

You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check “Yes” if the statement is always true and check “No” otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

- (a) Every integer in the empty set is prime.
- (b) The language $\{0^m 1^n \mid m + n \leq 374\}$ is regular.
- (c) The language $\{0^m 1^n \mid m - n \leq 374\}$ is regular.
- (d) For all languages L , the language L^* is regular.
- (e) For all languages L , the language L^* is infinite.
- (f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.
- (g) For all languages L and L' , if $L \cap L' = \emptyset$ and L' is not regular, then L is regular.
- (h) Every regular language is recognized by a DFA with at least 374 accepting states.
- (i) Every regular language is recognized by an NFA with at most 374 accepting states.
- (j) Every context-free language has an infinite fooling set.

2. The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s. For example:

$$\text{parity}(\epsilon) = 0 \quad \text{parity}(0010100) = 0 \quad \text{parity}(00101110100) = 1$$

- (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do **not** refer to # or other functions defined in class.)
- (b) **Prove** that for every regular language L , the language $\text{ODDPARITY}(L) := \{w \in L \mid \text{parity}(w) = 1\}$ is also regular. regular
↓
- (c) **Prove** that for every regular language L , the language $\text{ADDPARITY}(L) := \{\text{parity}(w) \odot w \mid w \in L\}$ is also regular.

[Hint: Parts (b) and (c) can be solved without the full language transformation machinery needed for Homework 4.]

3. Let $L = \{w0^n w^R \mid w \in \{0,1\}^* \text{ and } n > 0\}$ where w^R denotes the reversal of string w .

- (a) **Prove** that L is *not* a regular language. ← fooling sets
- (b) Describe a context-free grammar for L .

4. For any string $w \in \{0,1\}^*$, let $\text{take2skip2}(w)$ denote the subsequence of w containing symbols at positions 1, 2, 5, 6, 9, 10, $\dots, 4i+1, 4i+2, \dots$. In other words, $\text{take2skip2}(w)$ takes the first two symbols of w , skips the next two, takes the next two, skips the next two, and so on. For example:

$$\begin{aligned}\text{take2skip2}(1) &= 1 \\ \text{take2skip2}(010) &= 01 \\ \text{take2skip2}(\widehat{0}\widehat{1}\widehat{00}\widehat{1}\widehat{1}\widehat{1}\widehat{00}\widehat{0}\widehat{1}\widehat{1}) &= \widehat{0}\widehat{1}\widehat{1}\widehat{00}\widehat{1}\end{aligned}$$

Let L be an arbitrary regular language over $\{0,1\}$.

- (a) **Prove** that the language $\text{TOOK2ADDED2}(L) := \{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.
- (b) **Prove** that the language $\text{TOOK2SKIPPED2}(L) := \{\text{take2skip2}(w) \mid w \in L\}$ is regular.
5. For each of the following languages L over the alphabet $\{0,1\}$, describe a DFA that accepts L **and** give a regular expression that represents L . You do **not** need to prove that your answers are correct, but you should give your DFA states mnemonic names.

- (a) All strings in which every run of 1s has even length and every run of 0s has odd length.
(Recall that a *run* is a maximal non-empty substring in which all symbols are equal.)
- (b) All strings in 0^*10^* whose length is a multiple of 3.

↑ product construction of DFA?
regular expression cases based on first 0^* ?

CS/ECE 374 A ✧ Fall 2026
🌀 Midterm 1 Practice 3 🌀
September 27, 2026

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- **Don't panic!**
 - You have 120 minutes to answer five questions. **The questions are described in more detail in a separate handout.**
 - If you brought anything except your writing implements, your **hand-written** double-sided $8\frac{1}{2}'' \times 11''$ cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
 - Please clearly print your name and your NetID in the boxes above.
 - Please also print your name at the top of every page of the answer booklet, except this cover page. We want to make sure that if a staple falls out, we can reassemble your answer booklet. (It doesn't happen often, but it does happen.)
 - Proofs or other justifications beyond those listed in the standard grading rubrics are required if and only if we explicitly ask for them, using the word ***prove*** or ***justify*** in bold italics. **In particular, DFAs need to have mnemonic state names.**
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- **Do not write outside the black boxes on each page.** These indicate the area of the page that our scanners will actually scan. If the scanner can't see your work, we can't grade it.
 - If you run out of space for an answer, feel free to use the overflow/scratch pages at the back of the answer booklet, but **clearly indicate where we should look**. If we can't find your work, we can't grade it.
 - **Only work that is written into the stapled answer booklet will be graded.** We **do not** recommend detaching pages from the answer booklet, because any work on those detached pages will not be graded (and it disrupts our process for uploading solutions to Gradescope.) Please let us know if you detach a page either on purpose or accidentally so we can reattach it.
 - We will provide additional blank scratch paper on request, but we will not grade anything written on it.
 - **Please return all paper with your answer booklet** including your question sheet, your cheat sheet, and all scratch paper. Please put this loose paper **inside** your answer booklet.
-

Is between
between is and between
or between between and is
or between and and or
or between or and between
or between between and and
or between or and and
or between or and or?

