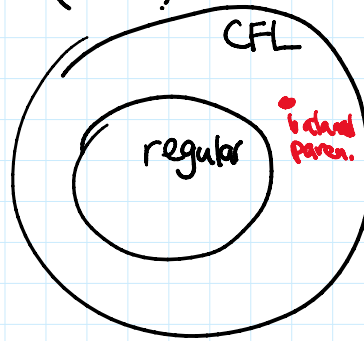


$\{ww : w \in \{0,1\}^*\}$
 $\{0^n 1^{2^n} : n \geq 0\}$

Last Time: Context-free languages (CFL)

e.g. all strings of balanced parentheses
not reg, but is CFL



$(() ()) ()$

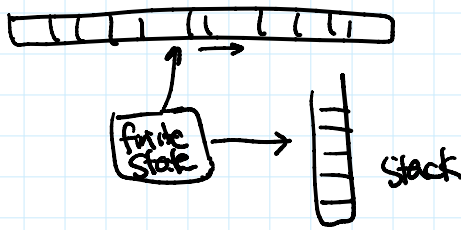
$$S \rightarrow (S) \mid SS \mid \epsilon$$

all strings with equal # 0's & # 1's

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \epsilon$$

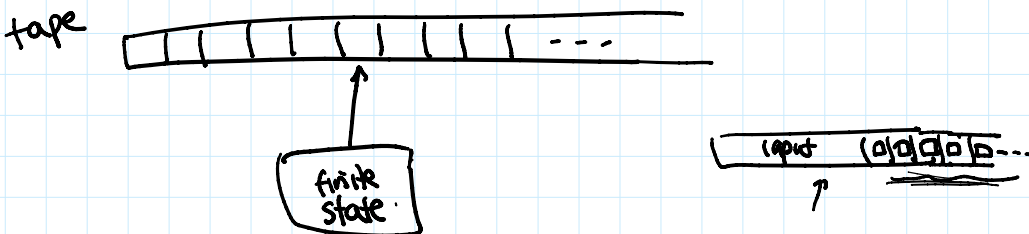
⋮

CFL \iff langs accepted by
nondet. pushdown automata (PDAs)



CFL not closed under complement.
not closed " intersection. (but CFL \cap reg is CFL)

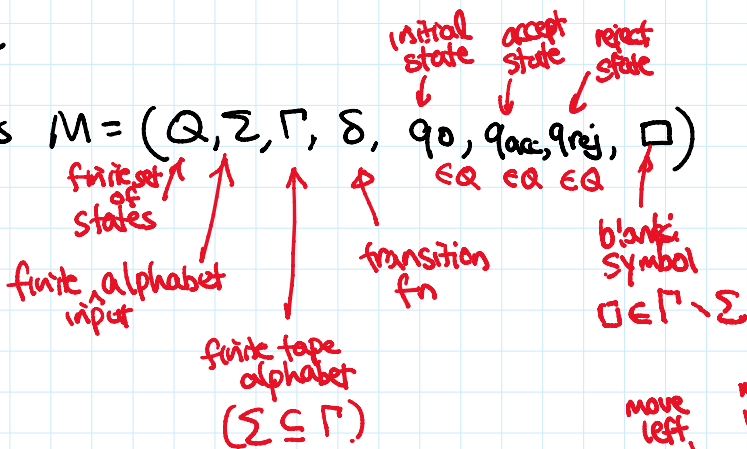
Turing Machines (TMs)



- read/write one char of current position (head)
- move head one position to right or left
- unbounded tape

Formal Def'n

A TM is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej}, \square)$

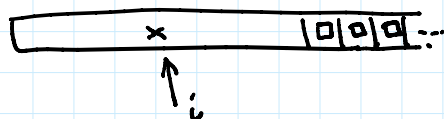


where

$$\delta : (Q - \{q_{acc}, q_{rej}\}) \times \Gamma \rightarrow Q \times \Gamma \times \{-1, +1\}$$

Def

A configuration is $C = (q, x, i) \in Q \times \Gamma^* \times \mathbb{N}$



We treat (q, x, i) as equiv to $(q, x\square, i)$

Def

$(q, a_0 a_1 \dots a_{i-1} a_i a_{i+1} \dots a_n, i)$

(single move) $\xrightarrow{M} (q', a_0 \dots a_{i-1} a'_i a_{i+1} \dots a_n, i + D)$
 if $\delta(q, a_i) = (q', a'_i, D)$

Def

$C \xrightarrow{k} M C'$ if $\exists C_1, \dots, C_{k-1}$ st.
 $C \xrightarrow{M} C_1 \xrightarrow{M} C_2 \xrightarrow{M} \dots \xrightarrow{M} C_{k-1} \xrightarrow{M} C'$
 $C \xrightarrow{*} M C'$ if $C \xrightarrow{k} M C'$ for some k

Def

Given $x \in \Sigma^*$,
 M accepts x if $(q_0, x, 0) \xrightarrow{*} M (q_{acc}, y, i)$

M accepts x if $(q_0, x, 0) \xrightarrow{M} (q_{acc}, y, i)$
for some y, i .

M rejects x if $(q_0, x, 0) \xrightarrow{*M} (q_{rej}, y, i)$
for some y, i .

(other possibilities: crash or not halt...)

