February 15, 2024 9:52 AM

10. reg expr containing 10 or 01 but not both

11111 000000

00000 1111111

~~0000000~~

~~1111111~~

$1^+0^+ + 0^+1^+$

or

$11^*00^* + 00^*11^*$

Var of 10. What if we want the complement?

Case. 00000

1111111

11111 0000 11111 ...

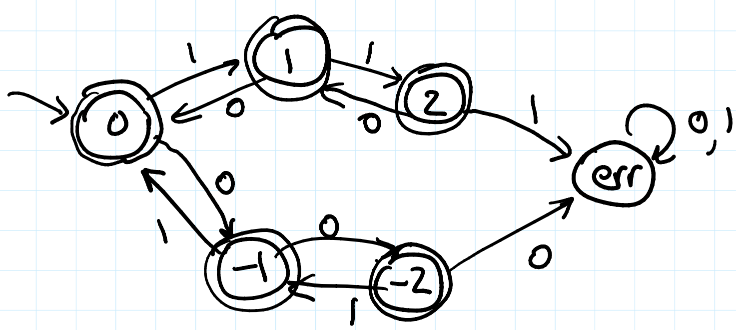
or 0000 1111 0000 ...

0^*
$+ 1^*$
$+ 1^+0^+1^+(0+1)^*$
$+ 0^+1^+0^+(0+1)^*$

29. In every prefix, $|\#1\text{'s} - \#0\text{'s}| \le 2$.

e.g. 10011011000 $\in L$

1100000 $\notin L$



state $i \in \{-2,-1,0,1,2\}$: curr value $\#1\text{'s} - \#0\text{'s} = i$

28. $\{w \in \{0,1\}^* : w^R$ is bin rep of # div. by 5$\}$

e.g. 5, 10 → 101 ∈ L
101, 1010   0101 ∈ L

2   add 2^3

State (v,t): v stands curr value mod 5
t stands for $\underset{\text{mod } 5}{2^i}$. i = curr pos

$$Q = \{0,1,2,3,4\} \times \{0,1,2,3,4\}$$

$$\delta((v,t), a) = ((v + at) \bmod 5,\ 2t \bmod 5)$$
$\forall a \in \{0,1\}$

$$A = \{(v,t) : v = 0\}.$$

39. balanced parentheses $\Sigma = \{(,)\}$
not regular.   eg. (( ) ( )) ( )

Pf: Fooling sets.

Let $F = \{ (^i : i \geq 1 \}$

Given 2 arbitrary strings $x, y \in F$, $x \neq y$,
$x = (^i$, $y = (^j$, for some $i \neq j$
(w.l.o.g. $i < j$)

Choose $z =)^i$.

alternate: $\cancel{z =)^j}$
$\cancel{z = ()^{i+1}}$

$xz = (^i)^i \in L$
$yz = (^j)^i \notin L$ because $i \neq j$

$\therefore$ F is fooling set for L, & infinite.

$\therefore$ Not reg. $\square$

~~18~~ 77. If L reg. then $\text{Evens}(L) = \{\text{evens}(x) : x \in L\}$ reg.

~~78~~ 77. If L reg. then $\text{Evens}(L)$ reg.

eg. $\text{evens}(0\underline{0}1\underline{0}1\underline{1}) = 001$

$\text{evens}(1\underline{1}0\underline{1}1\underline{0}) = 110$ ← your input

feed this in M — M' reading this

Let $M = (Q, \Sigma, s, \delta, A)$ be DFA accepting L.

Construct new NFA $M' = (Q', \Sigma, s', \delta', A')$ to accept $\text{Evens}(L)$.

$$Q' = Q \times \{\text{read}, \text{no-read}\}$$

$$s' = (s, \text{not-read})$$

$$\delta'((q, \text{not-read}), \varepsilon) = \{(\delta(q,0), \text{read}), (\delta(q,1), \text{read})\}$$

$$\delta'((q, \text{read}), a) = \{(\delta(q,a), \text{not-read})\}$$

$$A' = \{(q, \text{not-read}) : q \in A\} \cup \{(q, \text{read}) : q \in A\} = A \times \{\text{read}, \text{no-read}\}$$

alternate soln:

$Q' = Q$

$\delta'(q,a) = \{\delta(\delta(q,a),b) : b \in \Sigma\}$

$A' = A \cup \{q : \delta(q,b) \in A,\ b \in \Sigma\}$

131. T/F: If L_1, L_2 not reg then $L_1 \cap L_2$ is not reg.

False. Counterex: $L_1 = \{0^i 1^i : i \geq 1\}$ not reg

$L_2 = \{1^i 0^i : i \geq 1\}$ not reg

$L_1 \cap L_2 = \phi$ reg.

$A \to B$

$\neg B \to \neg A$

if $L_1 \cap L_2$ reg, L_1 reg or L_2 reg.

132. T/F: If L_1, L_2 not reg, then $L_1 \cup L_2$ is not reg.

False. Counterex: $L_1 = \{0^p : p \text{ prime}\}$ not reg

$L_2 =$ complement of L_1 not reg.

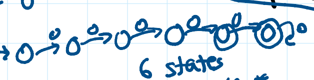
$L_1 \cup L_2 = \{0\}^*$

$L_1 \cup L_2 = \{0\}$

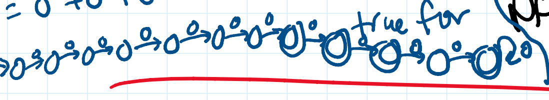
(or pick any non-reg L_1
pick $L_2 = \overline{L_1}$ non-reg.
$L_1 \cup L_2 = \Sigma^*$ reg)

182. T/F: If L has DFA with n states, L^* has DFA with $\leq n+1$ states.

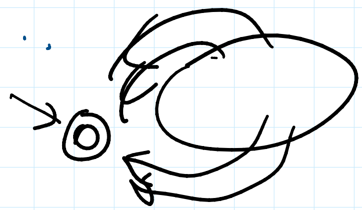
CounterEx:
$L = 0^4 + 0^5$

DFA

6 states

$L^* = 0^8 + 0^9 + 0^{10} + 0^{12} 0^*$ 12 states

DFA

true for NFA

False.



F'18 Q6. $L_1 + L_2 L_3^*$ $G_1 = (V_1, \quad)$

$S \to S_1 \mid S_2 T$

$T \to S_3 T \mid \varepsilon$