Emily Fox

For each statement below, check "Yes" if the statement is always true and check "No" otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

- (a) Every integer in the empty set is prime.

☒ Yes

☐ No

vacidly true (u.d. over $\emptyset$)

- (b) The language $\{0^m 1^n \mid m + n \leq 374\}$ is regular.

☒ Yes

☐ No

language is finite

- (c) The language $\{0^m 1^n \mid m - n \leq 374\}$ is regular.

☐ Yes

☒ No

$F = \{0^m \mid m \geq 375\}$

- (d) For all languages L , the language L^* is regular.

☐ Yes

☒ No

$L = \{0^n 1^n \mid n \geq 0\}$

- (e) For all languages L , the language L^* is infinite.

☐ Yes

☒ No

$L = \emptyset$

$L^* = \{\epsilon\}$

- (f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.

☒ Yes

☐ No

L is reg. $\Rightarrow \Sigma^* \setminus L$ is reg. $\Rightarrow$ accepted by DFA

accepted

- (g) For all languages L and L' , if $L \cap L' = \emptyset$ and L' is not regular, then L is regular.

☐ Yes

☒ No

Let $L = \Sigma^* \setminus L'$.

- (h) Every regular language is recognized by a DFA with at least 374 accepting states.

☒ Yes

☐ No

Can add 374 unreachable accepting states to any DFA.

- (i) Every regular language is recognized by an NFA with at most 374 accepting states.

☒ Yes

☐ No

Have any transition to an accepting state go to

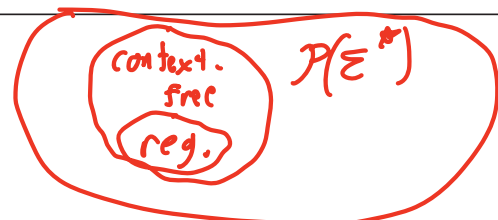
more detailed $\Rightarrow$ a new accepting state with no outgoing. $A' = (A \cap \{s\}) \cup \{nnu\}$

- (j) Every context-free language has an infinite fooling set.

☐ Yes

☒ No

than needed $L = \emptyset$



The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s.

- (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do **not** refer to # or other functions defined in class.)
- (b) **Prove** that for every regular language L , the language $\text{ODDPARITY}(L) := \{w \in L \mid \text{parity}(w) = 1\}$ is also regular.
- (c) **Prove** that for every regular language L , the language $\text{ADDPARITY}(L) := \{\text{parity}(w) \cdot w \mid w \in L\}$ is also regular.

$$(a) \text{parity}(w) := \begin{cases} 0 & \text{if } w = \epsilon \\ \text{parity}(x) & \text{if } w = 0 \cdot x \\ (1 + \text{parity}(x)) \bmod 2 & \text{if } w = 1 \cdot x \end{cases}$$

(b) $L_1 := \{w \mid \text{parity}(w) = 1\}$ is accepted by , and so it is regular.

$\text{OddParity}(L) = L \wedge L_1$, so it is regular, also.

(c) $\text{EvenParity}(L) := \{w \in L \mid \text{parity}(w) = 0\} = L \setminus \text{OddParity}(L)$ is regular.

$\{0\} + \{1\}$ are regular.

$\text{AddParity}(L) = \{0\} \circ \text{EvenParity}(L) \cup \{1\} \circ \text{OddParity}(L)$ is regular.

Emily Fox

Let $L = \{w0^n w^R \mid w \in \{0,1\}^* \text{ and } n > 0\}$ where w^R denotes the reversal of string w .

(a) **Prove** that L is not a regular language.

(b) Describe a context-free grammar for L .

(a) Let $F = \{1^i 0 \mid i \geq 1\}$. *playing it safe*

Let x, y be distinct members of F .

$x = 1^i 0$ + $y = 1^j 0$ for some $i, j \geq 1$ + $i \neq j$.

Let $z = 1^i$.

Then $xz = 1^i 0 1^i \in L$, but $yz = 1^j 0 1^i \notin L$, because $i \neq j$.

So F is an infinite fooling set for L . L is not reg.

