

Give regular expressions for each of the following languages over the alphabet  $\{0, 1\}$ .

1. All strings containing the substring **000**.
2. All strings *not* containing the substring **000**.
3. All strings in which every run of **0s** has length at least 3.
4. All strings in which **1** does not appear after a substring **000**.
5. All strings containing at least three **0s**.
6. Every string except **000**. [*Hint: Don't try to be clever.*]
7. All strings  $w$  such that *in every prefix of  $w$* , the number of **0s** and **1s** differ by at most 1.
- \*8. All strings containing at least two **0s** and at least one **1**.
- \*9. All strings  $w$  such that *in every prefix of  $w$* , the number of **0s** and **1s** differ by at most 2.
- ★10. All strings in which the substring **000** appears an even number of times.  
(For example, **0001000** and **0000** are in this language, but **00000** is not.)