Def Given transition fn δ,
define its extended transition fn $\delta^*: Q \times \Sigma^* \to Q$ inductively:

(i) $\delta^*(q, \varepsilon) = q$

(ii) $\delta^*(q, x) = \delta^*(\delta(q,a), y)$ if $x = ay$ with $a \in \Sigma$, $y \in \Sigma^*$

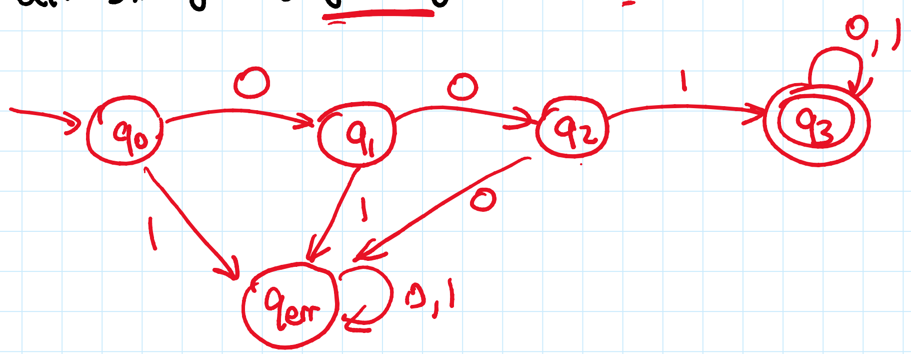
Def M accepts x iff $\delta^*(q_0, x) \in A$.

Define $L(M) = \{x \in \Sigma^* : M \text{ accepts } x\}$.

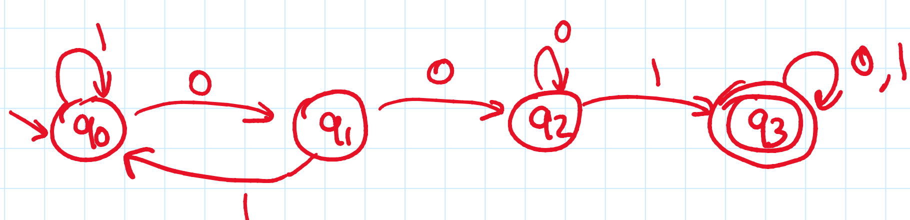
lang accepted by M

Exs $(\Sigma = \{0,1\})$.

a) all strings beginning with 001



b) all strings containing 001 as substring



q_3: found 001

q_2: just seen 00 but not found 001

q_1: just seen 0 but not in q_2, q_3

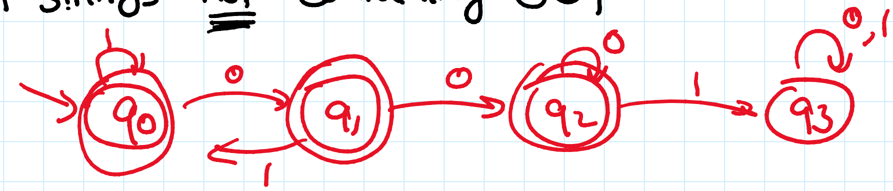
q_0: none of above

(clarification: drop 6 lowest means 4 written HW probs + 2 GPSs)

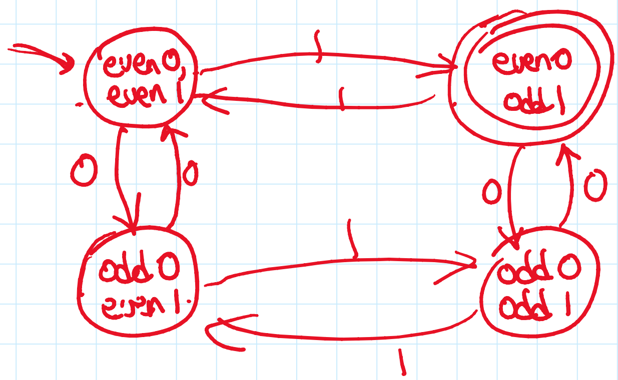
+ 2 GPSs
(see course web page)

