Programming Languages and Compilers (CS 421)

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Why Data Types?

- Data types play a key role in:
  - *Data abstraction* in the design of programs
  - *Type checking* in the analysis of programs
  - *Compile-time code generation* in the translation and execution of programs
    - Data layout (how many words; which are data and which are pointers) dictated by type

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Terminology

- Type: A type t defines a set of possible data values
  - E.g. short in C is $\{x \mid 2^{15} - 1 \geq x \geq -2^{15}\}$
  - A value in this set is said to have type t

- Type system: rules of a language assigning types to expressions

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Types as Specifications

- Types describe properties
- Different type systems describe different properties, eg
  - Data is read-write versus read-only
  - Operation has authority to access data
  - Data came from "right" source
  - Operation might or could not raise an exception
- Common type systems focus on types describing same data layout and access methods

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Sound Type System

- If an expression is assigned type t, and it evaluates to a value v, then v is in the set of values defined by t

- SML, OCAML, Scheme and Ada have sound type systems
- Most implementations of C and C++ do not

Strongly Typed Language

- When no application of an operator to arguments can lead to a run-time type error, language is *strongly typed*
  - Eg: 1 + 2.3;;
- Depends on definition of "type error"

Strongly Typed Language

- C++ claimed to be "strongly typed", but
  - Union types allow creating a value at one type and using it at another
  - Type coercions may cause unexpected (undesirable) effects
  - No array bounds check (in fact, no runtime checks at all)
- SML, OCAML "strongly typed" but still must do dynamic array bounds checks, runtime type case analysis, and other checks

Static vs Dynamic Types

- *Static type*: type assigned to an expression at compile time
- *Dynamic type*: type assigned to a storage location at run time
- *Statically typed language*: static type assigned to every expression at compile time
- *Dynamically typed language*: type of an expression determined at run time

Type Checking

- When is op(arg1,…,argn) allowed?
- *Type checking* assures that operations are applied to the right number of arguments of the right types
  - Right type may mean same type as was specified, or may mean that there is a predefined implicit coercion that will be applied
- Used to resolve overloaded operations

Type Checking

- Type checking may be done *statically* at compile time or *dynamically* at run time
- Dynamically typed (aka untyped) languages (eg LISP, Prolog) do only dynamic type checking
- Statically typed languages can do most type checking statically

Dynamic Type Checking

- Performed at run-time before each operation is applied
- Types of variables and operations left unspecified until run-time
  - Same variable may be used at different types

Dynamic Type Checking

- Data object must contain type information
- Errors aren't detected until violating application is executed (maybe years after the code was written)
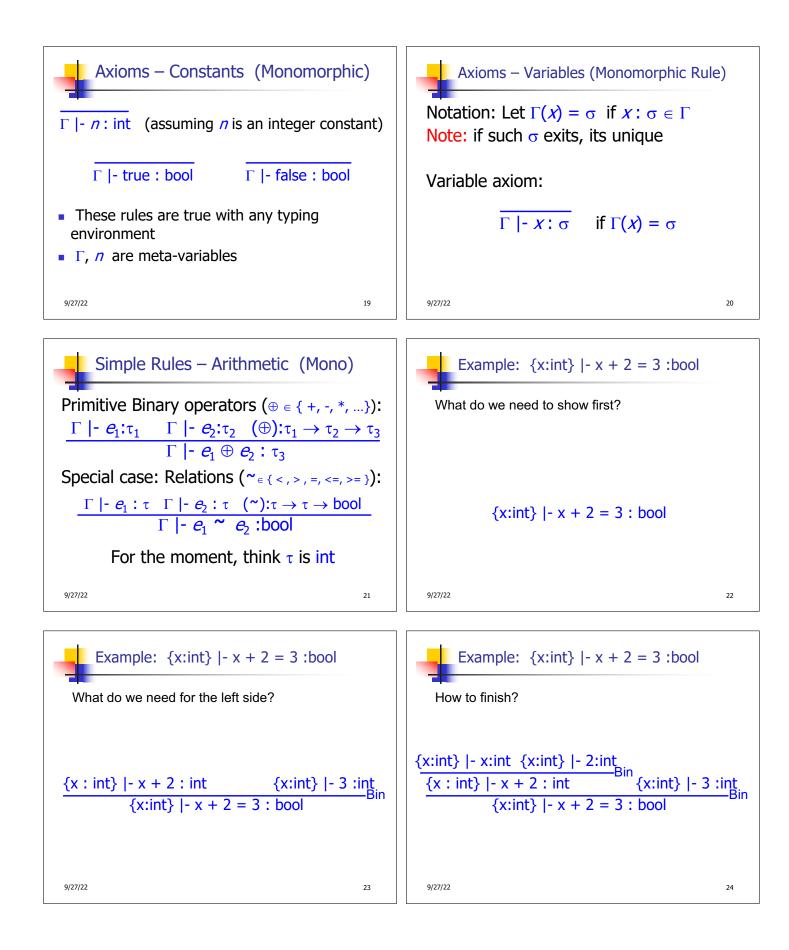
Static Type Checking

