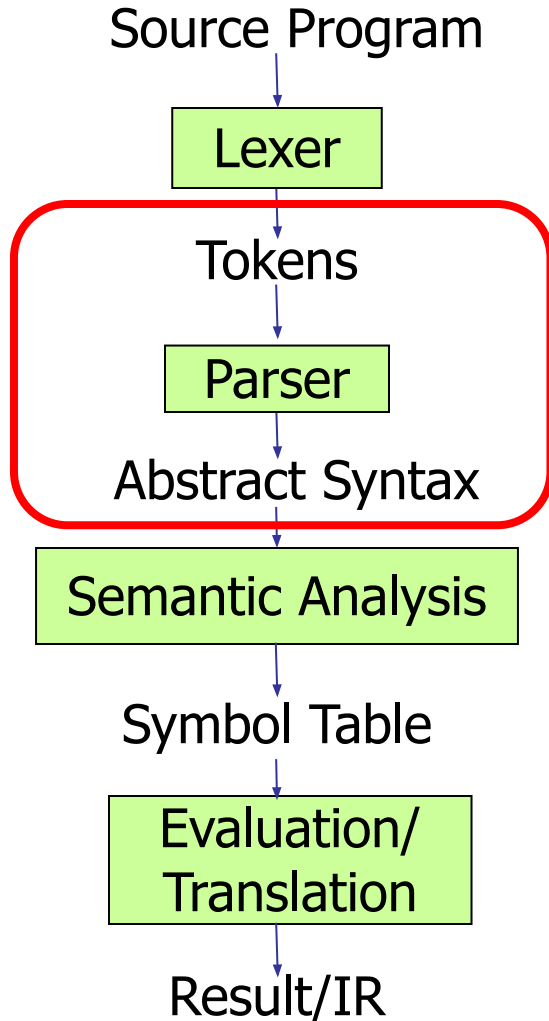
Parsing, Continued

Lexing and Parsing



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Lexing and Parsing



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The output, an **abstract syntax tree**, will track not just categories, but also **structure**.



Parse Trees

- **Abstract syntax tree** with **more detail**
- **Graphical representation** of derivation
- Each **node** labeled with either nonterminal or terminal
 - If node is labeled with a **terminal**, then it is a **leaf** (no sub-trees)
 - If node is labeled with a **nonterminal**, then it has **one branch for each element** in the **right-hand side** of rule used to substitute for it



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Example

Consider grammar:

$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle \mid \langle \text{factor} \rangle + \langle \text{factor} \rangle$

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$\langle \text{bin} \rangle ::= 0 \mid 1$

Problem: Build parse tree for $1 * 1 + 0$ as an $\langle \text{exp} \rangle$



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|
 $\langle \text{factor} \rangle$

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$\langle \text{factor} \rangle$

now derive $\langle \text{factor} \rangle$

Example

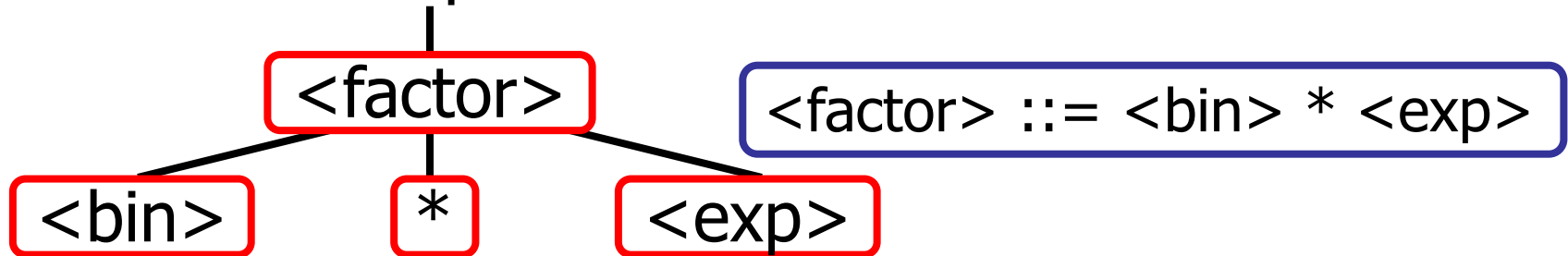
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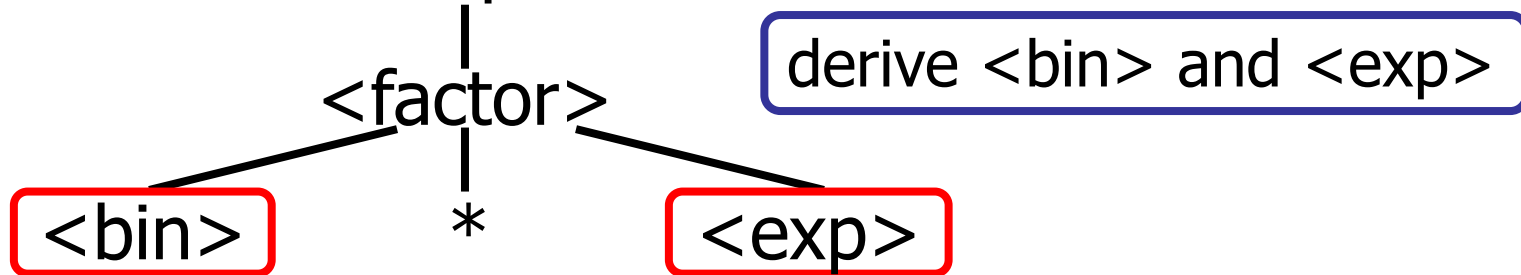
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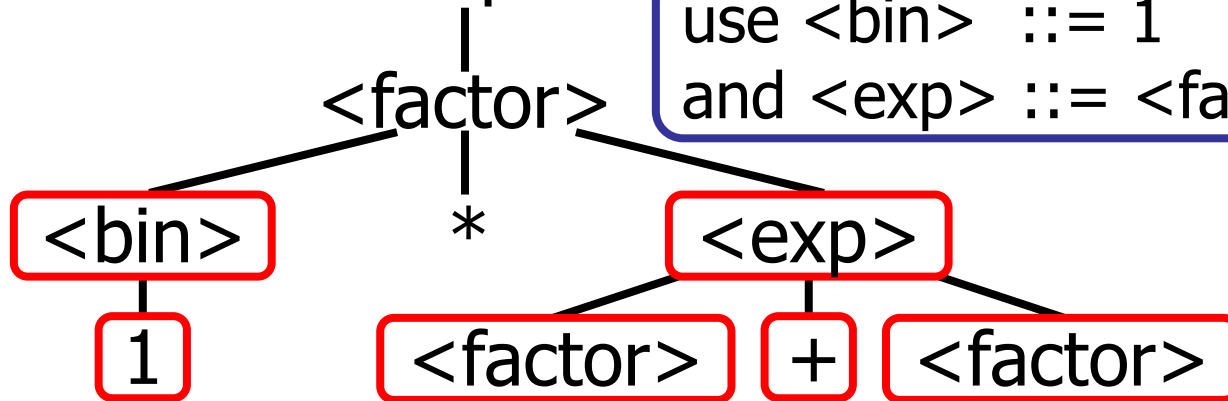


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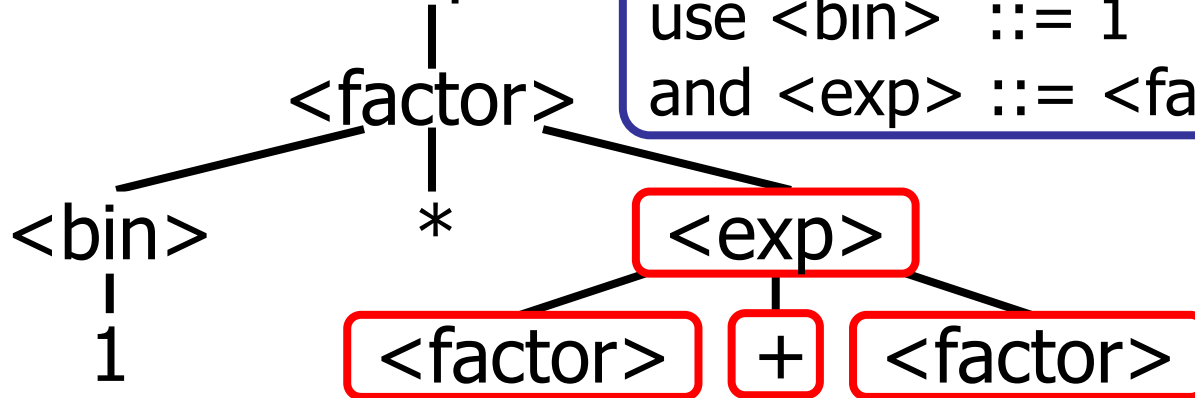
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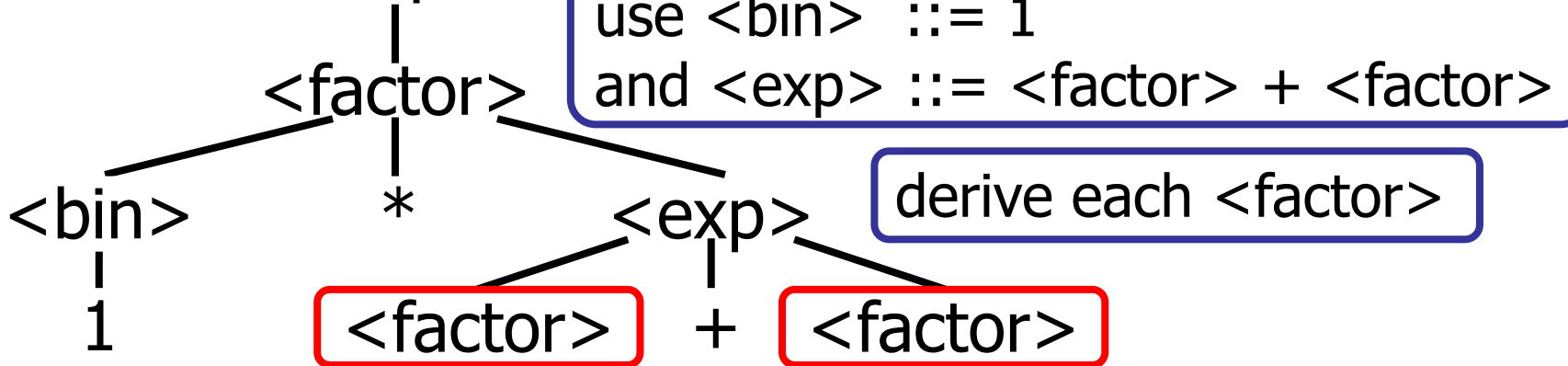
1 is terminal

