CS/ECE 374 A ✧ Spring 2026

Homework 2

Due Tuesday, February 3, 2026 at 9pm Central Time

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1. For each of the following languages, describe an equivalent regular expression, and ***briefly explain in English*** why your regular expression is correct. Unless specified otherwise, all languages are over the binary alphabet $\Sigma = \{0, 1\}$. There are infinitely many correct regular expressions for each language.

   (a) All strings that start with `0011`, end with `0011`, and whose length is a multiple of 5.

   (b) All strings that do not begin with `000` or `111`, but do end with `1010`.

   (c) All strings that have an odd number of `1`s before the first `0`, contain an even number of `0`s, and contain `101` as a substring.

   (d) All strings where every run of `0`s with odd length is immediately followed by a run of `1`s with odd length. A run is a consecutive sequence of the same alphabet, *e.g.,* `000` is a run of `0`s with odd length, `111111` is a run of `1`s with even length, and the string `000100` has *one* run of `0`s with odd length (which is immediately followed by one run of `1`s with odd length.)

   *(e) ***Practice only. Do not submit solutions.***
   All strings over the alphabet $\{0, 1, 2, 3\}$ in which every pair of adjacent symbols differs by exactly 1.

   [Hint: As a sanity check: Which of these languages contain the empty string? What about the strings `0` and `1`?]

2. For each of the following languages over the binary alphabet $\Sigma = \{0, 1\}$, describe a DFA that accepts the language, and ***briefly explain in English*** the purpose of each state. You can describe your DFA using a drawing, formal mathematical notation, or a product construction; see the standard DFA rubric.

   (a) All strings that start with `001100` and where the number of `1`s is divisible by 3.

   (b) All even length strings that do not start with `000` or `111`.

   (c) All strings such that ***exactly one*** of the following is true:
   - Every run of `0`s with length divisible by 374 is immediately followed by a run of `1`s with length divisible by 374.
   - The substring `01` appears a number of times divisible by 374.

   [Hint: You might find product constructions and mathematical notation descriptions useful for one or all of them. In fact, don't even try *to draw anything for the last one.]*

3. ***Practice only. Do not submit solutions.***

This question asks about strings over the set of *pairs* of bits, which we will write vertically. Let Σ_2 denote the set of all bit-pairs:

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

We can interpret any string w of bit-pairs as a $2 \times |w|$ matrix of bits; each row of this matrix is the binary representation of some non-negative integer, possibly with leading 0s. Let $hi(w)$ and $lo(w)$ respectively denote the *numerical values* of the top and bottom row of this matrix. For example, $hi(\varepsilon) = lo(\varepsilon) = 0$, and if

$$w = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0011 \\ 0101 \end{bmatrix}$$

then $hi(w) = 3$ and $lo(w) = 5$.

(a) Describe a DFA that accepts the language $L_{+1} = \{w \in \Sigma_2^* \mid hi(w) = lo(w) + 1\}$.
For example, $w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1100 \\ 1011 \end{bmatrix} \in L_{+1}$, because $hi(w) = 12$ and $lo(w) = 11$.

(b) Describe a regular expression for L_{+1}.

(c) Describe a DFA that accepts the language $L_{\times 3} = \{w \in \Sigma_2^* \mid hi(w) = 3 \cdot lo(w)\}$.
For example, $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1001 \\ 0011 \end{bmatrix} \in L_3$, because $hi(w) = 9$ and $lo(w) = 3$.

(d) Describe a regular expression for $L_{\times 3}$.

*(e) Describe a DFA that accepts the language $L_{\times 3/2} = \{w \in \Sigma_2^* \mid 2 \cdot hi(w) = 3 \cdot lo(w)\}$.
For example, $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1001 \\ 0110 \end{bmatrix} \in L_{\times 3/2}$, because $hi(w) = 9$ and $lo(w) = 6$.

(Don't bother with the regular expression for this one.)

