HW 10 today

GPS 10 due tomorrow..

Final exam Fri Dec 12th 8-11am.

Conflict form available.

Mon Dec 15th?

Based on forms filled by Fri Dec 5th.

DRES sign up for 12th or 15th ASAP.

P: decision problems with poly time algs

NP: " " can verify yes in poly time

NP-hard: Each problem in NP reduces to each NP-hard problem in poly time.

NP-complete: NP-hard & in NP (3SAT).

$\Rightarrow$ An NP-complete problem has a poly time alg iff $P = NP$.

To prove Y is NP-hard, reduce any know NP-hard problem X to Y in poly time.

So far: Circuit SAT, SAT, & 3SAT

Max Ind Set: Given undirected $G=(V,E)$.
Want max # of vertices that don't share edges.

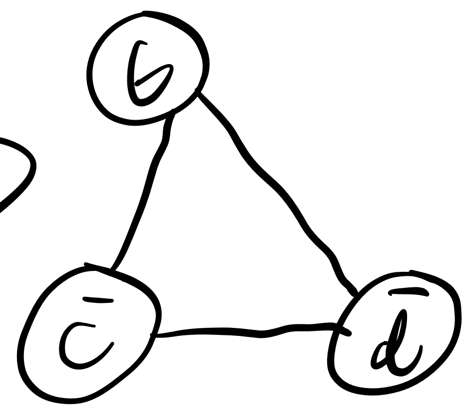
Thm: Max Ind Set is NP-hard. (By reduction from 3SAT.)

Proof:

Given 3CNF Boolean formula ϕ like
$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge \ldots$

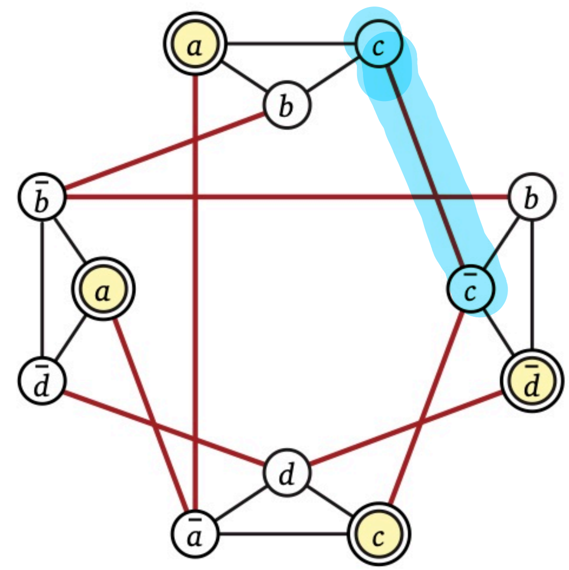
will build undirected $G=(V,E)$.

k: # clauses in ϕ

clause gadget $(b \vee \bar{c} \vee \bar{d}) \Rightarrow$ 

for each clause ($3k$ vertices)

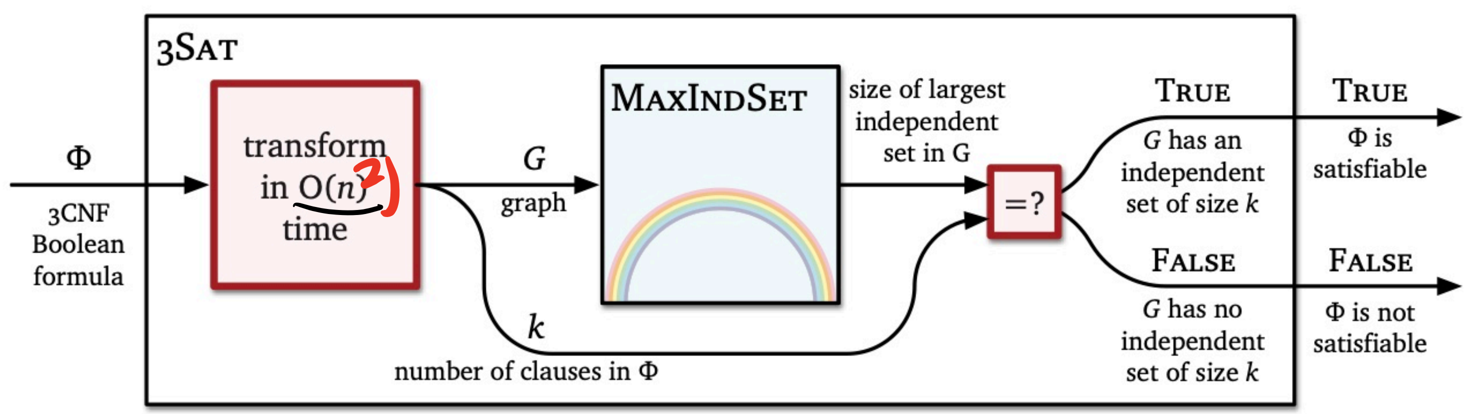
add edges between any literal vertex & its negation