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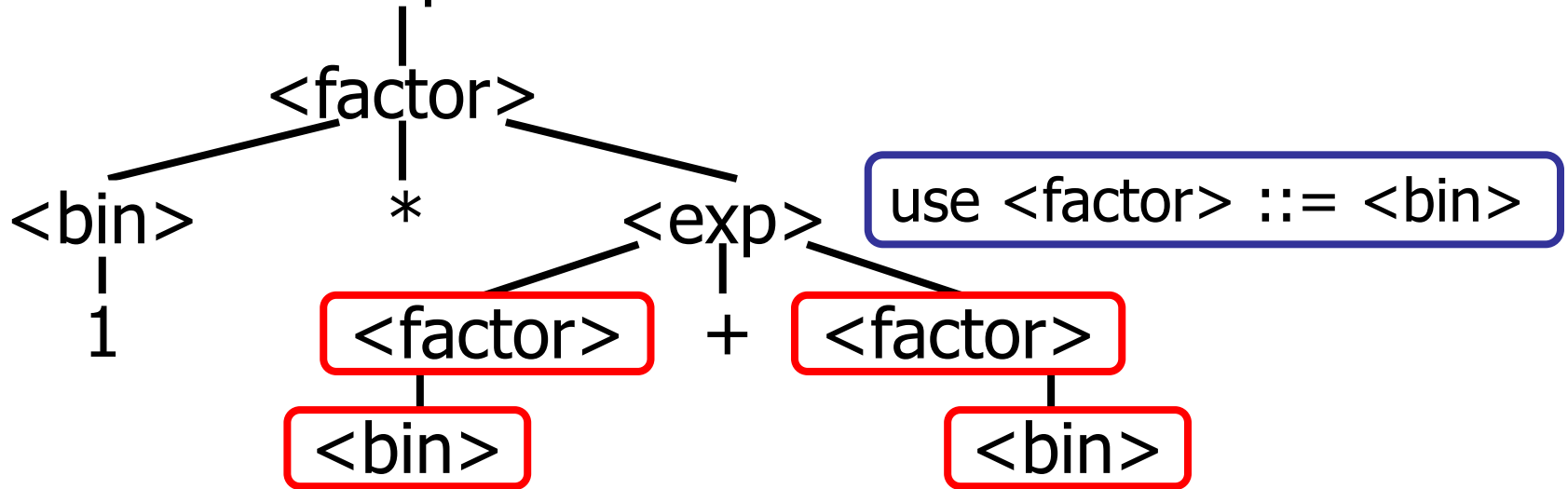


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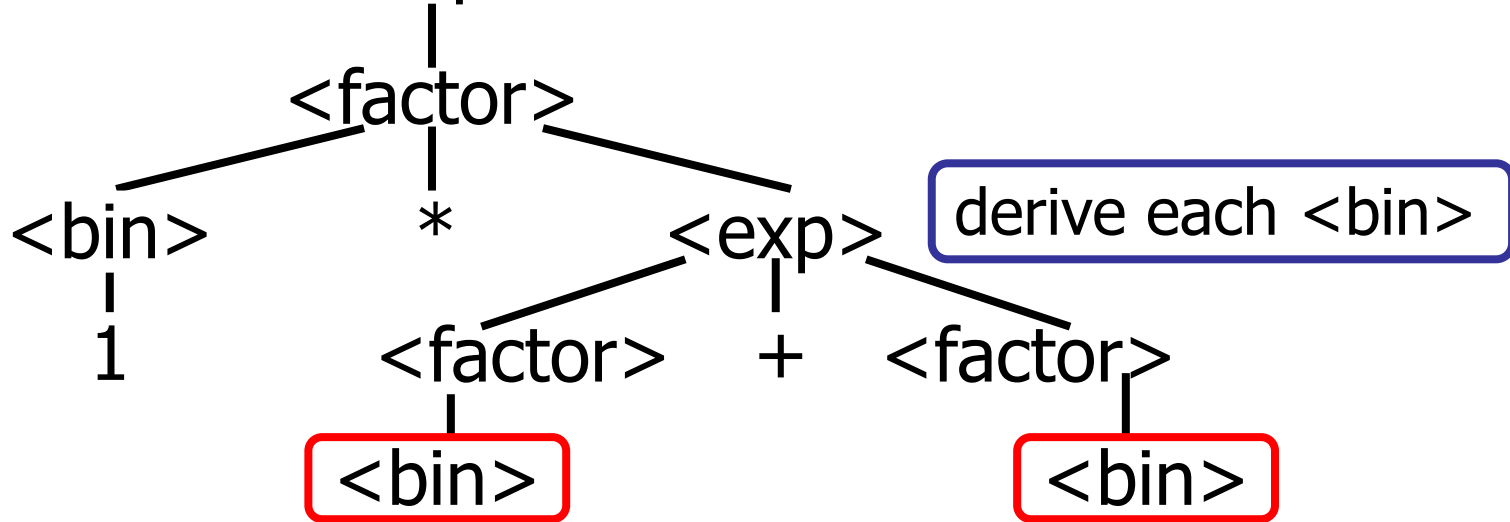
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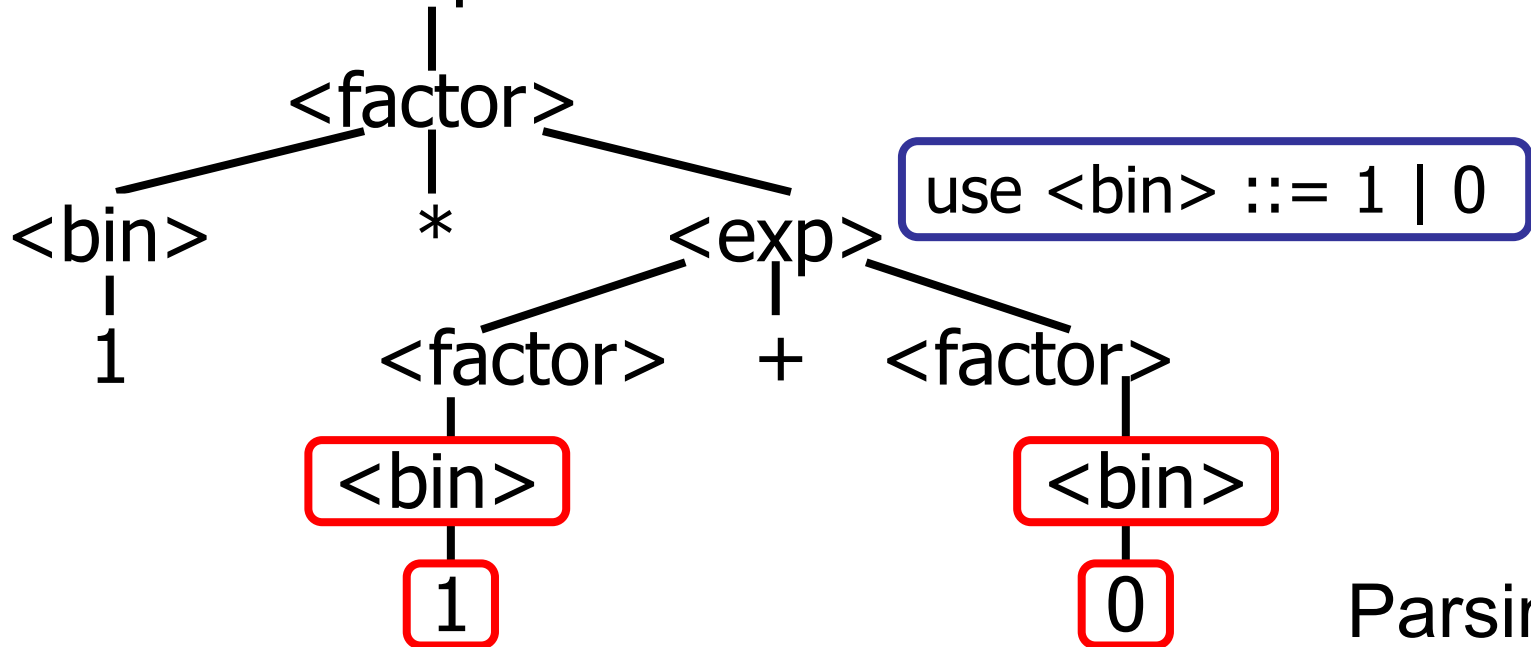
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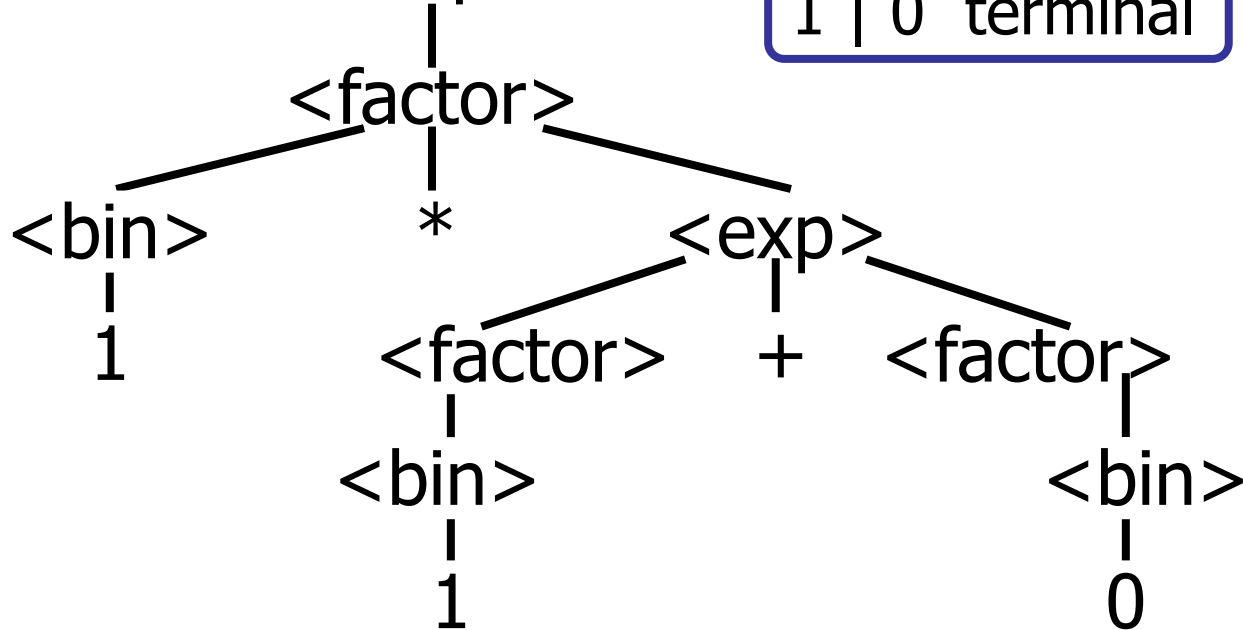
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1 | 0 terminal