Given $L \subseteq \Sigma^*$,

\rightarrow M accepts L if $L = \{x \in \Sigma^* : M \text{ accepts } x\}$

\rightarrow M decides L if $\forall x \in L, M$ accepts x
and $\forall x \notin L, M$ rejects x .

Def L is decidable / recursive if L is decided by some TM.
 L is recursively enumerable if L is accepted by some TM.

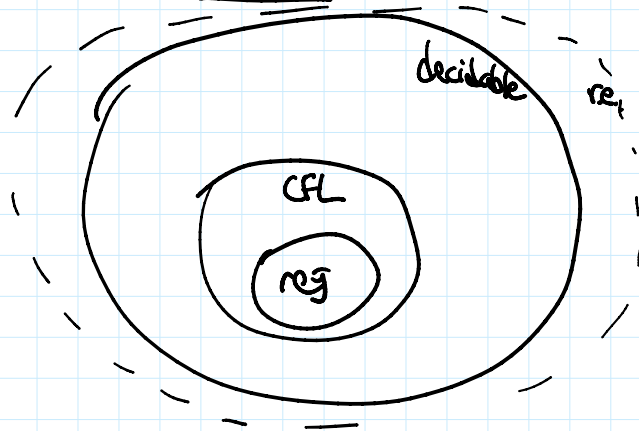
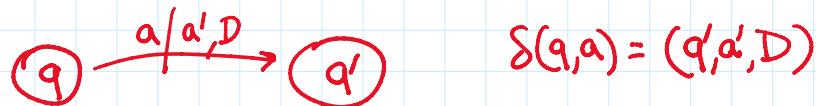
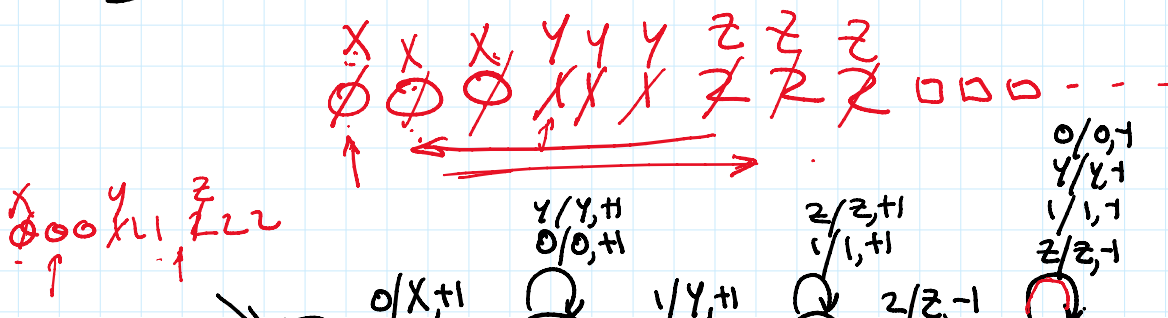
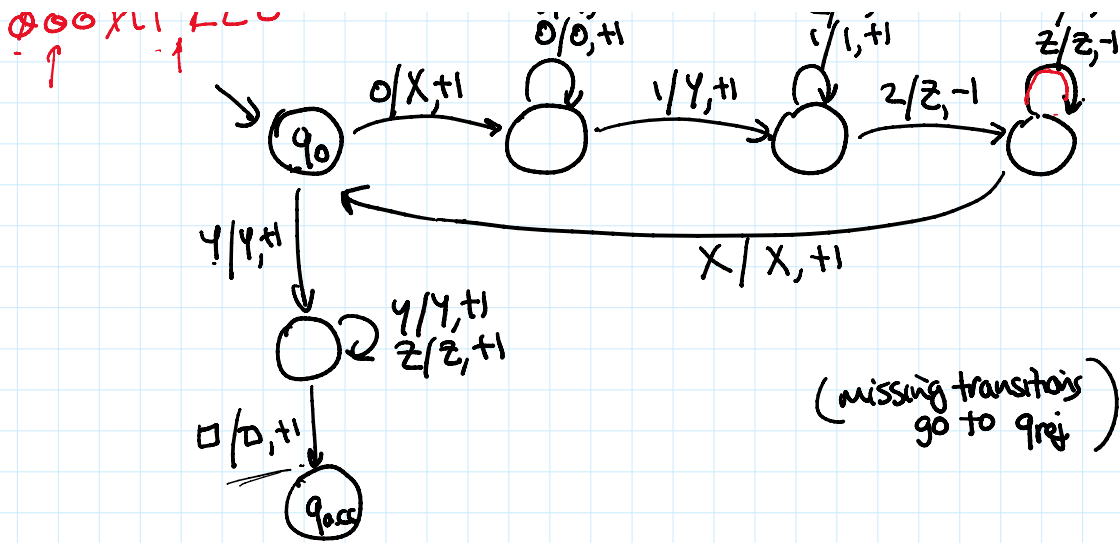


Diagram notation



Ex $\{0^n 1^n 2^n : n \geq 1\}$





$\{ 0^n : n \text{ prime} \}$

- can do:
- loops
 - function calls
 - arrays
 - anything doable in C/Java/python

can simulate itself

\exists universal TM :
 input: ^{encoding of} any TM M , string x
 accept iff M accepts x

Variations of TM:

- multiple tapes, heads, ...
- nondet.

all equiv.

Alternative models .. RAM, λ -calculus, unrestricted grammars

\Rightarrow Church-Turing thesis:
 anything computable is computable on TMs.