Solved problem

4. ***C comments*** are the set of strings over alphabet $\Sigma = \{$`*`, `/`, `A`, `◇`, `↲`$\}$ that form a proper comment in the C program language and its descendants, like C++ and Java. Here `↲` represents the newline character, `◇` represents any other whitespace character (like the space and tab characters), and `A` represents any non-whitespace character other than `*` or `/`.[1] There are two types of C comments:

   - Line comments: Strings of the form `//` ⋯ `↲`
   - Block comments: Strings of the form `/*` ⋯ `*/`

   Following the C99 standard, we explicitly disallow ***nesting*** comments of the same type. A line comment starts with `//` and ends at the first `↲` after the opening `//`. A block comment starts with `/*` and ends at the the first `*/` completely after the opening `/*`; in particular, every block comment has at least two `*`s. For example, each of the following strings is a valid C comment:

   `/***/` `//◇//◇↲` `/*///◇*◇↲**/` `/*◇//◇↲◇*/`

   On the other hand, *none* of the following strings is a valid C comment:

   `/*/` `//◇//◇↲◇↲` `/*◇/*◇*/◇*/`

   (Questions about C comments start on the next page.)

[1]The actual C commenting syntax is considerably more complex than described here, because of character and string literals.

- The opening `/*` or `//` of a comment must not be inside a string literal (`"`⋯`"`) or a (multi-)character literal (`'`⋯`'`).
- The opening double-quote of a string literal must not be inside a character literal (`'"'`) or a comment.
- The closing double-quote of a string literal must not be escaped (`\"`)
- The opening single-quote of a character literal must not be inside a string literal (`"`⋯`'`⋯`"`) or a comment.
- The closing single-quote of a character literal must not be escaped (`\'`)
- A backslash escapes the next symbol if and only if it is not itself escaped (`\\`) or inside a comment.

For example, the string `"/*\\\"*/"/*"/*\"/*"*/` is a valid string literal (representing the 5-character string `/*\"\*/`, which is itself a valid block comment!) followed immediately by a valid block comment. ***For this homework question, just pretend that the characters `'`, `"`, and `\` don't exist.***

Commenting in C++ is even more complicated, thanks to the addition of *raw* string literals. Don't ask.

Some C and C++ compilers do support nested block comments, in violation of the language specification. A few other languages, like OCaml, explicitly allow nesting block comments.

(a) Describe a regular expression for the set of all C comments.

Solution:

$$\texttt{//}(\texttt{/} + \texttt{*} + \texttt{A} + \diamond)^*\hookleftarrow \quad + \quad \texttt{/*}\Big(\texttt{/} + \texttt{A} + \diamond + \hookleftarrow + \texttt{*}\texttt{*}^*(\texttt{A} + \diamond + \hookleftarrow)\Big)^* \; \texttt{*}^*\texttt{*/}$$

The first subexpression matches all line comments, and the second subexpression matches all block comments. Within a block comment, we can freely use any symbol other than $\texttt{*}$, but any run of $\texttt{*}$s must be followed by a character in $(\texttt{A} + \diamond + \hookleftarrow)$ or by the closing slash of the comment. ■

Rubric: Standard regular expression rubric. This is not the only correct solution.

(b) Describe a regular expression for the set of all strings composed entirely of blanks ($\diamond$), newlines ($\hookleftarrow$), and C comments.

Solution:

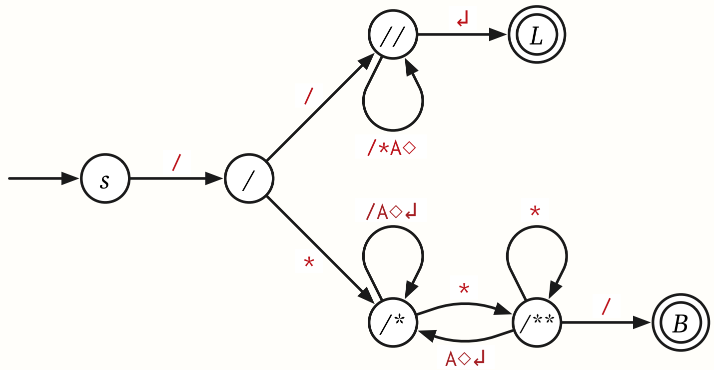
$$\Big(\diamond + \hookleftarrow \; + \; \texttt{//}(\texttt{/} + \texttt{*} + \texttt{A} + \diamond)^*\hookleftarrow + \texttt{/*}(\texttt{/} + \texttt{A} + \diamond + \hookleftarrow + \texttt{*}\texttt{*}^*(\texttt{A} + \diamond + \hookleftarrow))^* \, \texttt{*}\texttt{*}^*\texttt{/}\Big)^*$$