$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

Return Yes iff G has an ind. set of size k.

Runs in poly time.



$\boxed{\text{Lemma: There is a satisfying assignment for } \Phi \text{ iff } G \text{ has an ind. set of size } k.}$

$\Rightarrow$ Suppose Φ is sat.

Fix a sat. assignment to variables

Pick one true literal per clause & let S be those vertices.

$|S| = k$.

No edge goes between the literal vertices because we don't use a literal & its negation.

$\Leftarrow$ Suppose G has an ind. set S of size k.

All k vertices are from different
triangles.
Set S's literals to true.
Will sat. each triangle's clause.
Edges between inverse literals ⇒
the assignment is consistent.

⇒ Max Clique &
Min Vertex Cover are
NP-hard

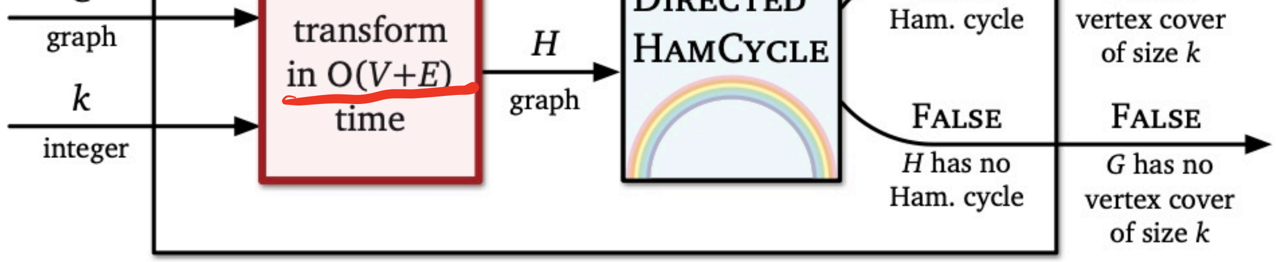
Thm: DirHamCycle is NP-hard

(actually NP-complete).

(from decision version of Vertex Cover).

Proof: Given undirected graph $G = (V, E)$ + an integer k. Is there a vertex cover of size k (at most k vertices that touch every edge).

Build <u>directed</u> graph H.



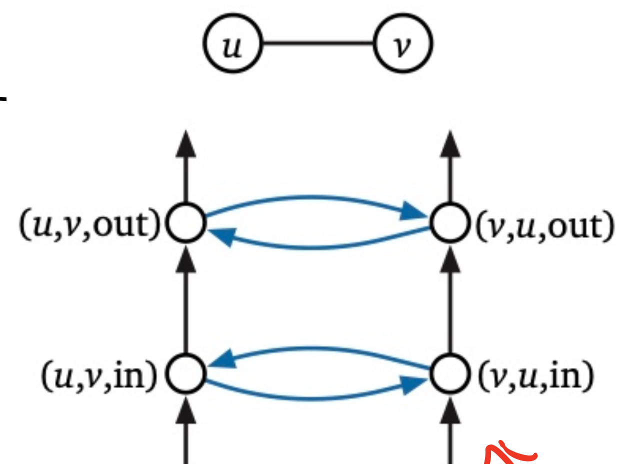
For each edge uv in G: an edge gadget with four vertices + six edges for H.

