CS/ECE 374 A ✧ Spring 2026

Homework 11

Due Tuesday, April 28, 2026 at 9pm Central Time

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1. Consider the DominatingSet decision problem:

   Input: an undirected graph $G = (V, E)$ and an integer $k \geqslant 0$.

   Output: Yes if and only if there exists a subset $S \subseteq V$ of vertices with size at most k such that every vertex $u \in V$ is either in S or is adjacent to at least one vertex from S. Formally, $|S| \leqslant k$ and for each $u \in V$, either $u \in S$ or exists $v \in S$ such that $u - v \in E$.

   One can prove DominatingSet is NP-complete using a reduction from VertexCover. **(We are *not* asking you to prove the previous statement.)**

   Now consider its relaxation, the 374-hop-DominatingSet problem:

   Input: an undirected graph $G' = (V', E')$ and an integer $k' \geqslant 0$.

   Output: Yes if and only if there exists a subset $S' \subseteq V'$ of vertices with size at most k' such that every vertex $u \in V'$ is at most 374 distance away from the closest vertex in S'. Formally, $|S'| \leqslant k'$ and for each $u \in V$, there exists $v \in S'$ such that $dist_{G'}(u, v) \leqslant 374$ where $dist_{G'}(u, v)$ is the shortest path distance (by number of edges) between u and v in G'.

   Prove 374-hop-DominatingSet is NP-*hard* **by describing a reduction from DominatingSet**.

   Aside: Note that, the reduction extends to h-hop-DominatingSet for $1 \leqslant h < O(\sqrt{n})$.

2. Consider the following decision problem Colorful-Walk:

   Input: a directed graph $G = (V, E)$, a vertex $s \in V$, a color $c(e) \in \{1, \ldots, k\}$ for each edge $e \in E$, and a number ℓ.

   Output: Yes if and only if there exists a walk that starts at s and encounters at least ℓ distinct colors.

   Note that the walk need not be simple. That is, it can repeat vertices and edges. Show that Colorful-Walk is NP-**complete**.

Some useful NP-hard problems. You are welcome to use any of these in your own NP-hardness proofs, except of course for the specific problem you are trying to prove NP-hard.

3Sat: Given a boolean formula in conjunctive normal form, with exactly three distinct literals per clause, does the formula have a satisfying assignment?

IndependentSet: Given an undirected graph G and an integer k, is there a subset of k vertices in G that have no edges among them?

Clique: Given an undirected graph G and an integer k, is there a subset of k vertices in G where there is an edge between every pair of them?

VertexCover: Given an undirected graph G and an integer k, is there a subset of k vertices that touch every edge in G?

DominatingSet: Given an undirected graph G and an integer k, is there a subset of k vertices such that every vertex of G is either in the set or adjacent to a member of the set?

SetCover: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S and an integer k, is there a subcollection of k of these subsets whose union is S?

HittingSet: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S and an integer k, is there a subset of S of size k that intersects every subset S_i?

3Color: Given an undirected graph G, can its vertices be colored with three colors so that every edge touches vertices with two different colors?

HamiltonianPath: Given graph G (either directed or undirected), is there a path in G that visits every vertex exactly once?

HamiltonianCycle: Given a graph G (either directed or undirected), is there a cycle in G that visits every vertex exactly once?

TravelingSalesman: Given a graph G (either directed or undirected) with weighted edges and a number L, is there a Hamiltonian path/cycle of weight at most L in G?

LongestPath: Given a graph G (either directed or undirected, possibly with weighted edges) and a number L, is there a simple path of length at least L in G?

SteinerTree: Given an undirected graph G with some of the vertices marked and an integer k, is there a subtree of G with at most k edges that contains every marked vertex?

SubsetSum: Given a set X of positive integers and an integer k, does X have a subset whose elements sum to k?

Partition: Given a set X of positive integers, can X be partitioned into two subsets with the same sum?

3Partition: Given a set X of $3n$ positive integers, can X be partitioned into n three-element subsets, all with the same sum?

Draughts: Given an $n \times n$ international draughts configuration and an integer k, is there a move that can (and therefore must) capture at least k pieces in a single move?