This regular expression has the form $(\langle\text{whitespace}\rangle + \langle\text{comment}\rangle)^*$, where $\langle\text{whitespace}\rangle$ is the regular expression $\diamond + \hookleftarrow$ and $\langle\text{comment}\rangle$ is the regular expression from part (a). ■

Rubric: Standard regular expression rubric. This is not the only correct solution.

(c) Describe a DFA that accepts the set of all C comments.

Solution: The following eight-state DFA recognizes the language of C comments. All missing transitions lead to a hidden reject state.



The states are labeled mnemonically as follows:

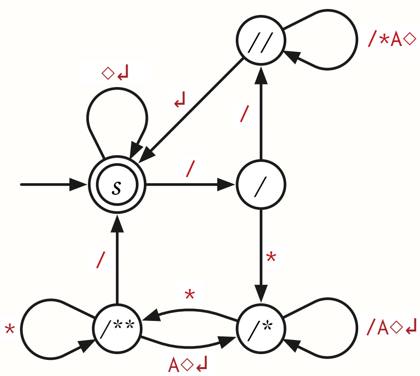
- *s* — We have not read anything.
- / — We just read the initial /.
- // — We are reading a line comment.
- *L* — We have just read a complete line comment.
- /* — We are reading a block comment, and we did not just read a * after the opening /*.
- /** — We are reading a block comment, and we just read a * after the opening /*.
- *B* — We have just read a complete block comment.

■

Rubric: Standard DFA design rubric. This is not the only correct solution, or even the simplest correct solution. (We don't need two distinct accepting states.)

(d) Describe a DFA that accepts the set of all strings composed entirely of blanks (◇), newlines (↲), and C comments.

Solution: By merging the accepting states of the previous DFA with the start state and adding white-space transitions at the start state, we obtain the following six-state DFA. Again, all missing transitions lead to a hidden reject state.



The states are labeled mnemonically as follows:

- *s* — We are between comments.
- / — We just read the initial / of a comment.
- // — We are reading a line comment.
- /* — We are reading a block comment, and we did not just read a * after the opening /*.
- /** — We are reading a block comment, and we just read a * after the opening /*.

■

Rubric: Standard DFA design rubric. This is not the only correct solution, but it is the simplest correct solution.

*5. Recall that the reversal w^R of a string w is defined recursively as follows:

$$w^R := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ x^R \bullet a & \text{if } w = a \cdot x \end{cases}$$

The reversal L^R of any *language* L is the set of reversals of all strings in L:

$$L^R := \left\{ w^R \mid w \in L \right\}.$$

Prove that the reversal of every regular language is regular.

Solution: Let r be an arbitrary regular expression. We want to derive a regular expression r' such that $L(r') = L(r)^R$.

Assume for every regular expression s smaller than r that there is a regular expression s' such that $L(s') = L(s)^R$.

There are five cases to consider (mirroring the definition of regular expressions).

