This is a review of context-free grammars from the lecture; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$$S \to \varepsilon \mid S\texttt{(}S\texttt{)} \qquad\qquad \text{properly nested parentheses}$$

  Here is a different grammar for the same language:

$$S \to \varepsilon \mid \texttt{(}S\texttt{)} \mid SS \qquad\qquad \text{properly nested parentheses}$$

- $\{\texttt{0}^m\texttt{1}^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of $\texttt{0}$s followed by a different number of $\texttt{1}$s.

$$
\begin{aligned}
S &\to A \mid B & \{\texttt{0}^m\texttt{1}^n \mid m \neq n\} \\
A &\to \texttt{0}A \mid \texttt{0}C & \{\texttt{0}^m\texttt{1}^n \mid m > n\} \\
B &\to B\texttt{1} \mid C\texttt{1} & \{\texttt{0}^m\texttt{1}^n \mid m < n\} \\
C &\to \varepsilon \mid \texttt{0}C\texttt{1} & \{\texttt{0}^m\texttt{1}^n \mid m = n\}
\end{aligned}
$$

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1. $\{\texttt{0}^{2n}\texttt{1}^n \mid n \geq 0\}$

2. $\{\texttt{0}^m\texttt{1}^n \mid m \neq 2n\}$

   [Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string $\texttt{01}$.]

3. $\{\texttt{0}, \texttt{1}\}^* \setminus \{\texttt{0}^{2n}\texttt{1}^n \mid n \geq 0\}$

   [Hint: Extend the previous grammar. What's missing?]

Work on these later:

4. $\left\{ w \in \{\texttt{0}, \texttt{1}\}^* \;\middle|\; \#(\texttt{0}, w) = 2 \cdot \#(\texttt{1}, w) \right\}$ — Binary strings where the number of $\texttt{0}$s is exactly twice the number of $\texttt{1}$s.

5. $\{\texttt{0}, \texttt{1}\}^* \setminus \{ww \mid w \in \{\texttt{0}, \texttt{1}\}^*\}$.

   *[Anti-hint: The language $\{ww \mid w \in \texttt{0}, \texttt{1}^*\}$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]*

6. Prove that every regular language is context free.