Example

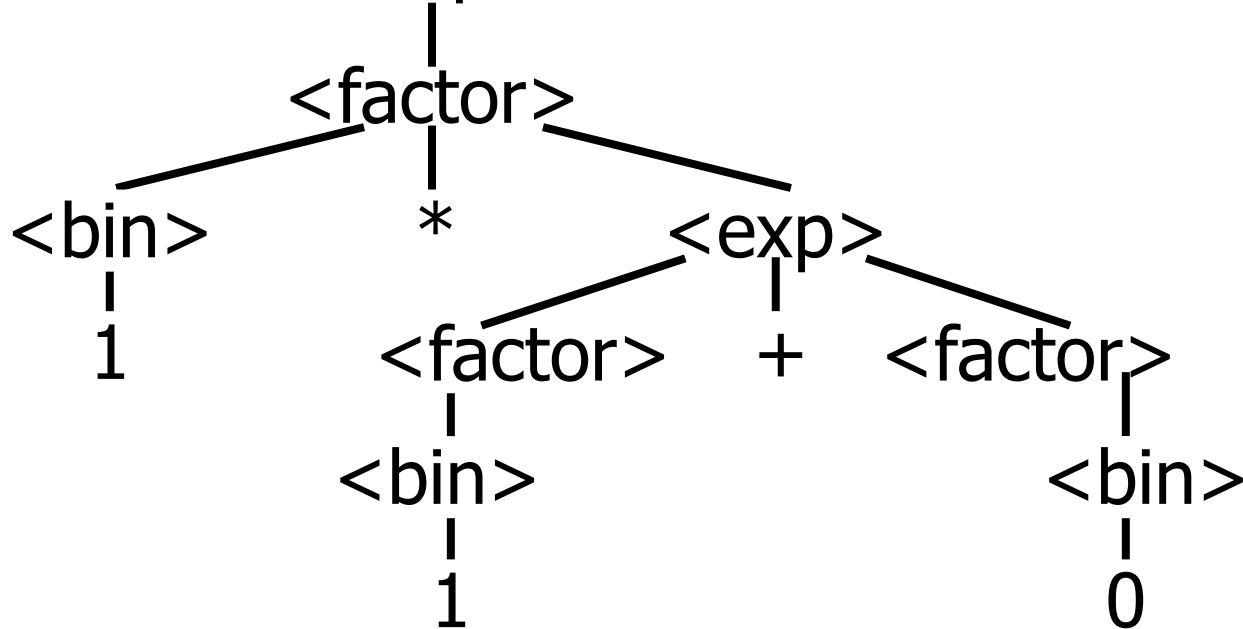
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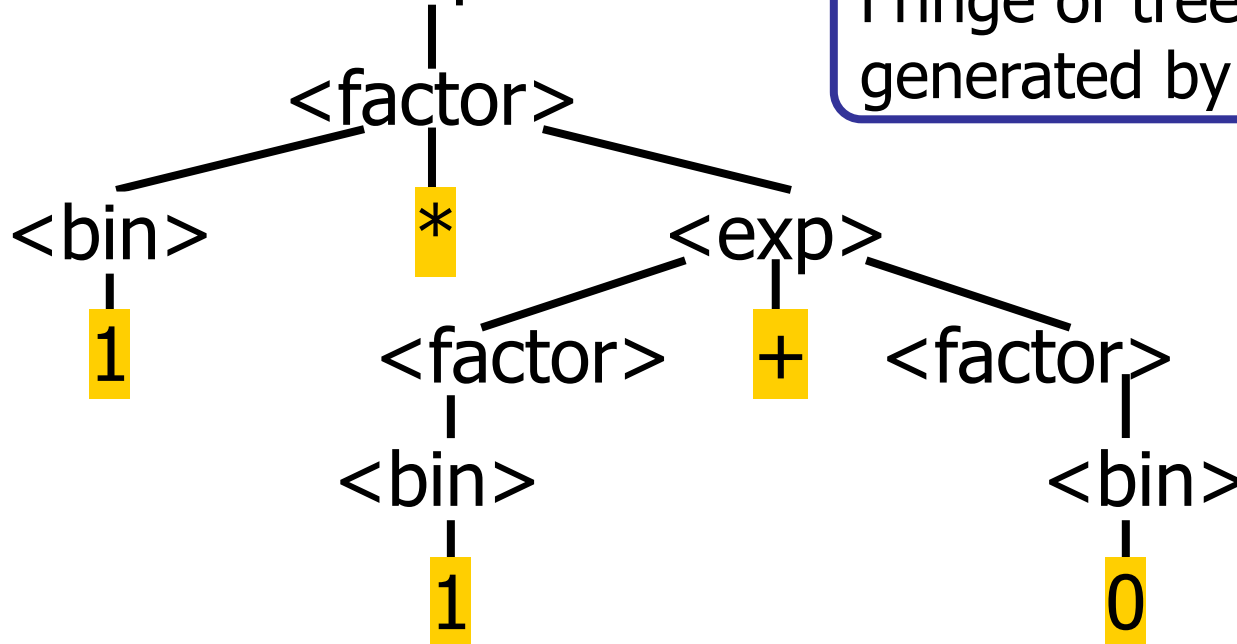


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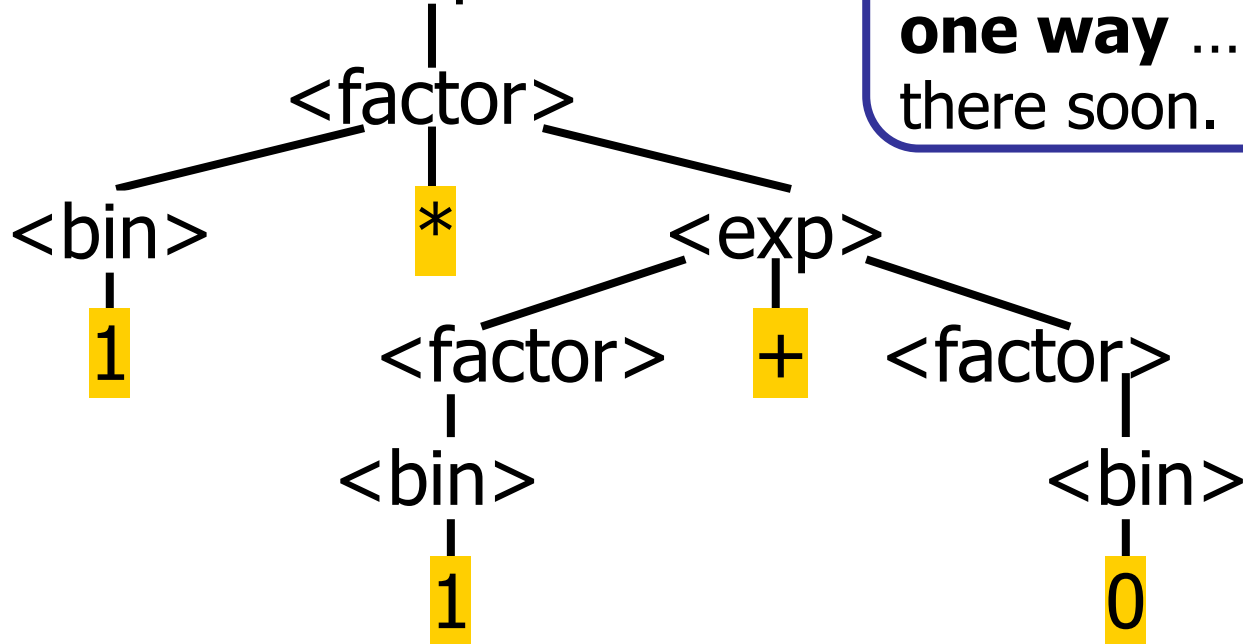
Fringe of tree is string generated by grammar

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Questions so far?



