1. It's almost time to show off your flippin' sweet dancing skills! Tomorrow is the big dance contest you've been training for your entire life, except for that summer you spent with your uncle in Alaska hunting wolverines. You've obtianed an advance copy of the list of n songs that the judges will play during the contest, in chronological order. Yesssssssss!

   You know all the songs, all the judges, and your own dancing ability extremely well. For each integer k, you know that if you dance to the kth song on the schedule, you will awarded exactly $Score[k]$ points, but then you will be physically unable to dance for the next $Wait[k]$ songs (that is, you cannot dance to songs $k+1$ through $k+Wait[k]$). The dancer with the highest total score at the end of the night wins the contest, so you want your total score to be as high as possible.

   Describe an algorithm to compute the maximum total score you can achieve. The input to your sweet algorithm is the pair of arrays $Score[1..n]$ and $Wait[1..n]$.

2. The City Council of Sham-Poobanana needs to partition Purple Street into voting districts. A total of n people live on Purple Street, at consecutive addresses $1, 2, \ldots, n$. Each voting district must be a contiguous interval of addresses $i, i+1, \ldots, j$ for some $1 \le i < j \le n$. By law, each Purple Street address must lie in exactly one district, and the number of addresses in each district must be between k and $2k$, where k is some positive integer parameter.

   Every election in Sham-Poobanana is between two rival factions: Oceania and Eurasia. A majority of the City Council are from Oceania, so they consider a district to be *good* if more than half the residents of that district voted for Oceania in the previous election. Naturally, the City Council has complete voting records for all n residents.

   For example, the figure below shows a legal partition of 22 addresses into 4 good districts and 3 bad districts, where $k = 2$. Each O indicates a vote for Oceania, and each X indicates a vote for Eurasia.



   Describe an algorithm to find the largest possible number of *good* districts in a legal partition. Your input consists of the integer k and a boolean array $\textsc{GoodVote}[1..n]$ indicating which residents previously voted for Oceania (TRUE) or Eurasia (FALSE). You can assume that a legal partition exists. Analyze the running time of your algorithm in terms of the parameters n and k.

3. Suppose we want to split an array $A[1..n]$ of integers into k contiguous intervals that partition the sum of the values as evenly as possible. Specifically, define the *cost* of such a partition as the maximum, over all k intervals, of the sum of the values in that interval; our goal is to minimize this cost. Describe and analyze an algorithm to compute the minimum cost of a partition of A into k intervals, given the array A and the integer k as input.

   For example, given the array $A = [1, 6, -1, 8, 0, 3, 3, 9, 8, 8, 7, 4, 9, 8, 9, 4, 8, 4, 8, 2]$ and the integer $k = 3$ as input, your algorithm should return the integer 37, which is the cost of the following partition:

$$[\overbrace{1, 6, -1, 8, 0, 3, 3, 9, 8}^{37} \mid \overbrace{8, 7, 4, 9, 8}^{36} \mid \overbrace{9, 4, 8, 4, 8, 2}^{35}]$$

   The numbers above each interval show the sum of the values in that interval.

4. A sequence of integers is *mostly odd* if strictly more than half of its elements are odd. Describe an algorithm that computes the length of the longest *mostly odd* increasing subsequence of a given array $A[1..n]$ of integers. (You can assume that A has at least one mostly-odd increasing subsequence.)

   For example, given the input array

$$[2, 4, 6, 8, 10, \mathbf{1}, \mathbf{7}, 12, \mathbf{13}, 14, \mathbf{19}, \mathbf{25}, 16, 18, 20, 22, 24],$$

   your algorithm should output the integer 7, which is the length of the mostly-odd increasing subsequence $[4, 6, \mathbf{7}, \mathbf{13}, 14, \mathbf{19}, \mathbf{25}]$. (This is not the only mostly-odd subsequence of length 7.) The increasing subsequence $[2, 4, 6, \mathbf{7}, 12, \mathbf{13}, 14, \mathbf{19}, 20, 22, 24]$ is longer, but it is not mostly odd.