not in q_2, q_3
q_0 : none of above

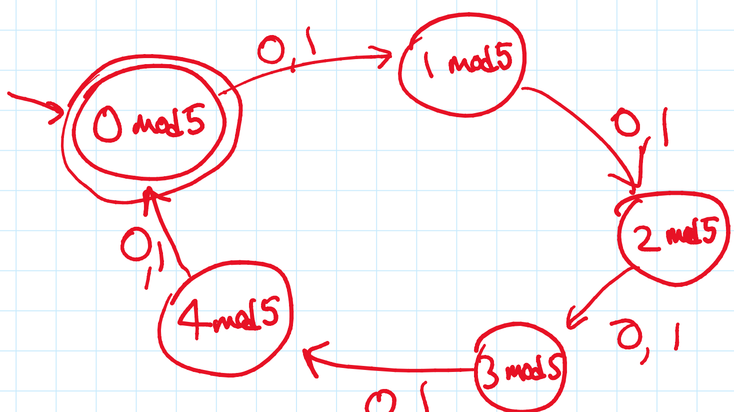
c) all strings not containing 001



d) all strings with even # 0's
and odd # 1's



e) strings with length divisible by 5



f) binary representation of all numbers
divisible by 5

e.g. ε, 101, 1010, 1111, ...

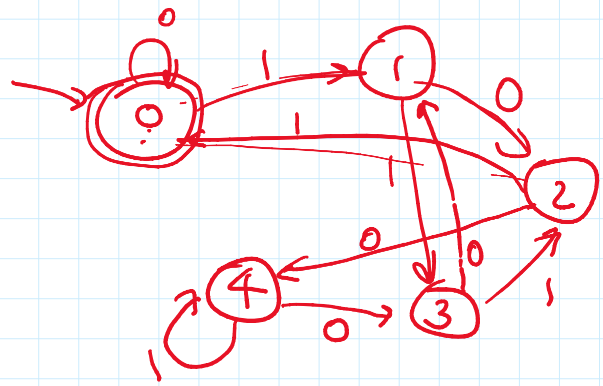
($Q = \{0, 1, 2, 3, 4\}$

$$\begin{cases} Q = \{0,1,2,3,4\} \\ A = \{0\}. \\ \delta(i,a) = (2i+a) \bmod 5 \end{cases}$$

$\underbrace{\boxed{}}_{i}\, a$

$\underbrace{101}\ 1$

$5 \times 2 + 1$



000101

g) all strings where 5th last symbol is 0.

e.g. 0110110100 1 1 - - - -

$$\begin{cases} Q = \{\text{ all strings of length } 5\} & |Q| = 32 \\ s = 11111 & \\ \delta(a_1a_2a_3a_4a_5, a) = a_2a_3a_4a_5a & \forall a_1,..,a_5 \in \{0,1\} \\ & a \in \{0,1\} \\ A = \{a_1a_2a_3a_4a_5 : a_1 = 0\}. & \end{cases}$$

Closure Properties

Thm If L is accepted by some DFA M, then its complement $\bar{L}$ is also accepted by some DFA M'.

Pf: idea - take complement of accepting states

Given $M = (Q, \Sigma, s, A, \delta)$,

Given $M = (Q, \Sigma, s, A, \delta)$,

Construct $M' = (Q, \Sigma, s, A', \delta)$

where $A' = Q \setminus A$

Then $x \in L(M') \iff \delta^*(s,x) \in A'$

$\iff \delta^*(s,x) \notin A$

$\iff x \notin L(M) = L.$

$\therefore L(M') = \overline{L}.$ $\square$

Thm If L_1 is accepted by DFA M_1,
L_2 " " " " M_2,
then $L_1 \cap L_2$ is also accepted by some DFA M'.