Parse Tree Data Structures

- Parse trees may be represented by OCaml **datatypes**
 - One **datatype** for each **nonterminal**
 - One **constructor** for each **rule**
 - Defined as **mutually recursive** collection of datatype declarations

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Example

type exp = Factor of factor | Plus of factor * factor
and factor = Bin of bin | Mult of bin * exp
and bin = Zero | One

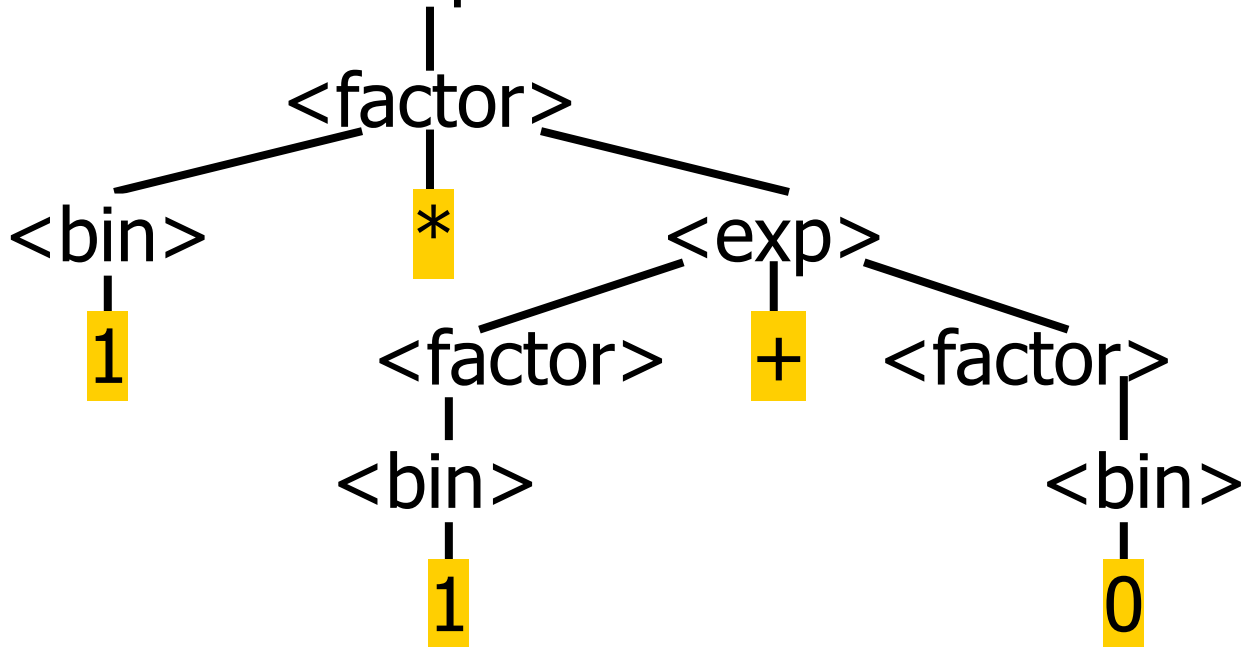
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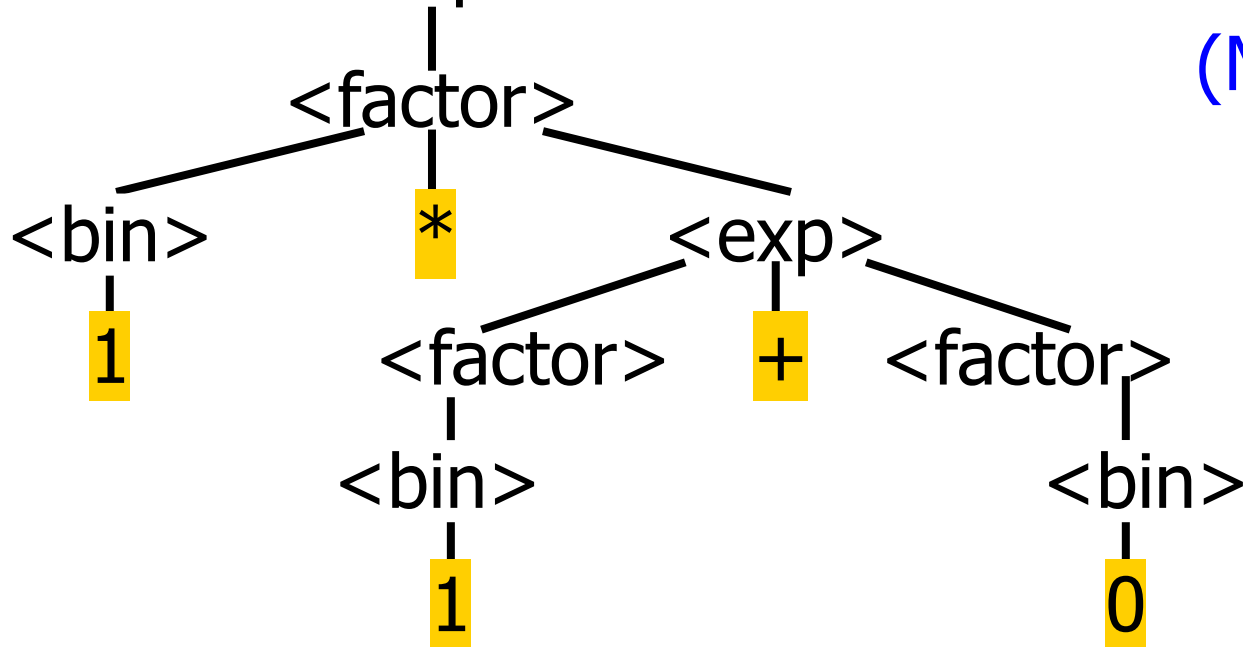
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1 * 1 + 0 : <exp> can be written as Factor



(Mult
(One,
Plus
(Bin One,
Bin Zero)))

Parsing



Questions so far?



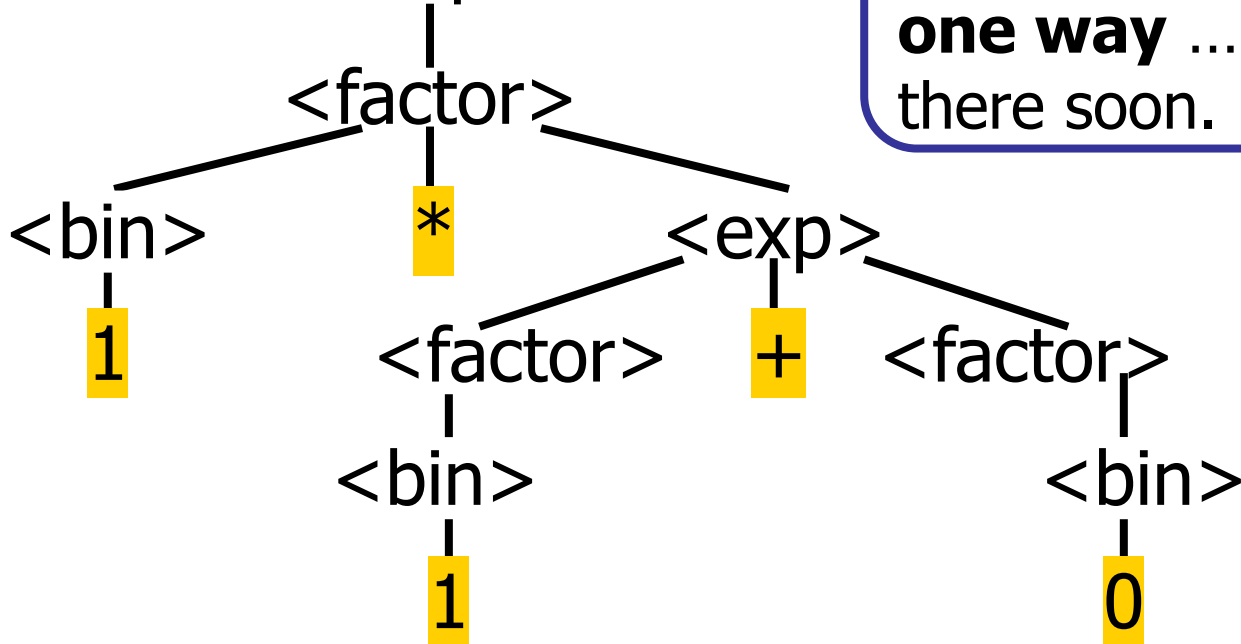
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Ambiguous Grammars and Languages

