GPS 3 due <u>Mon</u> Sep 14
HW 3 due <u>Tue</u> Sep 15

Kleene's theorem: every regular language is accepted by a DFA.

So, if language L has <u>no</u> DFA, it is not regular.

Thm: $L = \{0^n 1^n \mid n \geq 0\}$ is not regular!

Suppose L is regular, then there is a DFA $M = (Q, \delta, s, A)$ s.t. $L = L(M)$,

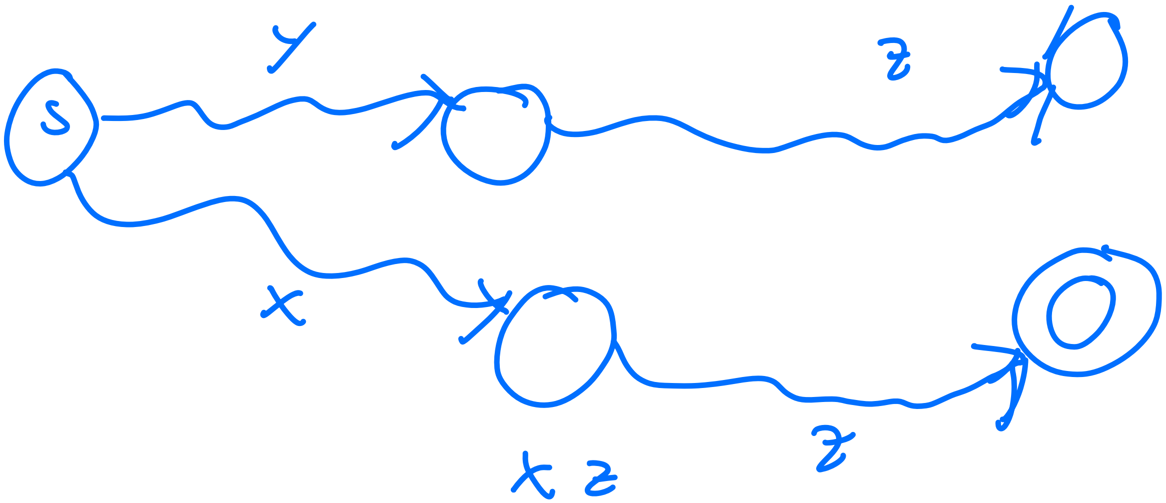
Consider two strings $x = 0^i$ & $y = 0^j$ with $i \neq j$

$x: 0000$ $y: 0000000$

Let $z = 1^i$.

$xz = 0^i 1^i \in L$.

$yz = 0^j 1^i \notin L$.



$\delta^*(\delta^*(s,x), z) \in A$ $\delta^*(\delta^*(s,y), z) \notin A$

$\Rightarrow \delta^*(s,x) \neq \delta^*(s,y)$

So $|Q| \geq 2$.

Let $F = \{0^i \mid i \geq 0\}$

Any two members of F take you to distinct states from s.

So Q is not finite! Not a DFA.

Fix an alphabet Σ, a language L over Σ, & two string $x, y \in \Sigma^*$.

A string $z \in \Sigma^*$ is a distinguishing suffix of x & y with respect to L if exactly one of xz or yz is in L.

A <u>fooling set</u> F for L is a set of string over Σ for which <u>every</u> pair of strings in F has a distinguishing suffix.

All $F \subseteq \Sigma^*$ with $\leq$ one member is a fooling set.

Ex: $L = (0+1)^*(00+11)(0+1)^*$

$F = \{\varepsilon, 0, 1, 00\}$

ε + 00 are distinguished by ε : $\varepsilon\varepsilon = \varepsilon \notin L$

$00\varepsilon = 00 \in L$

0 + 00 by 1 : $01 \notin L$ + $001 \in L$

1 & 00 by 0: $10 \notin L$ & $000 \in L$

ε & 0 by 0: $\varepsilon 0 = 0 \notin L$ & $00 \in L$

ε & 1 by 1: $1 \notin L$ & $11 \in L$

0 & 1 by 0: $00 \in L$ & $10 \notin L$

Any DFA for L has ≥ 4 states.

Thm (Myhill-Nerode): For any language L, the minimum # of states for a DFA accepting is exactly the maximum size of any fooling set for L.

$\implies$ If L has an infinite fooling set, there is no DFA for L &

$\boxed{L \text{ not regular.}}$

Thm: $L = \{0^n 1^n \mid n \geq 0\}$ is not regular.

Proof: Consider the infinite set $F = \{0^i \mid i \geq 0\}$

give a set

Let x & y be distinct strings in F.

By definition of F, $x = 0^i$ & $y = 0^j$ for some integer $i \neq j$.

consider arbitrary pairs of distinct members

Let $z = 1^i$.

give a d. suffix

Then $xz = 0^i 1^i \in L$.

But $yz = 0^j 1^i \notin L$, because $i \neq j$.

So z distiguishes x & y.

argue z is dist...

Because x & y are arbitrary strings in F, every pair of strings in F has a distinguishing suffix.

So F is a fooling set for L.

F is also infinite, so L is not regular.

conclude

Ex: The set of palindromes $L = \{w \mid w = w^R\}$

reversal

is not regular.

Proof: Consider the infinite set $F = \{0^i 1 \mid i \geq 0\}$.

Let x & y be distinct string in F.

Then $x = 0^i 1$ & $y = 0^j 1$ for some $i \neq j$.

Let $z = 0^i$.

Then $xz = 0^i 1 0^i \in L$.

But $yz = 0^j 1 0^i \notin L$, because $i \neq j$.

So z distiguishes x & y.

So F is a fooling set.

F is infinite, so L is not regular.

Ex: The language $L = \{0^{2^n} \mid n \geq 0\}$

Proof: Consider the infinite set $F = L$.

$= \{0^{2^i} \mid i \geq 0\}$

Let x & y be distinct strings in F.

Then $x = 0^{2^i}$ & $y = 0^{2^j}$ for some $i \neq j$.

Let $z = 0^{2^i}$

Then $xz = 0^{2^i} 0^{2^i} = 0^{2^{(i+1)}} \in L$.

But $yz = 0^{2^j} 0^{2^i} \notin C$.

So z distinguishes x & y

So F is an infinite fooling set & L is not regular.

Reductions from a known nonregular language.

Thm: The language $L = \Sigma^* \setminus \{0^n 1^n \mid n \geq 0\}$ is not regular.

Proof: Suppose L is regular.

Then $\overline{L} = \Sigma^* \setminus (\Sigma^* \setminus \{0^n 1^n \mid n \geq 0\})$

$= \{0^n 1^n \mid n \geq 0\}$

is regular.

But $\overline{L}$ is not regular, giving a contradiction.

So L was not regular after all.

If you can build a nonregular language from L by doing things that preserve regularity, L is not regular.

Ex: The language L of binary strings with an equal # of 0's & 1's.

$\{w \mid \#(0,w) = \#(1,w)\}$.

Proof: Suppose that L is regular.

Then $L \cap L(0^*1^*) = \{0^n1^n \mid n \geq 0\}$

is regular, a contradiction.

L is not regular after all.

Bad Ex: $C = \{0^n 1^n \mid n \geq 0\}$ & $\overline{C}$ are both not regular.

But $C \cup \overline{C} = \Sigma^*$ is regular.

So reduce in the right direction!

If you use the pumping lemma, cite your source & prove it!

Same for homomorphisms.