(a) If $r = \varnothing$, then we set $r' = \varnothing$, so that

$$\begin{aligned} L(r)^R &= L(\varnothing)^R && \text{because } r = \varnothing \\ &= \varnothing^R && \text{because } L(\varnothing) = \varnothing \\ &= \varnothing && \text{because } \varnothing^R = \varnothing \\ &= L(\varnothing) && \text{because } L(\varnothing) = \varnothing \\ &= L(r') && \text{because } r = \varnothing \end{aligned}$$

(b) If $r = w$ for some string $w \in \Sigma^*$, then we set $r' := w^R$, so that

$$\begin{aligned} L(r)^R &= L(w)^R && \text{because } r = w \\ &= \{w\}^R && \text{because } L(\langle\text{string}\rangle) = \{\langle\text{string}\rangle\} \\ &= \{w^R\} && \text{by definition of } L^R \\ &= L(w^R) && \text{because } L(\langle\text{string}\rangle) = \{\langle\text{string}\rangle\} \\ &= L(r') && \text{because } r = w^R \end{aligned}$$

(c) Suppose $r = s^*$ for some regular expression s. The inductive hypothesis implies a regular expressions s' such that $L(s') = L(s)^R$. Let $\boldsymbol{r' = (s')^*}$; then we have

$$\begin{aligned} L(r)^R &= L(s^*)^R && \text{because } r = s^* \\ &= (L(s)^*)^R && \text{by definition of } {}^* \\ &= (L(s)^R)^* && \text{because } (L^R)^* = (L^*)^R \\ &= (L(s'))^* && \text{by definition of } s' \\ &= L((s')^*) && \text{by definition of } {}^* \\ &= L(r') && \text{by definition of } r' \end{aligned}$$

(d) Suppose $r = s + t$ for some regular expressions s and t. The inductive hypothesis implies regular expressions s' and t' such that $L(s') = L(s)^R$ and $L(t') = L(t)^R$.

Set $\boldsymbol{r' := s' + t'}$; then we have

$$
\begin{aligned}
L(r)^R &= L(s+t)^R && \text{because } r = s+t \\
&= (L(s) \cup L(t))^R && \text{by definition of } + \\
&= \{w^R \mid w \in (L(s) \cup L(t))\} && \text{by definition of } L^R \\
&= \{w^R \mid w \in L(s) \text{ or } w \cup L(t)\} && \text{by definition of } \cup \\
&= \{w^R \mid w \in L(s)\} \ \cup \ \{w^R \mid w \cup L(t)\} && \text{by definition of } \cup \\
&= L(s)^R \cup L(t)^R && \text{by definition of } L^R \\
&= L(s') \cup L(t') && \text{by definition of } s' \text{ and } t' \\
&= L(s' + t') && \text{by definition of } + \\
&= L(r') && \text{by definition of } r'
\end{aligned}
$$

(e) Suppose $r = s \bullet t$ for some regular expressions s and t. The inductive hypothesis implies regular expressions s' and t' such that $L(s') = L(s)^R$ and $L(t') = L(t)^R$. Set $\boldsymbol{r' = t' \bullet s'}$; then we have

$$
\begin{aligned}
L(r)^R &= L(st)^R && \text{because } r = s+t \\
&= (L(s) \bullet L(t))^R && \text{by definition of } \bullet \\
&= \{w^R \mid w \in (L(s) \bullet L(t))\} && \text{by definition of } L^R \\
&= \{(x \bullet y)^R \mid x \in L(s) \text{ and } y \in L(t)\} && \text{by definition of } \bullet \\
&= \{y^R \bullet x^R \mid x \in L(s) \text{ and } y \in L(t)\} && \text{concatenation reversal} \\
&= \{y' \bullet x' \mid x' \in L(s)^R \text{ and } y' \in L(t)^R\} && \text{by definition of } L^R \\
&= \{y' \bullet x' \mid x' \in L(s') \text{ and } y' \in L(t')\} && \text{by definition of } s' \text{ and } t' \\
&= L(t') \bullet L(s') && \text{by definition of } \bullet \\
&= L(t' \bullet s') && \text{by definition of } \bullet \\
&= L(r') && \text{by definition of } r'
\end{aligned}
$$

In all five cases, we have found a regular expression r' such that $L(r') = L(r)^R$. It follows that $L(r)^R$ is regular. ■

Rubric: Standard induction rubric!!