- A BNF grammar is **ambiguous** if its language **contains strings** for which there is **more than one parse tree**
 - Common sources of ambiguity:
 - Lack of determination of operator **precedence**
 - Lack of determination of operator **associativity**
 - Not the only sources of ambiguity
- If *all* **BNFs** for a language are ambiguous, then the language is **inherently ambiguous**
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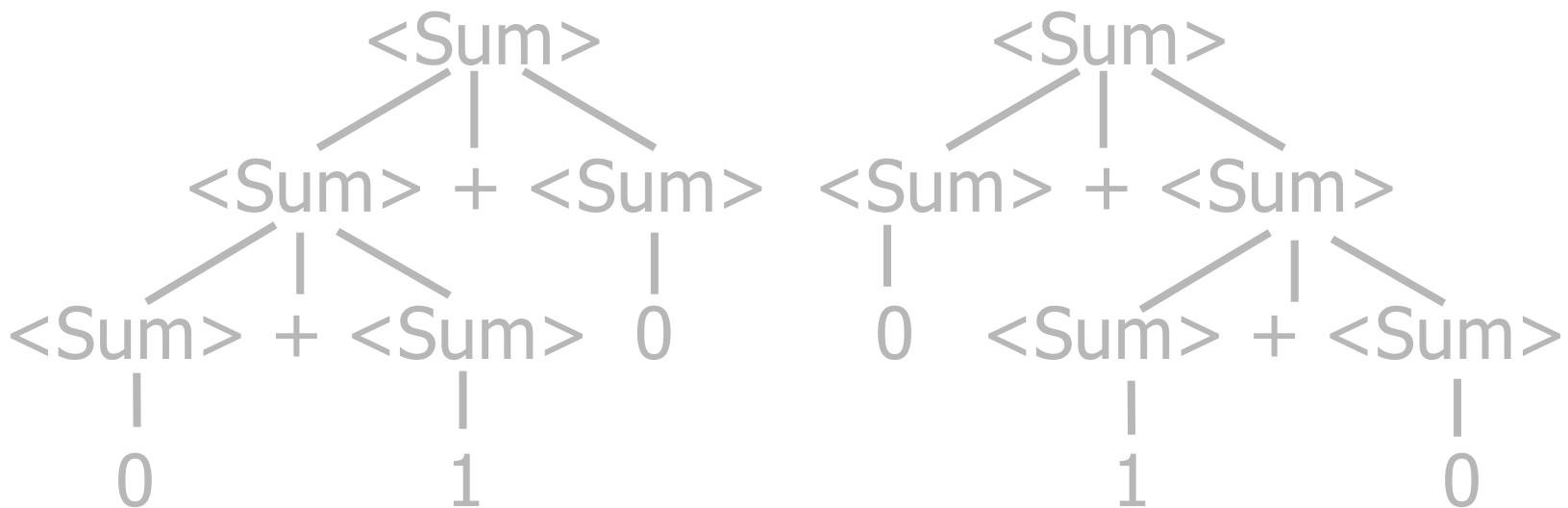
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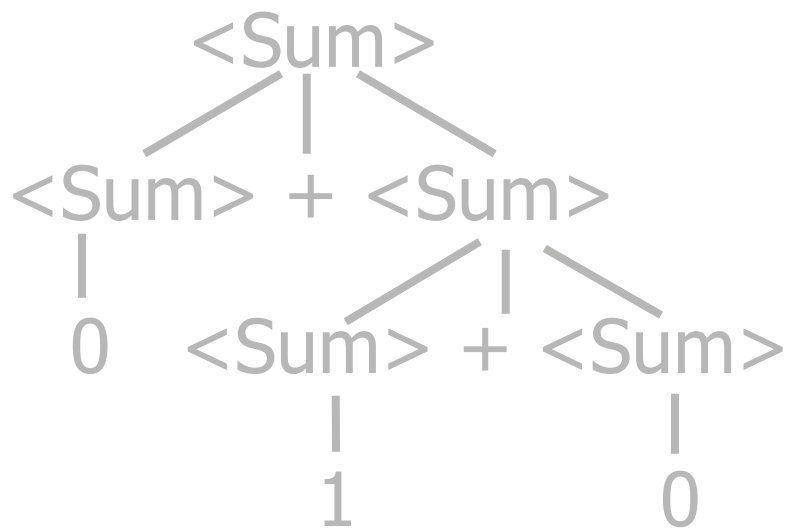
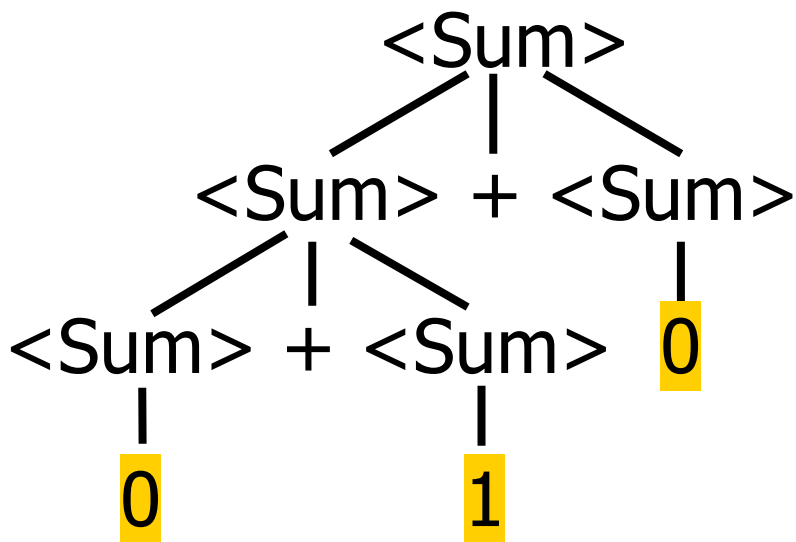
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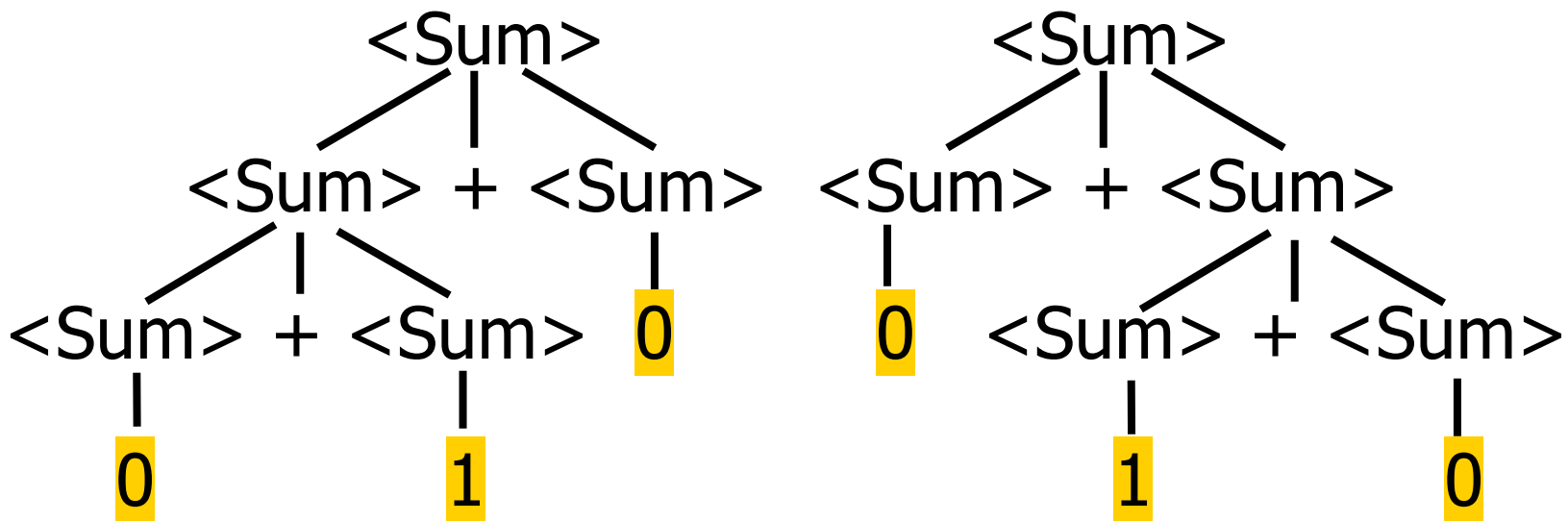
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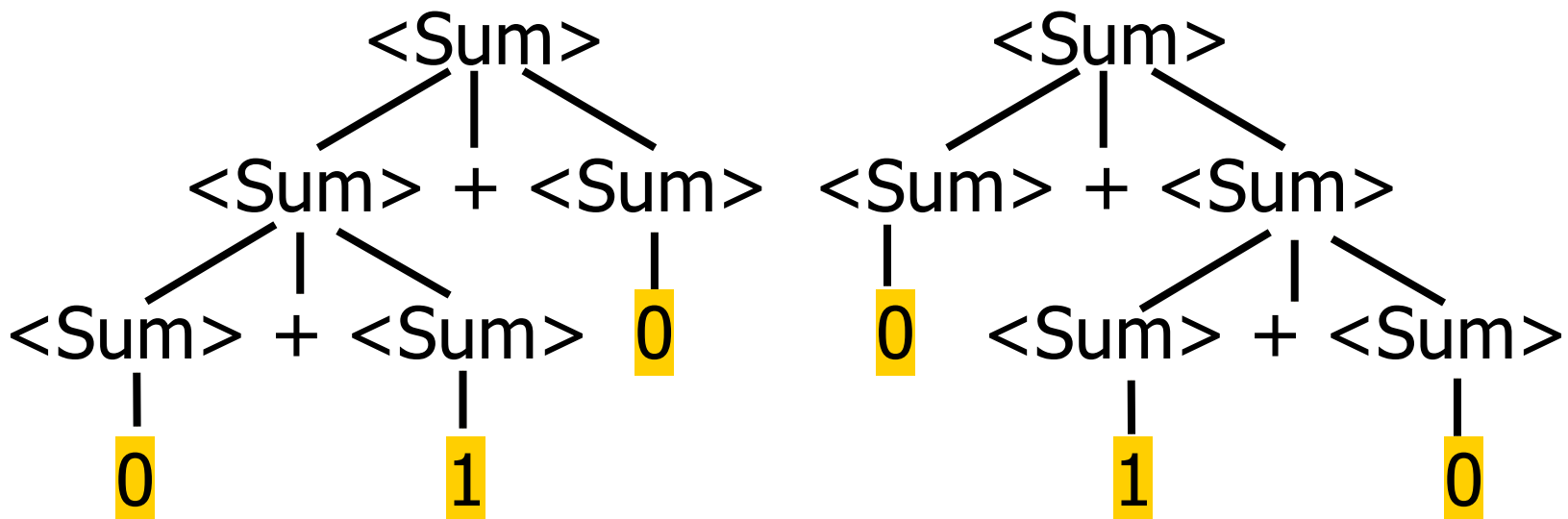


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How do we disambiguate?

