

- 1** Let L be an arbitrary regular language. Prove that the language $reverse(L) := \{w^R \mid w \in L\}$ is regular. *Hint*: Consider a DFA M that accepts L and construct a NFA that accepts $reverse(L)$.
- 2** Let L be an arbitrary regular language. Prove that the language $insert1(L) := \{x1y \mid xy \in L\}$ is regular. Intuitively, $insert1(L)$ is the set of all strings that can be obtained from strings in L by inserting exactly one **1**. For example, if $L = \{\varepsilon, \mathbf{OOK!}\}$, then $insert1(L) = \{\mathbf{1}, \mathbf{1OOK!}, \mathbf{O1OK!}, \mathbf{OO1K!}, \mathbf{OOK1!}, \mathbf{OOK11}\}$.

Work on these later:

- 3** Let L be an arbitrary regular language. Prove that the language $delete1(L) := \{xy \mid x1y \in L\}$ is regular. Intuitively, $delete1(L)$ is the set of all strings that can be obtained from strings in L by deleting exactly one **1**. For example, if $L = \{\mathbf{101101}, \mathbf{00}, \varepsilon\}$, then $delete1(L) = \{\mathbf{01101}, \mathbf{10101}, \mathbf{10110}\}$.
- 4** Consider the following recursively defined function on strings:

$$stutter(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ aa \bullet stutter(x) & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

Intuitively, $stutter(w)$ doubles every symbol in w . For example:

- $stutter(\mathbf{PRESTO}) = \mathbf{PPRRRESSTTOO}$
- $stutter(\mathbf{HOCUS} \diamond \mathbf{POCUS}) = \mathbf{HHOCCUUSS} \diamond \mathbf{PPOCCUUSS}$

Let L be an arbitrary regular language.

1. Prove that the language $stutter^{-1}(L) := \{w \mid stutter(w) \in L\}$ is regular.
2. Prove that the language $stutter(L) := \{stutter(w) \mid w \in L\}$ is regular.

- 5** Consider the following recursively defined function on strings:

$$evens(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ \varepsilon & \text{if } w = a \text{ for some symbol } a \\ b \cdot evens(x) & \text{if } w = abx \text{ for some symbols } a \text{ and } b \text{ and some string } x \end{cases}$$

Intuitively, $evens(w)$ skips over every other symbol in w . For example:

- $evens(\mathbf{EXPELLIARMUS}) = \mathbf{XELAMS}$
- $evens(\mathbf{AVADA} \diamond \mathbf{KEDAVRA}) = \mathbf{VD} \diamond \mathbf{EAR}$.

Once again, let L be an arbitrary regular language.

1. Prove that the language $evens^{-1}(L) := \{w \mid evens(w) \in L\}$ is regular.
2. Prove that the language $evens(L) := \{evens(w) \mid w \in L\}$ is regular.