

You have 120 minutes to answer five questions.

Write your answers in the separate answer booklet.

Please return this question sheet and your cheat sheet with your answers.

- Let $\text{compress0s}(w)$ be a function that takes a string w as input, and returns the string formed by compressing every run of 0s in w by half. Specifically, every run of $2n$ 0s is compressed to length n , and every run of $2n + 1$ 0s is compressed to length $n + 1$. For example:

$$\text{compress0s}(\underline{00000}1\underline{10001}) = \underline{000}1\underline{1001}$$

$$\text{compress0s}(11\underline{000010}) = 11\underline{0010}$$

$$\text{compress0s}(11111) = 11111$$

Let L be an arbitrary regular language.

- Prove** that $\text{DECOMPRESSED}(L) := \{w \in \Sigma^* \mid \text{compress0s}(w) \in L\}$ is regular.
 - Prove** that $\text{COMPRESSED}(L) := \{\text{compress0s}(w) \mid w \in L\}$ is regular.
- Let L be the language of all strings over $\{0, 1\}$ that contain at least 374 consecutive 1s.
 - Give a regular expression that matches L .
You may use the notation R^k to denote the concatenation of k copies of the regular expression R ; for example,

$$(1 + 01)^5 = (1 + 01)(1 + 01)(1 + 01)(1 + 01)(1 + 01)$$
 - Describe a DFA whose language is L . [Hint: Do not try to **draw** your DFA!]
 - Prove** that any DFA whose language is L must have at least 375 states. [Hint: What must be true if L has a fooling set of some size ℓ ?]
 - Consider the following recursive function Bond , which doubles the length of any run of 0s in its input string.

$$\text{Bond}(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ 00 \cdot \text{Bond}(x) & \text{if } w = 0 \cdot x \text{ for some string } x \\ 1 \cdot \text{Bond}(x) & \text{if } w = 1 \cdot x \text{ for some string } x \end{cases}$$

- Prove** that $|\text{Bond}(w)| \geq |w|$ for all strings w .
- Prove** that $\text{Bond}(x \cdot y) = \text{Bond}(x) \cdot \text{Bond}(y)$ for all strings x and y .

As usual, you can assume any result proved in class, in the lecture notes, in labs, or in homework solutions.

4. Let L be the language $\{0^a 1^b 0^c \mid a = b \text{ or } a = c \text{ or } b = c\}$
- (a) **Prove** that L is *not* a regular language.
 - (b) Describe a context-free grammar for L . You do not need to justify your answer.
5. For each statement below, check “True” if the statement is always true and check “False” otherwise, and give a brief (one short sentence) explanation of your answer. Read these statements very carefully—small details matter!

For any string $w \in (0 + 1)^*$, let w^c denote the *bitwise complement* of w , obtained by flipping every 0 in w to a 1, and vice versa. For example, $\varepsilon^c = \varepsilon$ and $000110^c = 111001$.

- (a) If $2 + 2 = 5$, then zero is odd.
- (b) $\{0^n 1 \mid n > 0\}$ is the only infinite fooling set for the language $\{0^n 1 0^n \mid n > 0\}$.
- (c) $\{0^n 1 0^n \mid n > 0\}$ is a context-free language.
- (d) The context-free grammar $S \rightarrow 00S \mid S11 \mid 01$ generates the language $\{0^n 1^n \mid n \geq 0\}$.
- (e) Every regular language is recognized by a DFA with exactly one accepting state.
- (f) Any language that can be decided by an NFA with ε -transitions can also be decided by an NFA without ε -transitions.
- (g) If L is a regular language over the alphabet $\{0, 1\}$, then $\{xy^c \mid x, y \in L\}$ is also regular.
- (h) If L is a regular language over the alphabet $\{0, 1\}$, then $\{ww^c \mid w \in L\}$ is also regular.
- (i) The regular expression $(00 + 11)^*$ represents the language of all strings over $\{0, 1\}$ of even length.
- (j) Let L_1 and L_2 be two regular languages. The language $(L_1 \cup L_2)^*$ is also regular.