Disambiguating a Grammar

- Given an ambiguous grammar G with start symbol S , find a grammar G' with the same start symbol S , such that **language of G = language of G'**
- **Not always possible**
- **No algorithm in general**
- But often in programming languages, when faced with an undecidable problem, we can either
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- **Idea:** Each nonterminal represents **all strings having some property**
- **Identify these properties** (often in terms of things that cannot happen)
- **Use these properties to inductively guarantee** every string in language has a unique parse
- We'll handle this in more detail later, but let's start with some examples



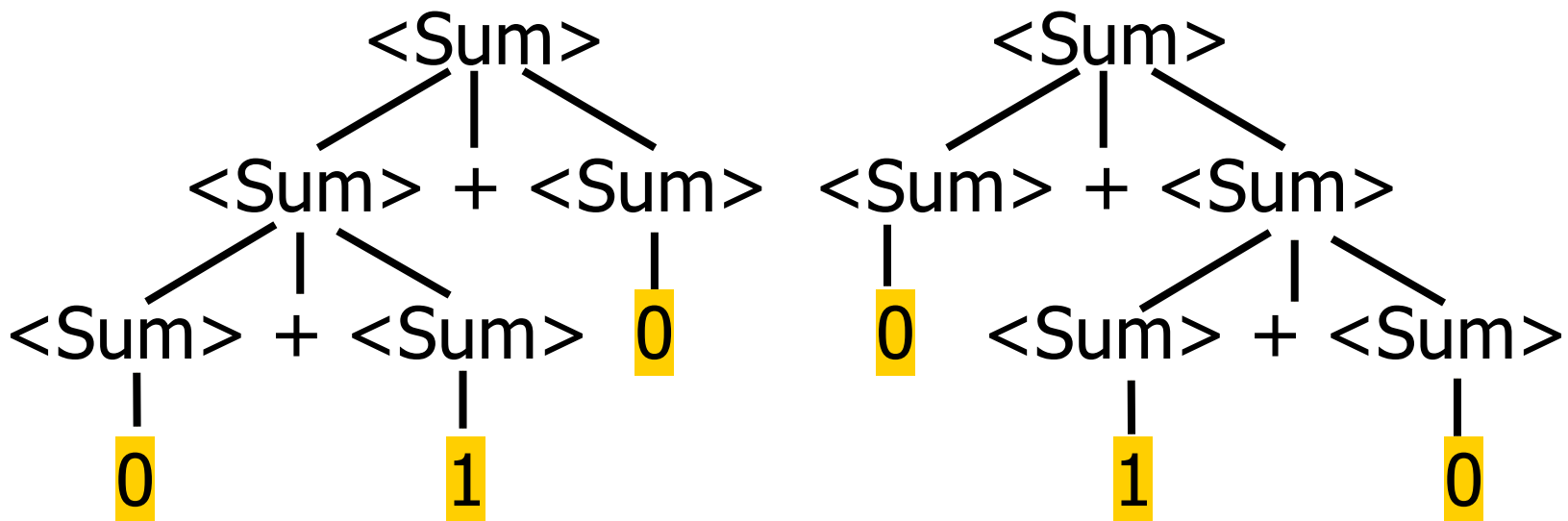
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Source of ambiguity: **associativity**



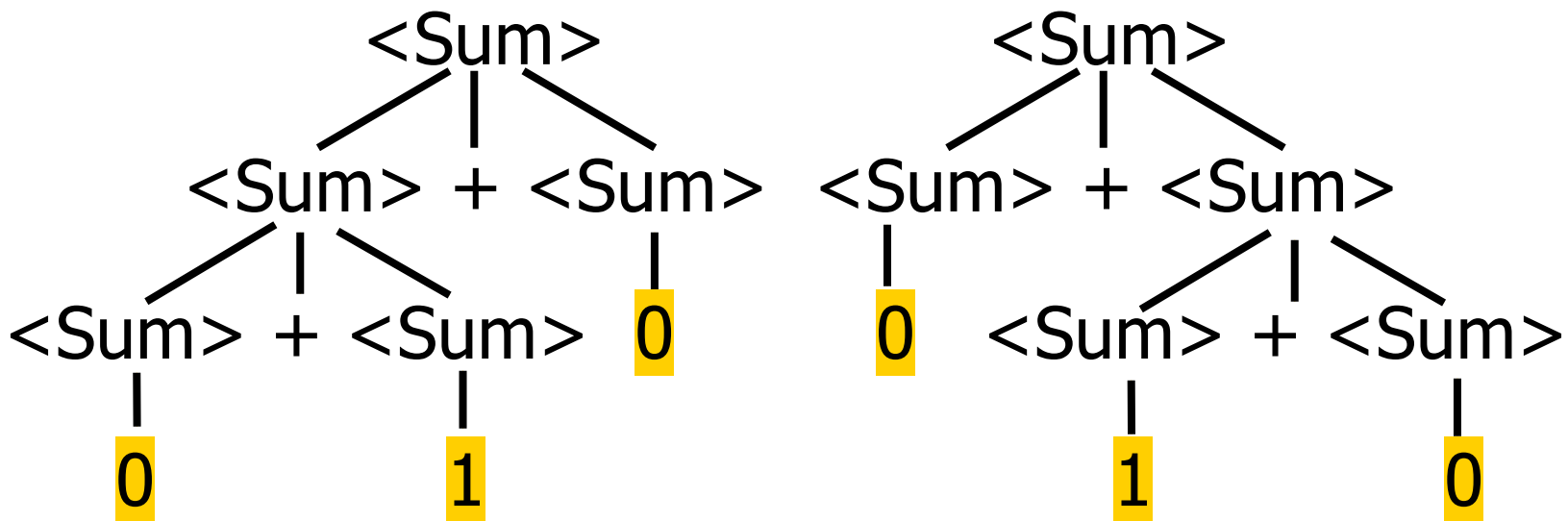
How to Enforce Associativity

- Have at most **one recursive call** per production
- When **two or more recursive calls** would be natural, to refactor:
 - Leave **rightmost call** for **right associativity**
 - Leave **leftmost call** for **left associativity**

Example: Disambiguation

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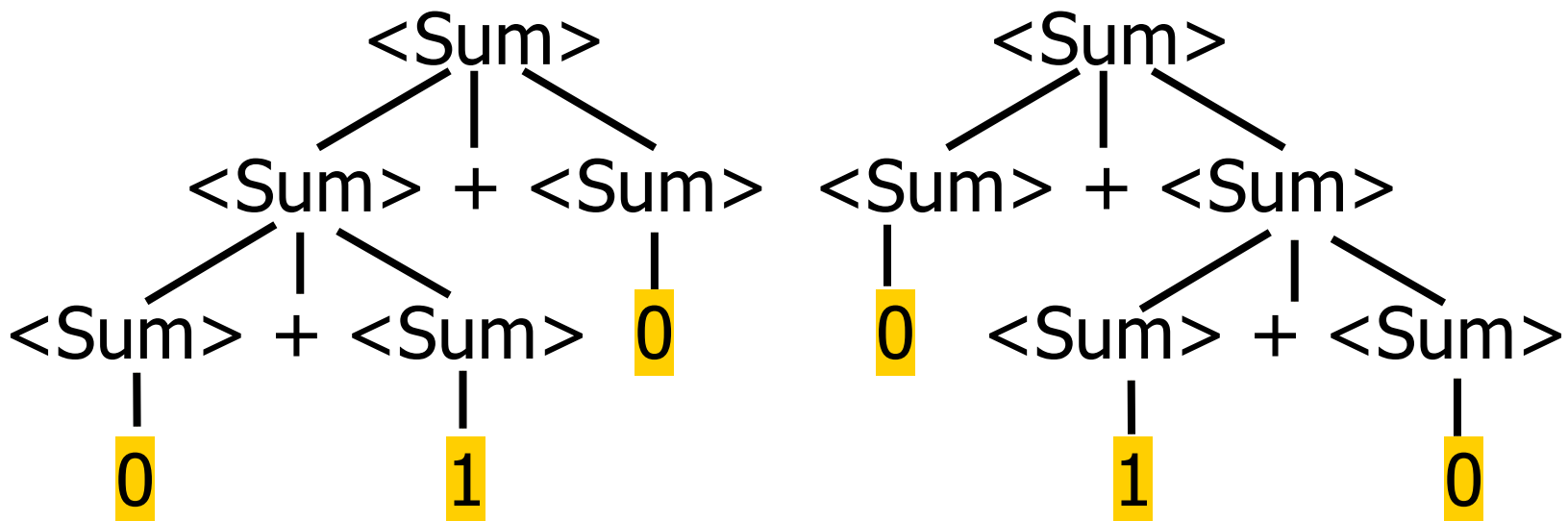


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Ambiguity



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Source of ambiguity: **associativity**



Example: Disambiguation

$\langle \text{Sum} \rangle ::= 0 \mid 1 \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \mid (\langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle ::= \langle \text{Num} \rangle \mid \langle \text{Num} \rangle + \langle \text{Sum} \rangle$