- Performed after parsing, before code generation
- Type of every variable and signature of every operator must be known at compile time

Static Type Checking

- Can eliminate need to store type information in data object if no dynamic type checking is needed
- Catches many programming errors at earliest point
- Can't check types that depend on dynamically computed values
  - Eg: array bounds

Static Type Checking

- Typically places restrictions on languages
  - Garbage collection
  - References instead of pointers
  - All variables initialized when created
  - Variable only used at one type
    - Union types allow for work-arounds, but effectively introduce dynamic type checks

Type Declarations

- *Type declarations*: explicit assignment of types to variables (signatures to functions) in the code of a program
  - Must be checked in a strongly typed language
  - Often not necessary for strong typing or even static typing (depends on the type system)

Type Inference

- *Type inference*: A program analysis to assign a type to an expression from the program context of the expression
  - Fully static type inference first introduced by Robin Miller in ML
  - Haskle, OCAML, SML all use type inference
    - Records are a problem for type inference

Format of Type Judgments

- A *type judgement* has the form
$$\Gamma \ |\text{-} \ exp : \tau$$
- Γ is a typing environment
  - Supplies the types of variables (and function names when function names are not variables)
  - Γ is a set of the form $\{ x : \sigma , \ldots \}$
  - For any x at most one σ such that $(x : \sigma \in \Gamma)$
- exp is a program expression
- τ is a type to be assigned to exp
- |- pronounced "turnstyle", or "entails" (or "satisfies" or, informally, "shows")

Axioms – Constants (Monomorphic)

$$\overline{\Gamma \mid\!- n : \text{int}} \quad \text{(assuming } n \text{ is an integer constant)}$$

$$\overline{\Gamma \mid\!- \text{true} : \text{bool}} \qquad \overline{\Gamma \mid\!- \text{false} : \text{bool}}$$

- These rules are true with any typing environment
- Γ, n are meta-variables

Axioms – Variables (Monomorphic Rule)

Notation: Let $\Gamma(x) = \sigma$ if $x : \sigma \in \Gamma$
Note: if such σ exits, its unique

Variable axiom:

$$\overline{\Gamma \mid\!- x : \sigma} \quad \text{if } \Gamma(x) = \sigma$$

Simple Rules – Arithmetic (Mono)

Primitive Binary operators ($\oplus \in \{ +, -, *, \ldots \}$):

$$\frac{\Gamma \mid\!- e_1{:}\tau_1 \quad \Gamma \mid\!- e_2{:}\tau_2 \quad (\oplus){:}\tau_1 \to \tau_2 \to \tau_3}{\Gamma \mid\!- e_1 \oplus e_2 : \tau_3}$$

Special case: Relations ($\sim \in \{ <, >, =, <=, >= \}$):

$$\frac{\Gamma \mid\!- e_1 : \tau \quad \Gamma \mid\!- e_2 : \tau \quad (\sim){:}\tau \to \tau \to \text{bool}}{\Gamma \mid\!- e_1 \sim e_2 : \text{bool}}$$

For the moment, think τ is int

Example: {x:int} |- x + 2 = 3 :bool

What do we need to show first?