Pf: idea - "remember" a pair of states

Given $M_1 = (Q_1, \Sigma, s_1, A_1, \delta_1)$
$M_2 = (Q_2, \Sigma, s_2, A_2, \delta_2)$

Construct $M' = (Q', \Sigma, s', A', \delta')$.

where $\boxed{Q' = Q_1 \times Q_2}$ $\quad |Q'| = |Q_1||Q_2|$

$s' = (s_1, s_2)$

$A' = A_1 \times A_2 = \{(q_1, q_2) : q_1 \in A_1 \text{ and } q_2 \in A_2\}$

$\delta' : Q' \times \Sigma \to Q'$:

$\boxed{\delta'((q_1,q_2), a) = (\delta_1(q_1,a), \delta_2(q_2,a))}$ $\quad q_1 \in Q_1, q_2 \in Q_2$

product construction

Lemma $\delta'^*((q_1,q_2), x) = (\delta_1^*(q_1,x), \delta_2^*(q_2,x))$

Pf: by induction (omitted). $\square$ $\leftarrow$

$$x \in L(M') \iff \underbrace{\delta'^*((s_1,s_2),x)}_{(\delta_1^*(s_1,x),\ \delta_2^*(s_2,x))} \in \underline{A_1 \times A_2}$$

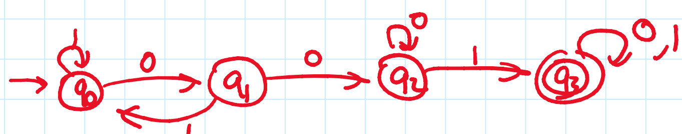
$$\iff \underbrace{\delta_1^*(s_1,x) \in A_1}\ \underline{\text{and}}\ \delta_2^*(s_2,x) \in A_2$$

$$\iff x \in \underbrace{L(M_1)}_{=L_1}\ \underline{\text{and}}\ x \in \underbrace{L(M_2)}_{=L_2}$$

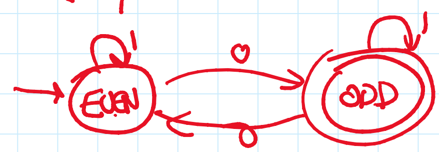
$$\iff x \in L_1 \cap L_2 . \quad \square$$

Ex all strings containing 001 <u>and</u> having odd # 0's.

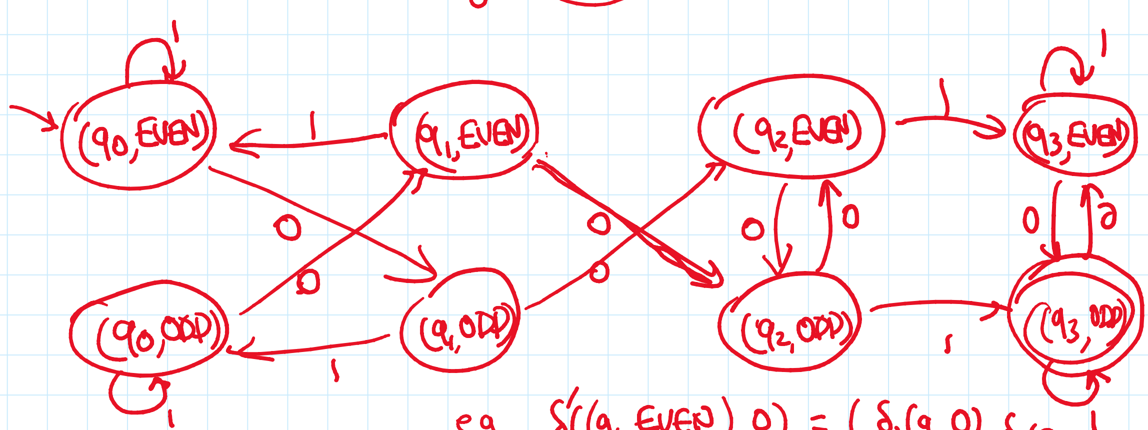
M_1:



M_2:



M':



e.g. $\delta'((q_1, EVEN), 0) = (\underbrace{\delta_1(q_1,0)}, \delta_2(EVEN, 0))$
$= (q_2, ODD)$

Cor If L_1 accepted by some DFA,
L_2 " " " ,
then so is $L_1 \cup L_2$ $\left(= \overline{\overline{L_1} \cap \overline{L_2}}\right)$ (de Morgan's law)
$L_1 \setminus L_2$
$= L_1 \cap \overline{L_2}$