

CS/ECE 374 A ♦ Fall 2026

Conflict Midterm 1 Problem 1 Solution

For each statement below, there are two boxes in the answer booklet labeled “Yes” and “No”. Check “Yes” if the statement is *always* true and “No” otherwise, and give a **brief** (at most one short sentence) explanation of your answer.

Read each statement *very* carefully; some of them are deliberately subtle!

- (a) The regular expression $(0 + 1 + \varepsilon)(00 + 11)^*$ represents a language with an infinite fooling set.

Yes	X
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The language is regular, and no regular language has an infinite fooling set.

- (b) The regular expression 0^*0^* represents the language $\{0^n \mid n \geq 0 \text{ and } n \text{ is even}\}$.

Yes	X
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0 matches 0^*0^* .

- (c) For any two languages L_1 and L_2 , if $L_1 \cap L_2$ is regular, then at least one of L_1 and L_2 is regular.

Yes	X
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Let $L_1 = \{0^n 1^n \mid n \geq 0\}$ and $L_2 = \bar{L}_1 = \Sigma^* \setminus L_1$.

- (d) For any two languages L_1 and L_2 , if $L_1 \cap L_2$ is nonregular, then at least one of L_1 and L_2 is nonregular.

X	No
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Regularity is closed under intersection.

- (e) For every regular language L , there is a DFA accepting L whose start state is accepting.

Yes	X
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The start state must not accept if $\varepsilon \notin L$.

- (f) For every regular language L , there is an NFA with ε -transitions accepting L whose start state is accepting.

Yes	X
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The start state must not accept if $\varepsilon \notin L$.

- (g) For every regular language L , the language $(L \bullet L \bullet L)^R$ (i.e., the reversal of concatenating three copies of L) is regular.

X	No
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Regular languages are closed under both concatenation and reversal.

- (h) Every context-free grammar has at most two non-terminals.

Yes	X
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In lecture and handouts, we saw grammars with more than two non-terminals.

- (i) For any two regular language L_1 and L_2 , their union $L_1 \cup L_2$ is context-free.

X	No
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The union of regular languages is regular and therefore context-free.

- (j) The language $\{0^n 1^m 0^n \mid n, m \geq 1\}$ is context-free.

X	No
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This is generated by a grammar with $S \rightarrow 0A0 \mid 0S0$ and $A \rightarrow 1 \mid 1A$.

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Conflict Midterm 1 Problem 2 Solution

Let $L = \{0^a 1^b 2^c 3^d \mid b, c, d \geq 0 \text{ and } a > d \text{ and } b = c\}$ be a language over $\{0, 1, 2, 3\}$.

(a) **Prove** that L is not a regular language.

Solution: Consider the set $F = \{0^i \mid i \geq 1\}$.

Let x and y be arbitrary distinct strings in F .

Then $x = 0^i$ and $y = 0^j$ for some non-negative integers $i, j \geq 1$ with $i \neq j$. Assume without loss of generality that $i < j$.

Let $z = 3^{j-1}$.

- $xz = 0^i 3^{j-1} = 0^i 1^0 2^0 3^{j-1} \notin L_a$, because $i \leq j - 1$.
- $yz = 0^j 3^{j-1} = 0^j 1^0 2^0 3^{j-1} \in L_a$.

Thus, z is a distinguishing suffix for x and y .

We conclude that F is an infinite fooling set for L , and L is not regular. ■

Rubric: 5 points: standard fooling set rubric (scaled).

- (b) Describe a context-free grammar for L . You do not need to prove your context-free grammar is correct.

Solution:

$$S \rightarrow \epsilon A \mid \epsilon S \mid \epsilon S 3$$

 L

$$A \rightarrow \epsilon \mid 1A2$$

$$\{1^b 2^c \mid b, c \geq 0 \text{ and } b = c\}$$

■

Rubric: 5 points, standard context free grammar rubric (scaled).

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Conflict Midterm 1 Problem 3 Solution

Consider the function bubble defined in the question handout.

