e.g. $L_1 = \{0, 00\}$, $L_2 = \{1, 01\}$

$L_1 L_2 = \{01, 0001, 001\}$

e.g. $L_1 = \{0, 00, 000, \ldots\} = \{0^i : i \geq 1\}$

$L_2 = \{1, 11, 111, \ldots\} = \{1^i : i \geq 1\}$

$L_1 L_2 = \cancel{\{0^i 1^i : i \geq 1\}}$?

$= \{0^i 1^j : i, j \geq 1\}$.

(OH starts this Sat ...)

(GPS1, HW1 available)

GPS1 due Tue 10am

HW1 due Thur 10am

c) i^{th} power: $L^i = \underbrace{LL \cdots L}_{i \text{ times}}$

i.e. $L^i = L \cdot L^{i-1}$

$L^0 = \{\varepsilon\}$

e.g. $\{1, 01\}^2 = \{11, 0101, 101, 011\}$

$\cancel{001}$

$\cancel{(01)^2 \neq 0^2 1^2}$

d) <u>Kleene star</u>

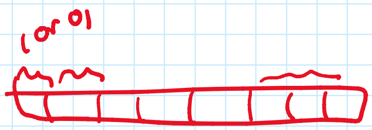
$$L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots$$

e.g. $\{01\}^* = \{\varepsilon, 01, 0101, 010101, \ldots\}$

$\{1, 01\}^* = \{\varepsilon, 1, 01, 11, 101, 011, 0101,$

$111, 1101, 1011, 10101$

$$\{1, 01\}^* = \{\varepsilon, 1, 01, 11, 101, 011, 0101, 111, 1101, 1011, 10101, 0111, 01101, 01011, 010101, \ldots\}$$



$= \{x \in \{0,1\}^* :$ x does not contain 00 as a substring & does not end in 0$\}$

$\{0,1\}^*$ = all binary strings

e) other ops:

$$L^+ = \bigcup_{i=1}^{\infty} L^i$$

e.g. $\{0,1\}^+$ = all nonempty binary strings

$$L^+ = L \cdot L^* = L^* \cdot L$$

$$= \begin{cases} L^* \setminus \{\varepsilon\} & \text{if } \varepsilon \notin L \\ L^* & \text{if } \varepsilon \in L \end{cases}$$

Regular Languages

all langs obtainable from union, concat, and star (starting from trivial base cases)

Formal Defn by Induction:

(i) $\emptyset$, $\{\varepsilon\}$, $\{a\}$ are regular langs $\forall a \in \Sigma$.

(ii) if L_1, L_2 are regular langs, then so are $L_1 \cup L_2$, $L_1 L_2$, and L_1^*

(iii) only langs obtained by finite # of applns of above rules are regular

Ex $(\Sigma = \{0,1\})$

a) $\{1001\}$ is regular.

$= \{1\}\cdot\{0\}\cdot\{0\}\cdot\{1\}$.

b) $\{1001, 10, 1\}$ is regular

$= \{1\}\cdot\{0\}\cdot\{0\}\{1\} \cup \{1\}\{0\} \cup \{1\}$.

c) all finite langs are regular.

d) $\{x \in \{0,1\}^* : |x| \text{ is odd}\}$ is regular.

$= \{00, 01, 10, 11\}^* \cdot \{0,1\}$

$= \left(\left(\{0\}\cup\{1\}\right)\cdot\left(\{0\}\cup\{1\}\right)\right)^* \cdot \left(\{0\}\cup\{1\}\right)$

Notation regular exprs

(i) ϕ, ε, a are regular exprs for ϕ, $\{\varepsilon\}$, $\{a\}$ $\forall a \in \Sigma$.

(ii) if r_1, r_2 are regular exprs for L_1, L_2 resp,

then $(r_1 + r_2)$ is reg expr for $L_1 \cup L_2$

$(r_1 r_2)$ " " " " $L_1 L_2$

(r_1^*) " " " L_1^*

Let $L(r)$ denotes lang. corresponding to expr r.

Ex d) $\left((0+1)(0+1)\right)^* (0+1)$

Rmk - omit unnecessary parentheses $\quad 0(1^*)$

(precedence order: $*, \cdot, +$)

- shorthand: $r^+ = r \cdot r^*$

- a lang may have diff reg expr

e.g. $(0+1)\cdot(00+01+10+11)^*$

etc.

Ex $(\Sigma = \{0,1\})$

a) all strings with 00 as a substring

$$(0+1)^*00(0+1)^*$$

b) all strings with 00 as a substring having even length

$$\boxed{((0+1)(0+1))^*\,00\,((0+1)(0+1))^* + (0+1)((0+1)(0+1))^*\,00\,(0+1)((0+1)(0+1))^*}$$

c) all strings with even # of 0's

$$(1^*01^*01^*)^* + 1^* \checkmark$$

$$(1^*01^*01^* + 1)^* \checkmark$$

or $(1^*)(01^*01^*)^*$ ✓

d) all strings not beginning with 00

$\cancel{00(0+1)^*}$

cases: begin with 1
or begin with 01

$$1(0+1)^* + 01(0+1)^* + \varepsilon + 0$$

$$+\ \underbrace{\varepsilon.\ +\ 0}_{}$$

watch out for
boundary cases