Hi, I'm Emily! (she/they)

CS/ECE 374 Section A

7-10 TAs, 16 CAs

2 Midterms
1 Final exam (cumulative)

https://courses.grainger.illinois.edu/cs374al1/fa2026/

Grading: 35%: Best 9 Guided Problem Sets (PrairieLearn)

out of 11

Mondays

Best 18 Homework Problems

out of 22

Tuesdays

65%: Exams

2 Midterms - 5 problems

1 Final - 7 problems

Final percentages needed on the website.

Resources:

- Textbook/lecture notes by Erickson
- My lecture notes
- Lectures
- Labs:
- office hours:
- homework groups:

- homework parties (2 per week)
- ANYTHING ELSE!
- Ed Discussion
- Discord

Σ: an <u>alphabet</u>: an arbitrary <u>finite</u> set

Ex: $\Sigma = \{A, B, \ldots, Z\}$ ← members are <u>symbols</u> or <u>characters</u>

$\Sigma = \{0, 1\}$ ←

A <u>string</u> (or <u>word</u>): is a <u>finite</u> sequence of ~~members of an alphabet~~, symbols from an alphabet.

Formally: w is a string over Σ if it is one of:

- the <u>empty string</u> ϵ ← epsilon
- an ordered pair (a, x) $a \in \Sigma$ &

$x \neq w$ is a string over Σ

Ex: $STRING = (S, (TRING))$

$= (S, (T, (RING)))$

$= \dots$

$= (S, (T, (R, (I, (N, (G, \varepsilon))))))$

Functions are defined recursively...

<u>length</u> $|w|$ is # symbols in w

$$|w| := \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = ax \end{cases}$$

i.e. (a, x)

Ex: $|HAT| = 1 + |AT|$

$$= 1 + (1 + |T|)$$

$$= 1 + (1 + (1 + |\epsilon|))$$

$$= 1 + (1 + (1 + 0))$$

$$= 3$$

and

<u>Concatenation</u> $w \bullet z$ of strings w & z

is

$$w \bullet z := \begin{cases} z & \text{if } w = \epsilon \\ (a, x \bullet z) & \text{if } w = ax \end{cases}$$

$$\text{Ex: } NOW \bullet HERE = N \cdot (OW \bullet HERE)$$

$$= N \cdot (O \cdot (W \bullet HERE))$$

$$= N \cdot (O \cdot (W \cdot (\varepsilon \bullet HERE)))$$

$$= N \cdot (O \cdot (W \cdot HERE))$$

$$\equiv (W, HERE)$$

$$\vDash NOWHERE$$

Induction:

- Consider an arbitrary object (probably of size n)
- Explicitely state a strong inductive hypothesis (_ _ _ for all $k < n$)
- Explicitely handle each case for the object
- Use inductive hypothesis when it's useful.
- End by pointing out you handled all

cases.

Ex: Lemma: For every string w, we have

$$w \bullet \epsilon = w.$$

Proof: Let w be an arbitrary string.

Assume, for all string x s.t. $|x| < |w|$,

that $x \bullet \epsilon = x$.

There are two cases:

Suppose $w = \epsilon$.

Then, $w \bullet \epsilon = \epsilon \bullet \epsilon$ $\quad (w = \epsilon)$

$= \epsilon$ $\quad$ (def. $\bullet$)

$= w$ $\quad (w = \epsilon)$

Suppose $w = ax$ for symbol a & string x.

Then,

$$
\begin{aligned}
w \bullet \varepsilon &= (a \cdot x) \bullet \varepsilon && (w = ax) \\
&= a \cdot (x \bullet \varepsilon) && (\text{def. } \bullet) \\
&= a \cdot x && (\text{IH}) \\
&= w && (w = ax)
\end{aligned}
$$

In all cases, $w \bullet \varepsilon = w$. $\blacksquare$

Lemma: For strings w and z:

$$|w \bullet z| = |w| + |z|$$

Suppose $|w|$ ~~is to~~ $+ |z|$ are large.. so $w = ax$

$$
\begin{aligned}
|w \bullet z| &= |(ax) \bullet z| \\
&= |a \cdot (x \bullet z)| \\
&= 1 + |(x \bullet z)|
\end{aligned}
$$

do induction on first string?

Proof: Let w *and* z be arbitrary strings.

Assume, for every string x such that $|x| < |w|$, that $|x \bullet z| = |x| + |z|$.[4]
There are two cases to consider.

- Suppose $w = \varepsilon$.
  Then,

$$\begin{aligned} |w \bullet z| &= |\varepsilon \bullet z| && w = \varepsilon \\ &= |z| && \text{def. concat.} \\ &= 0 + |z| && \text{arithmetic} \\ &= |\varepsilon| + |z| && \text{def. length} \\ &= |w| + |z|. && w = \varepsilon \end{aligned}$$

- Suppose $w = a \cdot x$ for some symbol a and string x.
  Then,[5]

$$\begin{aligned} |w \bullet z| &= |(a \cdot x) \bullet z| && w = a \cdot x \\ &= |a \cdot (x \bullet z)| && \text{def. concat.} \\ &= 1 + |x \bullet z| && \text{def. length} \\ &= 1 + |x| + |z| && \textbf{ind. hypo.} \\ &= |a \cdot x| + |z| && \text{def. length} \\ &= |w| + |z|. && w = a \cdot x \end{aligned}$$

In both cases, we conclude that $|w \bullet z| = |w| + |z|$. □