- (a) **Prove** that $\text{bubble}(x \bullet y) = \text{bubble}(x) \bullet \text{bubble}(y)$ for all strings x and y over $\{0, 1\}$.

Solution: Let x and y be arbitrary strings over $\{0, 1\}$.

Assume for every string $z \in \{0, 1\}^*$ such that $|z| < |x|$, we have $\text{bubble}(z \bullet y) = \text{bubble}(z) \bullet \text{bubble}(y)$.

There are three cases to consider.

- Suppose $x = \varepsilon$. Then,

$$\begin{aligned}
 \text{bubble}(x \bullet y) &= \text{bubble}(\varepsilon \bullet y) & x &= \varepsilon \\
 &= \text{bubble}(y) & \text{def. } (\bullet) \\
 &= \varepsilon \bullet \text{bubble}(y) & \text{def. } (\bullet) \\
 &= \text{bubble}(\varepsilon) \bullet \text{bubble}(y) & \text{def. bubble} \\
 &= \text{bubble}(x) \bullet \text{bubble}(y) & x &= \varepsilon
 \end{aligned}$$

- Suppose $x = 0 \cdot z$ for some string z . Then,

$$\begin{aligned}
 \text{bubble}(x \bullet y) &= \text{bubble}((0 \cdot z) \bullet y) & x &= 0 \cdot z \\
 &= \text{bubble}(0 \cdot (z \bullet y)) & \text{def. } (\bullet) \\
 &= 0 \cdot \text{bubble}(z \bullet y) & \text{def. bubble} \\
 &= 0 \cdot (\text{bubble}(z) \bullet \text{bubble}(y)) & \text{ind. hypo.} \\
 &= (0 \cdot \text{bubble}(z)) \bullet \text{bubble}(y) & \text{def. } (\bullet) \\
 &= \text{bubble}(0 \cdot z) \bullet \text{bubble}(y) & \text{def. bubble} \\
 &= \text{bubble}(x) \bullet \text{bubble}(y) & x &= 0 \cdot z
 \end{aligned}$$

- Suppose $x = 1 \cdot z$ for some string z . Then,

$$\begin{aligned}
 \text{bubble}(x \bullet y) &= \text{bubble}((1 \cdot z) \bullet y) & x &= 1 \cdot z \\
 &= \text{bubble}(1 \cdot (z \bullet y)) & \text{def. } (\bullet) \\
 &= 010 \cdot \text{bubble}(z \bullet y) & \text{def. bubble} \\
 &= 010 \cdot (\text{bubble}(z) \bullet \text{bubble}(y)) & \text{ind. hypo.} \\
 &= (010 \cdot \text{bubble}(z)) \bullet \text{bubble}(y) & \text{def. } (\bullet) \\
 &= \text{bubble}(1 \cdot z) \bullet \text{bubble}(y) & \text{def. bubble} \\
 &= \text{bubble}(x) \bullet \text{bubble}(y) & x &= 1 \cdot z
 \end{aligned}$$

In every case, $\text{bubble}(x \bullet y) = \text{bubble}(x) \bullet \text{bubble}(y)$. ■

Rubric: 5 points: standard induction rubric (scaled). This solution is more detailed than necessary for full credit.

(b) **Prove** that $\#(0, \text{bubble}(w)) \geq 2 \cdot \#(1, w)$ for all strings w over $\{0, 1\}$.

Solution: Let w be an arbitrary string over $\{0, 1\}$.

Assume for every string $x \in \{0, 1\}^*$ such that $|x| < |w|$, we have $\#(0, \text{bubble}(x)) \geq 2 \cdot \#(1, x)$.

There are three cases to consider.