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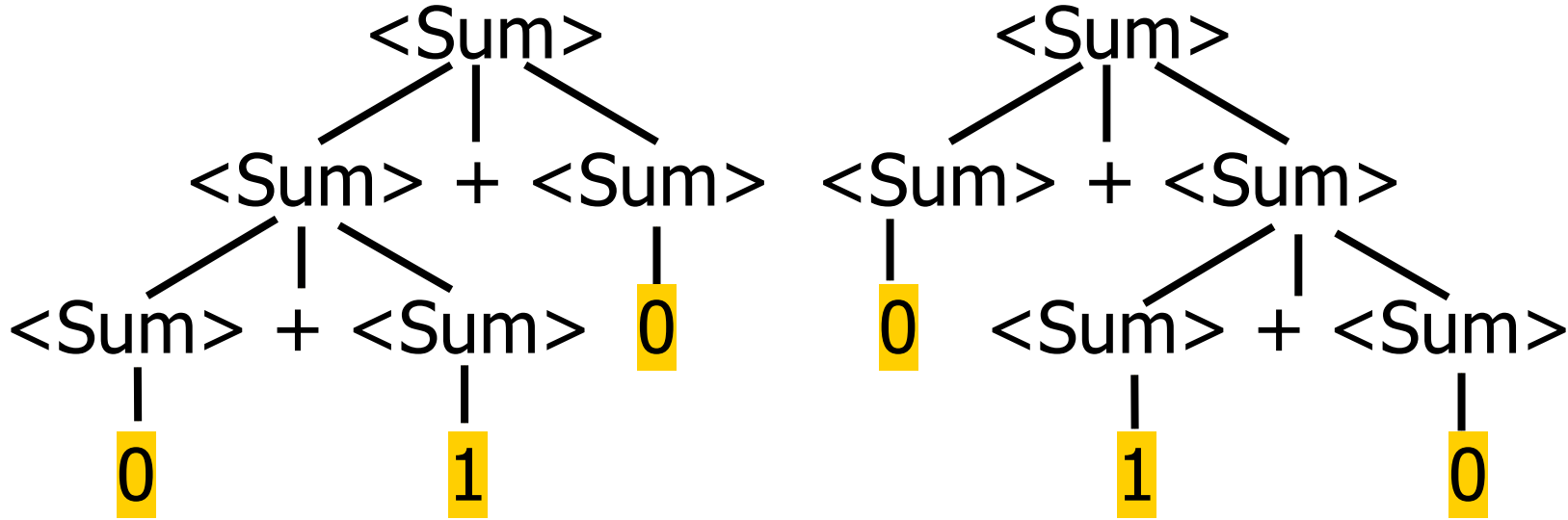
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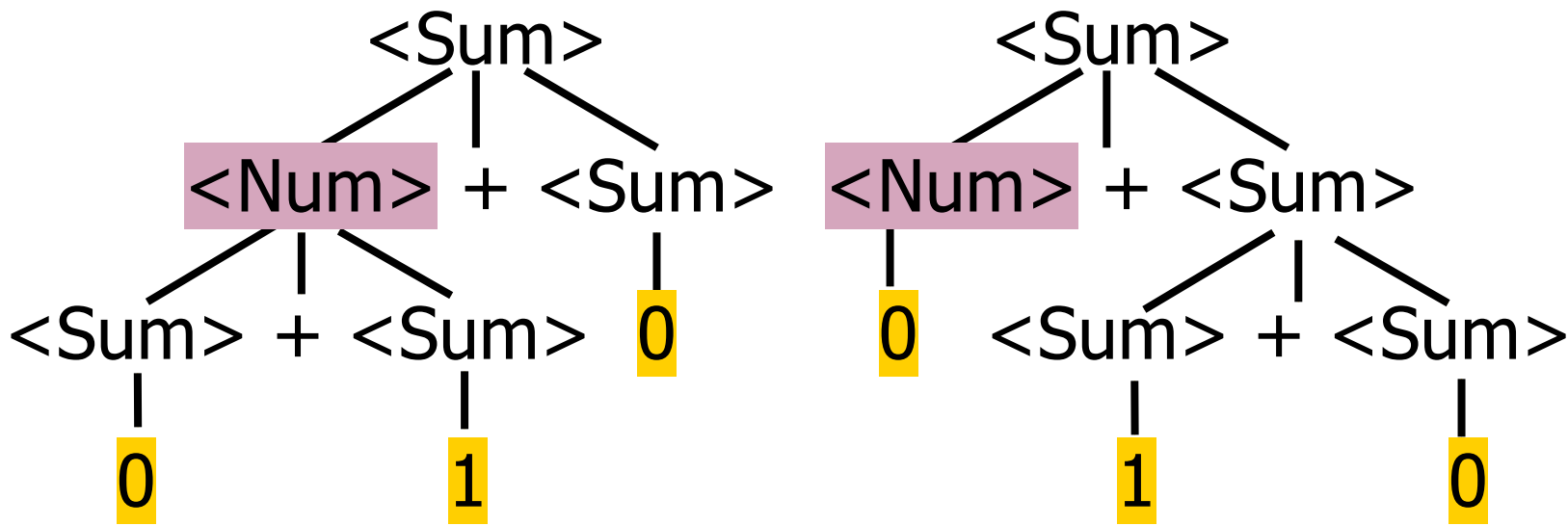


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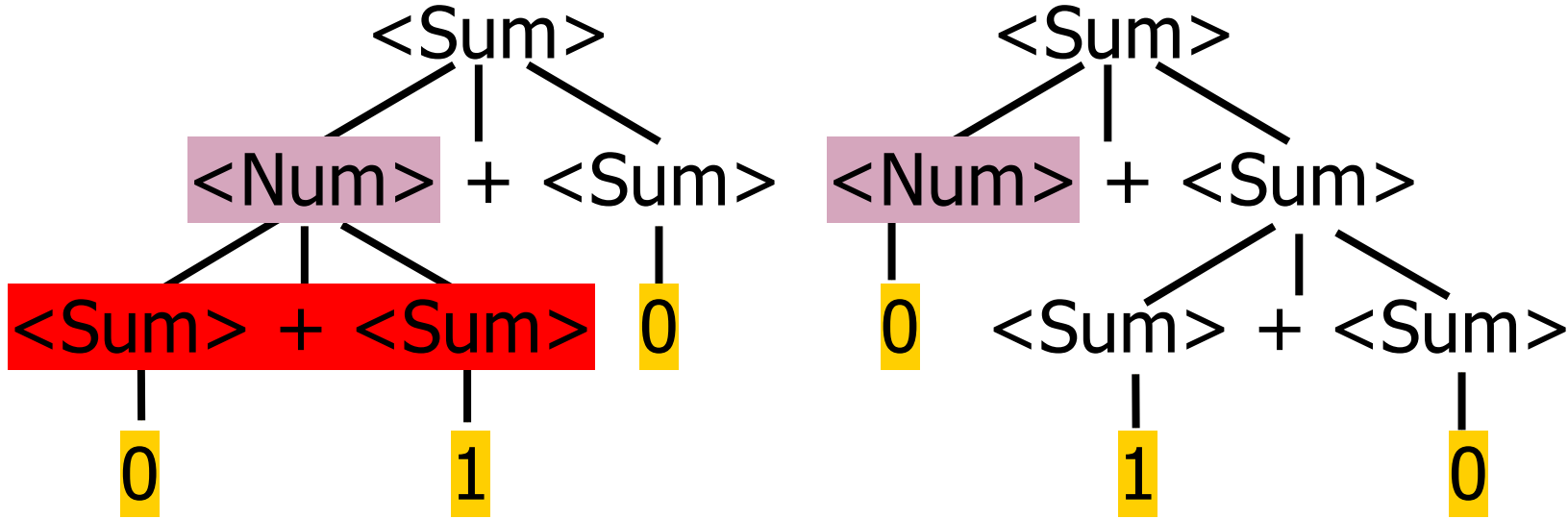


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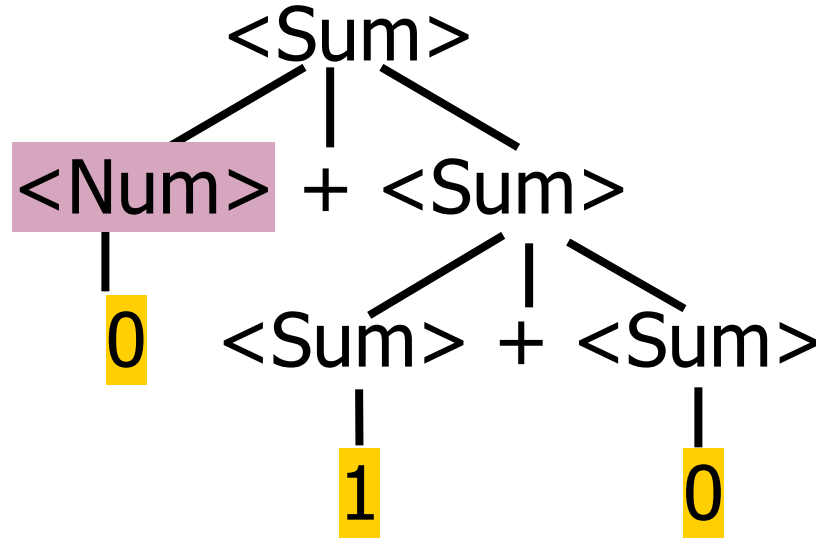