(b) $S \rightarrow A \mid 0S0 \mid 1S1$

$A \rightarrow 0 \mid 0A$

L

$\{0^n \mid n > 0\}$

For any string $w \in \{0,1\}^*$, let $\text{take2skip2}(w)$ denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \dots, 4i+1, 4i+2, \dots$. In other words, $\text{take2skip2}(w)$ takes the first two symbols of w , skips the next two, takes the next two, skips the next two, and so on. Let L be an arbitrary regular language over $\{0,1\}$.

(a) **Prove** that the language $\text{TOOK2ADDED2}(L) := \{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.

(b) **Prove** that the language $\text{TOOK2SKIPPED2}(L) := \{\text{take2skip2}(w) \mid w \in L\}$ is regular.

(a) Let $M = (Q, \delta, s, A)$ be a DFA accepting L .

Build an NFA $M' = (Q', \delta', s', A')$ accepting $\text{Took2Added2}(L)$.

$Q' = Q \times \{1, 2, 3, 4\}$ (state in M , index of next symbol "mod 4")

$s' = (s, 1)$

$A' = A \times \{1, 2, 3, 4\}$

$\delta'((q, 1), a) = \{(\delta(q, a), 2)\}$

$\forall q \in Q, a \in \{0, 1\}$

$\delta'((q, 2), a) = \{(\delta(q, a), 3)\}$

$\forall q \in Q, a \in \{0, 1\}$

$\delta'((q, 3), a) = \{(q, 4)\}$

$\forall q \in Q, a \in \{0, 1\}$

$\delta'((q, 4), a) = \{(q, 1)\}$

$\forall q \in Q, a \in \{0, 1\}$

M' simulates M only on symbols of index $1 + 2 \text{ mod } 4$.

(b) Let $M = (Q, \delta, s, A)$ be a DFA accepting L .

Build NFA $M' = (Q', \delta', s', A')$ with ϵ -transitions that accepts $\text{Took2Skipped2}(L)$.

$Q' = Q \times \{1, 2, 3, 4\} \leftarrow$ index of next symbol mod 4 for M

$s' = (s, 1)$

$A' = A \times \{1, 2, 3, 4\}$

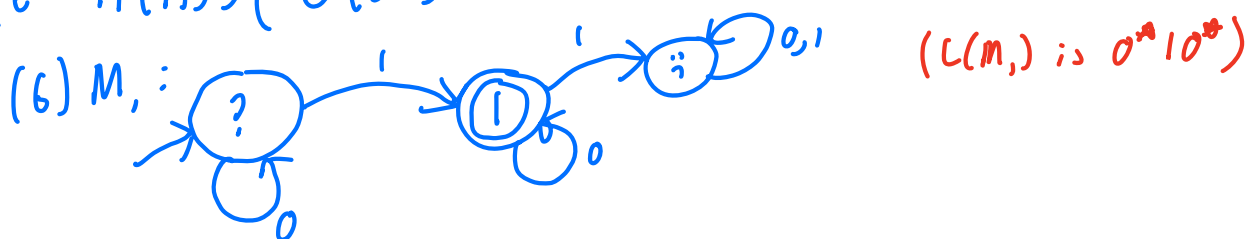
cont. on 6...

For each of the following languages L over the alphabet $\{0, 1\}$, describe a DFA that accepts L and give a regular expression that represents L . You do **not** need to prove that your answers are correct, but you should give your DFA states mnemonic names.

- (a) All strings in which every run of 1s has even length and every run of 0s has odd length. (Recall that a *run* is a maximal non-empty substring in which all symbols are equal.)
- (b) All strings in 0^*10^* whose length is a multiple of 3.



$$(e + 11(11)^*) (0(00)^* 11(11)^*)^* (e + 0(00)^*)$$



Machine M for L is product of M_1 & M_2 . Accept in states $A_1, \times A_2$ where $A_1, \times A_2$ are accepting states of $M_1, \times M_2$.

$$(000)^* 100(000)^* + 0(000)^* 10(000)^* + 00(000)^* 1(000)^*$$

(scratch paper)

4(6) cont.

$$\delta'((q, 1), a) = \{(\delta(q, a), 2)\} \quad \forall q \in Q, a \in \{0, 1\}$$

$$\delta'((q, 2), a) = \{(\delta(q, a), 3)\} \quad \forall q \in Q, a \in \{0, 1\}$$

$$\delta'((q, 3), \epsilon) = \{(\delta(q, 0), 4), (\delta(q, 1), 4)\} \quad \forall q \in Q$$

$$\delta'((q, 4), \epsilon) = \{(\delta(q, 0), 1), (\delta(q, 1), 1)\} \quad \forall q \in Q$$

All other transitions go to $\emptyset$.

M' simulates M on all symbols of w one at a time, including those that are skipped.

(scratch paper)

(scratch paper)