$$\{x{:}int\} \mid\!- x + 2 = 3 : \text{bool}$$

Example: {x:int} |- x + 2 = 3 :bool

What do we need for the left side?

$$\frac{\{x : int\} \mid\!- x + 2 : int \qquad \{x{:}int\} \mid\!- 3 : int}{\{x{:}int\} \mid\!- x + 2 = 3 : \text{bool}}\text{Bin}$$

Example: {x:int} |- x + 2 = 3 :bool

How to finish?

$$\frac{\dfrac{\{x{:}int\} \mid\!- x{:}int \quad \{x{:}int\} \mid\!- 2{:}int}{\{x : int\} \mid\!- x + 2 : int}\text{Bin} \qquad \{x{:}int\} \mid\!- 3 : int}{\{x{:}int\} \mid\!- x + 2 = 3 : \text{bool}}\text{Bin}$$

Example: {x:int} |- x + 2 = 3 :bool

Complete Proof (type derivation)

$$\frac{\dfrac{\overline{\text{{x:int} |- x:int}}^{\ Var}\quad \dfrac{}{\text{{x:int} |- 2:int}}^{\ Const}}{\text{{x : int} |- x + 2 : int}}^{\ Bin}\quad \dfrac{}{\text{{x:int} |- 3 :int}}^{\ Const}}{\text{{x:int} |- x + 2 = 3 : bool}}^{\ Bin}$$

Simple Rules - Booleans

Connectives

$$\frac{\Gamma\ |\text{-}\ e_1 : bool \quad \Gamma\ |\text{-}\ e_2 : bool}{\Gamma\ |\text{-}\ e_1\ \&\&\ e_2 : bool}$$

$$\frac{\Gamma\ |\text{-}\ e_1 : bool \quad \Gamma\ |\text{-}\ e_2 : bool}{\Gamma\ |\text{-}\ e_1\ ||\ e_2 : bool}$$

Type Variables in Rules

- If_then_else rule:

$$\frac{\Gamma\ |\text{-}\ e_1 : bool \quad \Gamma\ |\text{-}\ e_2 : \tau \quad \Gamma\ |\text{-}\ e_3 : \tau}{\Gamma\ |\text{-}\ (if\ e_1\ then\ e_2\ else\ e_3) : \tau}$$

- τ is a type variable (meta-variable)
- Can take any type at all
- All instances in a rule application must get same type
- Then branch, else branch and if_then_else must all have same type

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Function Application

- Application rule:

$$\frac{\Gamma\ |\text{-}\ e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma\ |\text{-}\ e_2 : \tau_1}{\Gamma\ |\text{-}\ (e_1\ e_2) : \tau_2}$$

- If you have a function expression e_1 of type $\tau_1 \rightarrow \tau_2$ applied to an argument e_2 of type τ_1, the resulting expression $e_1 e_2$ has type τ_2

Fun Rule

- Rules describe types, but also how the environment Γ may change
- Can only do what rule allows!
- fun rule:

$$\frac{\{x : \tau_1\} + \Gamma\ |\text{-}\ e : \tau_2}{\Gamma\ |\text{-}\ fun\ x \text{->}\ e : \tau_1 \rightarrow \tau_2}$$

Fun Examples

$$\frac{\{y : int\} + \Gamma\ |\text{-}\ y + 3 : int}{\Gamma\ |\text{-}\ fun\ y \text{->}\ y + 3 : int \rightarrow int}$$

$$\frac{\{f : int \rightarrow bool\} + \Gamma\ |\text{-}\ f\ 2 :: [true] : bool\ list}{\Gamma\ |\text{-}\ (fun\ f \text{->}\ (f\ 2) :: [true])\ : (int \rightarrow bool) \rightarrow bool\ list}$$

(Monomorphic) Let and Let Rec

- let rule:

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \{x : \tau_1\} + \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (\text{let } x = e_1 \text{ in } e_2) : \tau_2}$$

- let rec rule:

$$\frac{\{x : \tau_1\} + \Gamma \vdash e_1 : \tau_1 \quad \{x : \tau_1\} + \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (\text{let rec } x = e_1 \text{ in } e_2) : \tau_2}$$