

Design Turing machines $M = (Q, \Sigma, \Gamma, \delta, \text{start}, \text{accept}, \text{reject})$ for each of the following tasks, either by listing the states Q , the tape alphabet Γ , and the transition function δ (in a table), or by drawing the corresponding labeled graph.

Each of these machines uses the input alphabet $\Sigma = \{1, \#\}$; the tape alphabet Γ can be any superset of $\{1, \#, \square, \triangleright\}$ where \square is the blank symbol and \triangleright is a special symbol marking the left end of the tape. Each machine should **reject** any input not in the form specified below.

- 1 On input 1^n , for any non-negative integer n , write $1^n \# 1^n$ on the tape and **accept**.
- 2 On input $\#^n 1^m$, for any non-negative integers m and n , write 1^m on the tape and **accept**. In other words, delete all the $\#$ s and shift the 1 s to the start of the tape.
- 3 On input $\# 1^n$, for any non-negative integer n , write $\# 1^{2n}$ on the tape and **accept**. (**Hint**: Modify the Turing machine from problem 1.)
- 4 On input 1^n , for any non-negative integer n , write 1^{2^n} on the tape and **accept**. (**Hint**: Use the three previous Turing machines as subroutines.)