- Suppose $w = \varepsilon$. Then,

$$\begin{aligned}
 \#(0, \text{bubble}(w)) &= \#(0, \text{bubble}(\varepsilon)) & w = \varepsilon \\
 &= \#(0, \varepsilon) & \text{def. bubble} \\
 &= 0 & \text{def. } (\#) \\
 &= 2 \cdot 0 & \text{arith.} \\
 &= 2 \cdot \#(1, \varepsilon) & \text{def. } (\#) \\
 &= 2 \cdot \#(1, w) & w = \varepsilon
 \end{aligned}$$

- Suppose $w = 0 \cdot x$ for some string x . Then,

$$\begin{aligned}
 \#(0, \text{bubble}(w)) &= \#(0, \text{bubble}(0 \cdot x)) & w = 0 \cdot x \\
 &= \#(0, 0 \cdot \text{bubble}(x)) & \text{def. bubble} \\
 &= 1 + \#(0, \text{bubble}(x)) & \text{def. } (\#) \\
 &\geq 1 + 2 \cdot \#(1, x) & \text{ind. hypo.} \\
 &> 2 \cdot \#(1, x) & \text{arith.} \\
 &= 2 \cdot \#(1, 0 \cdot x) & \text{def. } (\#) \\
 &= 2 \cdot \#(1, w) & w = 0 \cdot x
 \end{aligned}$$

- Suppose $w = 1 \cdot x$ for some string x . Then,

$$\begin{aligned}
 \#(0, \text{bubble}(w)) &= \#(0, \text{bubble}(1 \cdot x)) & w = 1 \cdot x \\
 &= \#(0, 010 \cdot \text{bubble}(x)) & \text{def. bubble} \\
 &= 2 + \#(0, \text{bubble}(x)) & \text{def. } (\#) \times 3 \\
 &\geq 2 + 2 \cdot \#(1, x) & \text{ind. hypo.} \\
 &= 2 \cdot (1 + \#(1, x)) & \text{arith.} \\
 &= 2 \cdot \#(1, 1 \cdot x) & \text{def. } (\#) \\
 &= 2 \cdot \#(1, w) & w = 1 \cdot x
 \end{aligned}$$

In every case, $\#(0, \text{bubble}(w)) \geq 2 \cdot \#(1, w)$. ■

Rubric: 5 points: standard induction rubric (scaled). This solution is more detailed than necessary for full credit.

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Conflict Midterm 1 Problem 4 Solution

For any string $w \in \{0, 1\}^*$, let $\text{Delete2nd}(w)$ denote the string obtained by deleting the second symbol in w . If $|w| < 2$, then $\text{Delete2nd}(w) = w$. Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. **Prove** that the following languages are also regular.

(a) $\text{INSERTED2ND}(L) = \{w \in \Sigma^* \mid \text{Delete2nd}(w) \in L\}$

Solution: Given an arbitrary DFA $M = (Q, \delta, s, A, \delta)$ for L , we construct an NFA $M' = (Q', \delta', s', A')$ for $\text{INSERTED2ND}(L)$ as follows.

M' will simulate M with the first symbol in its input string, ignore the second symbol, and then continue simulating M faithfully.

$$Q' = Q \times \{\text{none}, \text{one}, \text{more}\}$$

$$s' = (s, \text{none})$$

$$A' = A \times \{\text{none}, \text{one}, \text{more}\}$$

$$\delta'((q, \text{none}), a) = \{(\delta(q, a), \text{one})\} \quad \text{for all } q \in Q, a \in \{0, 1\}$$

$$\delta'((q, \text{one}), a) = \{(q, \text{more})\} \quad \text{for all } q \in Q, a \in \{0, 1\}$$

$$\delta'((q, \text{more}), a) = \{(\delta(q, a), \text{more})\} \quad \text{for all } q \in Q, a \in \{0, 1\}$$

More specifically, the empty string ε takes M' to $s' = (s, \text{none})$. String a for $a \in \{0, 1\}$ takes M' to state (q, one) if and only if a takes M to q . Finally, string x with $|x| \geq 2$ takes M' to state (q, more) if and only if $\text{Delete2nd}(x)$ takes M to state q .

Yes, all states in $(Q \setminus \{s\}) \times \{\text{none}\}$ and all but at most two states in $Q \times \{\text{one}\}$ are unreachable.

Yes, this could have been a DFA. ■

Rubric: 5 points: standard language transformation rubric (scaled).

