

You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check “Yes” if the statement is always true and check “No” otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

- (a) Every integer in the empty set is prime.
- (b) The language $\{0^m 1^n \mid m + n \leq 374\}$ is regular.
- (c) The language $\{0^m 1^n \mid m - n \leq 374\}$ is regular.
- (d) For all languages L , the language L^* is regular.
- (e) For all languages L , the language L^* is infinite.
- (f) For all languages $L \subseteq \Sigma^*$, if L can be represented by a regular expression, then $\Sigma^* \setminus L$ is recognized by a DFA.
- (g) For all languages L and L' , if $L \cap L' = \emptyset$ and L' is not regular, then L is regular.
- (h) Every regular language is recognized by a DFA with at least 374 accepting states.
- (i) Every regular language is recognized by an NFA with at most 374 accepting states.
- (j) Every context-free language has an infinite fooling set.

2. The *parity* of a bit-string w is 0 if w has an even number of 1s, and 1 if w has an odd number of 1s. For example:

$$\text{parity}(\epsilon) = 0 \quad \text{parity}(0010100) = 0 \quad \text{parity}(00101110100) = 1$$

- (a) Give a *self-contained*, formal, recursive definition of the *parity* function. (In particular, do **not** refer to # or other functions defined in class.)
- (b) **Prove** that for every regular language L , the language $\text{ODDPARITY}(L) := \{w \in L \mid \text{parity}(w) = 1\}$ is also regular.
- (c) **Prove** that for every regular language L , the language $\text{ADDPARITY}(L) := \{\text{parity}(w) \cdot w \mid w \in L\}$ is also regular.

[Hint: Parts (b) and (c) can be solved without the full language transformation machinery needed for Homework 4.]

3. Let $L = \{w0^n w^R \mid w \in \{0, 1\}^* \text{ and } n > 0\}$ where w^R denotes the reversal of string w .
- Prove** that L is *not* a regular language.
 - Describe a context-free grammar for L .
4. For any string $w \in \{0, 1\}^*$, let $\text{take2skip2}(w)$ denote the subsequence of w containing symbols at positions $1, 2, 5, 6, 9, 10, \dots, 4i + 1, 4i + 2, \dots$. In other words, $\text{take2skip2}(w)$ takes the first two symbols of w , skips the next two, takes the next two, skips the next two, and so on. For example:

$$\begin{aligned}\text{take2skip2}(1) &= 1 \\ \text{take2skip2}(010) &= 01 \\ \text{take2skip2}(0100111100011) &= 0111001\end{aligned}$$

Let L be an arbitrary regular language over $\{0, 1\}$.

- Prove** that the language $\text{TOOK2ADDED2}(L) := \{w \in \Sigma^* \mid \text{take2skip2}(w) \in L\}$ is regular.
 - Prove** that the language $\text{TOOK2SKIPPED2}(L) := \{\text{take2skip2}(w) \mid w \in L\}$ is regular.
5. For each of the following languages L over the alphabet $\{0, 1\}$, describe a DFA that accepts L **and** give a regular expression that represents L . You do **not** need to prove that your answers are correct, but you should give your DFA states mnemonic names.
- All strings in which every run of **1**s has even length and every run of **0**s has odd length. (Recall that a *run* is a maximal non-empty substring in which all symbols are equal.)
 - All strings in 0^*10^* whose length is a multiple of 3.