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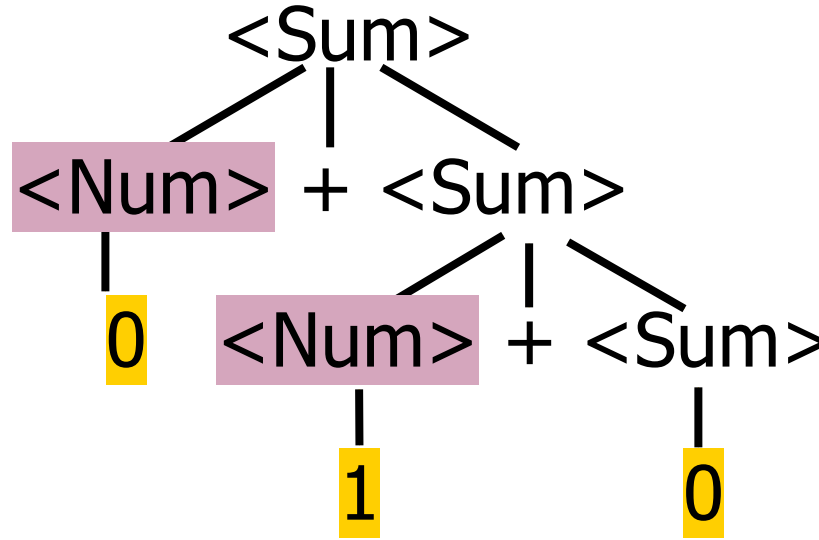


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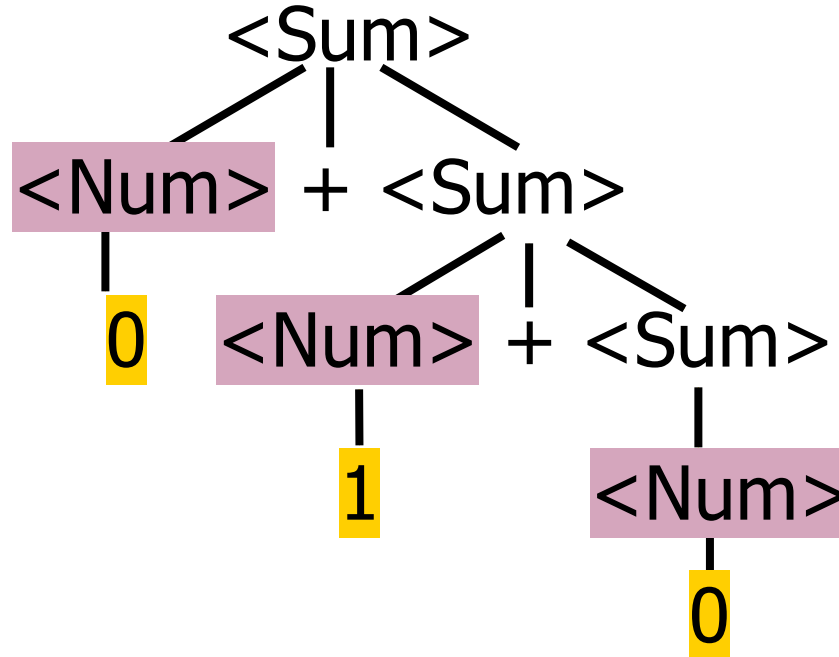


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Ambiguity ₆₄



Example: Disambiguation

Ambiguous grammar:

$\langle \text{exp} \rangle ::= 0 \mid 1 \mid \langle \text{exp} \rangle + \langle \text{exp} \rangle \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle$

Strings with more than one parse:

$0 + 1 + 0$

$1 * 1 + 1$

Sources of ambiguity:

associativity and precedence



Example: Disambiguation

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Strings with more than one parse:

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Sources of ambiguity:

associativity and precedence



Operator Precedence

- Operators of **highest precedence** evaluated **first** (that is, they bind more tightly)
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Precedence Table - Dated Sample

	Fortran	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, /, *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
lowest	+, -		+, -	+, -	::



Precedence in Grammar

- **Higher precedence** translates to **longer derivation chain**

- Example:

$\langle \text{exp} \rangle ::= 0 \mid 1 \mid \langle \text{exp} \rangle + \langle \text{exp} \rangle$
 $\quad \quad \quad \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle$

- Becomes

$\langle \text{exp} \rangle ::= \langle \text{mult_exp} \rangle$
 $\quad \quad \quad \mid \langle \text{exp} \rangle + \langle \text{mult_exp} \rangle$
 $\langle \text{mult_exp} \rangle ::= \langle \text{id} \rangle \mid \langle \text{mult_exp} \rangle * \langle \text{id} \rangle$
 $\langle \text{id} \rangle ::= 0 \mid 1$



Precedence in Grammar

- **Higher precedence** translates to **longer derivation chain**

- Example:

$$\begin{aligned} \langle \text{exp} \rangle ::= & 0 \mid 1 \mid \langle \text{exp} \rangle + \langle \text{exp} \rangle \\ & \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle \end{aligned}$$

- Becomes

$$\begin{aligned} \langle \text{exp} \rangle ::= & \langle \text{mult_exp} \rangle \\ & \mid \langle \text{exp} \rangle + \langle \text{mult_exp} \rangle \\ \langle \text{mult_exp} \rangle ::= & \langle \text{id} \rangle \mid \langle \text{mult_exp} \rangle * \langle \text{id} \rangle \\ \langle \text{id} \rangle ::= & 0 \mid 1 \end{aligned}$$

Precedence in Grammar

- **Higher precedence** translates to **longer derivation chain**

- Example:

$$\langle \text{exp} \rangle ::= 0 \mid 1 \mid \langle \text{exp} \rangle + \langle \text{exp} \rangle \\ \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle$$

- Becomes

$$\langle \text{exp} \rangle ::= \langle \text{mult_exp} \rangle$$
$$\mid \langle \text{exp} \rangle + \langle \text{mult_exp} \rangle$$
$$\langle \text{mult_exp} \rangle ::= \langle \text{id} \rangle \mid \langle \text{mult_exp} \rangle * \langle \text{id} \rangle$$
$$\langle \text{id} \rangle ::= 0 \mid 1$$

Ambiguity ₇₂



Questions so far?



Implementing Parsers



Parser Code

- `<grammar>.mly` defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point



Ocamlyacc Input

File format:

%{

<header>

%}

<declarations>

%%

<rules>

%%

<trailer>



Ocamlyacc *<header>*

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- *<footer>* similar. Possibly used to call parser



Ocamlyacc <declarations>

- **%token** *symbol ... symbol*
Declare given symbols as tokens
- **%token** <*type*> *symbol ... symbol*
Declare given symbols as token constructors, taking an argument of type <*type*>
- **%start** *symbol ... symbol*
Declare given symbols as entry points; functions of same names in <*grammar*>.ml



Ocamlyacc <declarations>

- **%type** <type> symbol ... symbol

Specify type of attributes for given symbols.

Mandatory for start symbols

- **%left** symbol ... symbol
- **%right** symbol ... symbol
- **%nonassoc** symbol ... symbol

Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)

Ocamlyacc <rules>

- *nonterminal* :
 symbol ... symbol { semantic_action }
 | ...
 | *symbol ... symbol { semantic_action }*
 ;
■ Semantic actions are arbitrary Ocaml expressions
■ Must be of same type as declared (or inferred) for *nonterminal*
■ Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...



Example - Base types

(* File: expr.ml *)

type expr =

- | Term_as_Expr of term
- | Plus_Expr of (term * expr)
- | Minus_Expr of (term * expr)

and term =

- | Factor_as_Term of factor
- | Mult_Term of (factor * term)
- | Div_Term of (factor * term)

and factor =

- | Id_as_Factor of string
- | Parenthesized_Expr_as_Factor of expr

Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }  
let numeric = ['0' - '9']  
let letter = ['a' - 'z' 'A' - 'Z']  
rule token = parse  
| "+" {Plus_token}  
| "-" {Minus_token}  
| "*" {Times_token}  
| "/" {Divide_token}  
| "(" {Left_parenthesis}  
| ")" {Right_parenthesis}  
| letter (letter|numeric|"_" )* as id {Id_token id}  
| [' ' '\t' '\n'] {token lexbuf}  
| eof {EOL}
```



Example - Parser (exprparse.mly)

```
%{ open Expr
```

```
%}
```

```
%token <string> Id_token
```

```
%token Left_parenthesis Right_parenthesis
```

```
%token Times_token Divide_token
```

```
%token Plus_token Minus_token
```

```
%token EOL
```

```
%start main
```

```
%type <expr> main
```

```
%%
```



Example - Parser (exprparse.mly)

expr:

| term { Term_as_Expr \$1 }

| term Plus_token expr { Plus_Expr (\$1, \$3) }

| term Minus_token expr { Minus_Expr (\$1, \$3) }

term:

| factor { Factor_as_Term \$1 }

| factor Times_token term { Mult_Term (\$1, \$3) }

| factor Divide_token term { Div_Term (\$1, \$3) }



Example - Parser (exprparse.mly)

factor:

| Id_token { Id_as_Factor \$1 }

| Left_parenthesis expr Right_parenthesis
{Parenthesized_Expr_as_Factor \$2 }

main:

| expr EOL { \$1 }



Example - Using Parser

```
# #use "expr.ml";;
```

```
...
```

```
# #use "exprparse.ml";;
```

```
...
```

```
# #use "exprlex.ml";;
```

```
...
```

```
# let test s =
```

```
  let lexbuf = Lexing.from_string (s^"\n") in  
    main token lexbuf;;
```



Example - Using Parser

```
# test "a + b";;
```

```
- : expr =
```

```
Plus_Expr
```

```
(Factor_as_Term
```

```
(Id_as_Factor "a"),
```

```
Term_as_Expr
```

```
(Factor_as_Term (Id_as_Factor "b"))))
```



Questions?



Next Class: Underlying Algorithm



Next Class

- **MP8** due next **Tuesday**
- **WA8** due next **Thursday**
- All deadlines can be found on **course website**
- Use **office hours** and **class forums** for help