(b) $\text{DELETED2ND}(L) = \{\text{Delete2nd}(w) \mid w \in L\}$

Solution: Given an arbitrary DFA $M = (Q, \delta, s, A, \delta)$ for L , we construct an NFA $M' = (Q', \delta', s', A')$ with ε -transitions for $\text{DELETED2ND}(L)$ as follows.

M' will simulate M with the first symbol in its input string, use an ε -transition to guess which of the two symbols was deleted, and then continue simulating M faithfully.

$$Q' = Q \times \{\text{none}, \text{one}, \text{more}\}$$

$$s' = (s, \text{none})$$

$$A' = A \times \{\text{none}, \text{one}, \text{more}\}$$

$$\delta'((q, \text{none}), a) = \{(\delta(q, a), \text{one})\} \quad \text{for all } q \in Q, a \in \{0, 1\}$$

$$\delta'((q, \text{none}), \varepsilon) = \emptyset \quad \text{for all } q \in Q$$

$$\delta'((q, \text{one}), a) = \emptyset \quad \text{for all } q \in Q, a \in \{0, 1\}$$

$$\delta'((q, \text{one}), \varepsilon) = \{(\delta(q, 0), \text{more}), (\delta(q, 1), \text{more})\} \quad \text{for all } q \in Q$$

$$\delta'((q, \text{more}), a) = \{(\delta(q, a), \text{more})\} \quad \text{for all } q \in Q, a \in \{0, 1\}$$

$$\delta'((q, \text{more}), \varepsilon) = \emptyset \quad \text{for all } q \in Q$$

More specifically, the empty string ε takes M' to $s' = (s, \text{none})$. String a for $a \in \{0, 1\}$ takes M' to state (q, one) if and only if a takes M to q . Finally, string $\text{Delete2nd}(x)$ with $|x| \geq 2$ takes M' to state (q, more) if and only if x takes M to state q .

Yes, all states in $(Q \setminus \{s\}) \times \{\text{none}\}$ and all but at most two states in $Q \times \{\text{one}\}$ are unreachable. ■

Rubric: 5 points: standard language transformation rubric (scaled).

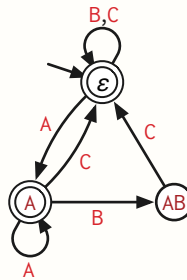
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Conflict Midterm 1 Problem 5 Solution

For each of the following languages L over the alphabet $\{A, B, C\}$,¹ describe a DFA that accepts L **and** give a regular expression that represents L . **Give your DFA states mnemonic names.** You do not need to do more to prove your answers are correct.

- (a) All strings in which every instance of the substring AB is immediately followed by a C .

Solution:

$$(B + C)^* (AA^*(BC + C) (B + C)^*)^* (\varepsilon + AA^*)$$



All missing transitions in the DFA go to a hidden dump state.

The states are labeled based on the last few symbols read assuming we have not yet failed to follow an instance of AB with a C :

- A : We just read A .
- AB : We just read AB .
- ε : We did not just read A or AB .

■

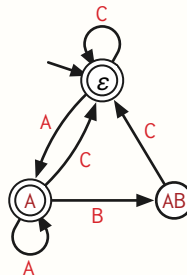
Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.

¹Easy as $\{1, 2, 3\}$

- (b) All strings in which every **B** is immediately **preceded** by a **A** and immediately **followed** by a **C** (i.e., every **B** belongs to an instance of the substring **ABC**.)

Solution:

$$(A + C)^* (ABC (A + C)^*)^* (A + C)^*$$



All missing transitions in the DFA go to a hidden dump state.

The states are labeled based on the last few symbols read assuming we have not yet failed to include a **B** in the substring **ABC**:

- **A**: We just read **A**.
- **AB**: We just read **AB**.
- **ε**: We did not just read **A** or **AB**.

Note that in contrast to the solution to part (a), a **B** takes state **ε** to the dump state. ■

Rubric: 5 points = 2½ for regular expression + 2½ for DFA, standard rubrics (scaled). The state labels and explanations are not required for credit.