(u,v,in), (u,v,out), (v,u,in), (v,u,out)

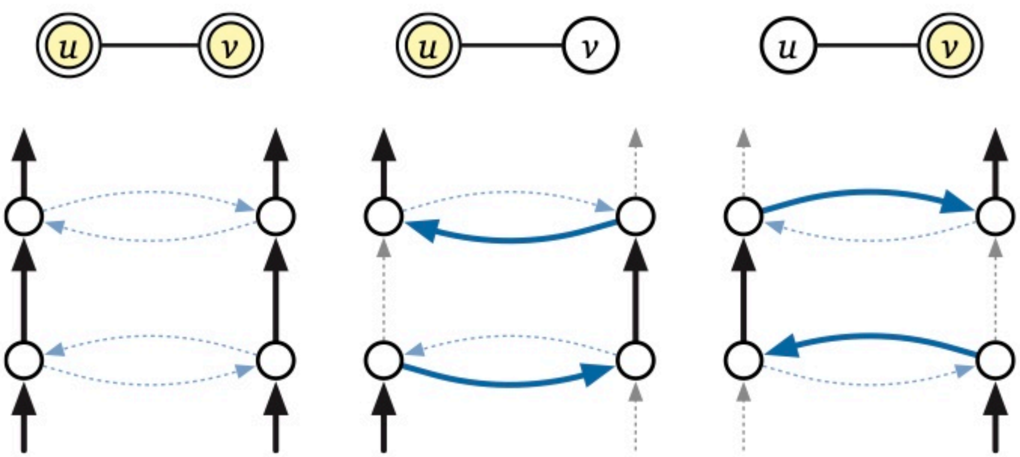
$(u,v,\text{in}) \to (u,v,\text{out})$ $(u,v,\text{in}) \to (v,u,\text{in})$ $(v,u,\text{in}) \to (u,v,\text{in})$

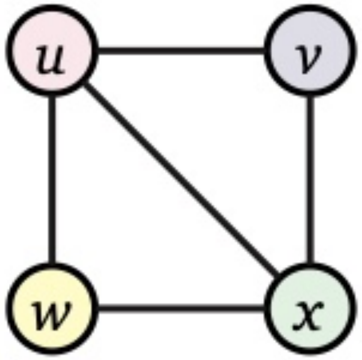
$(v,u,\text{in}) \to (v,u,\text{out})$ $(u,v,\text{out}) \to (v,u,\text{out})$ $(v,u,\text{out}) \to (u,v,\text{out})$

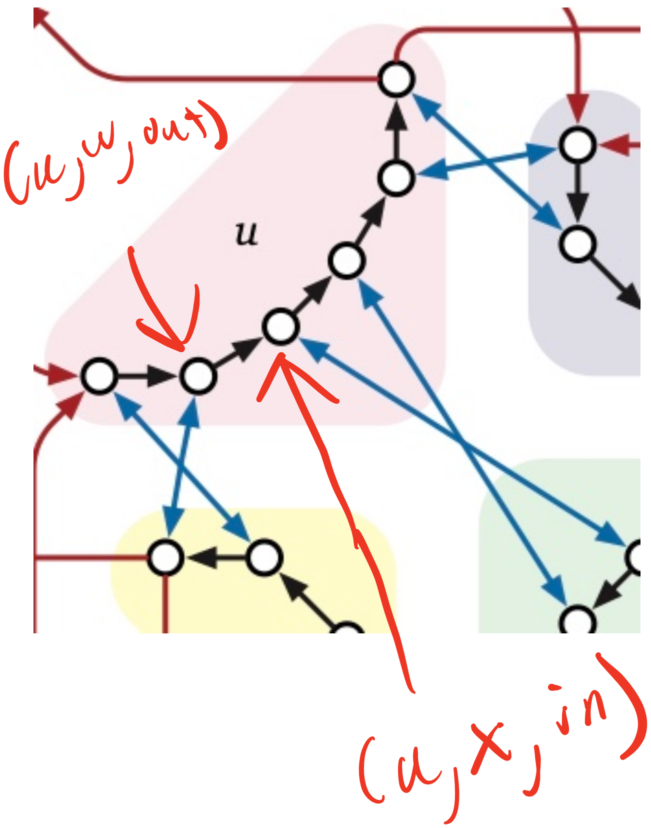
from G



for H







For each $u \in V$ with neighbors $v_1, v_2, \ldots, v_d$.

Build a <u>vertex chain</u>

by adding edges

$$(u, v_i, out) \to (u, v_{i+1}, in)$$

for all $i \in \{1, \ldots, d-1\}$

Cycle passes through edge gadgets along chain or touch its vertices through detours.

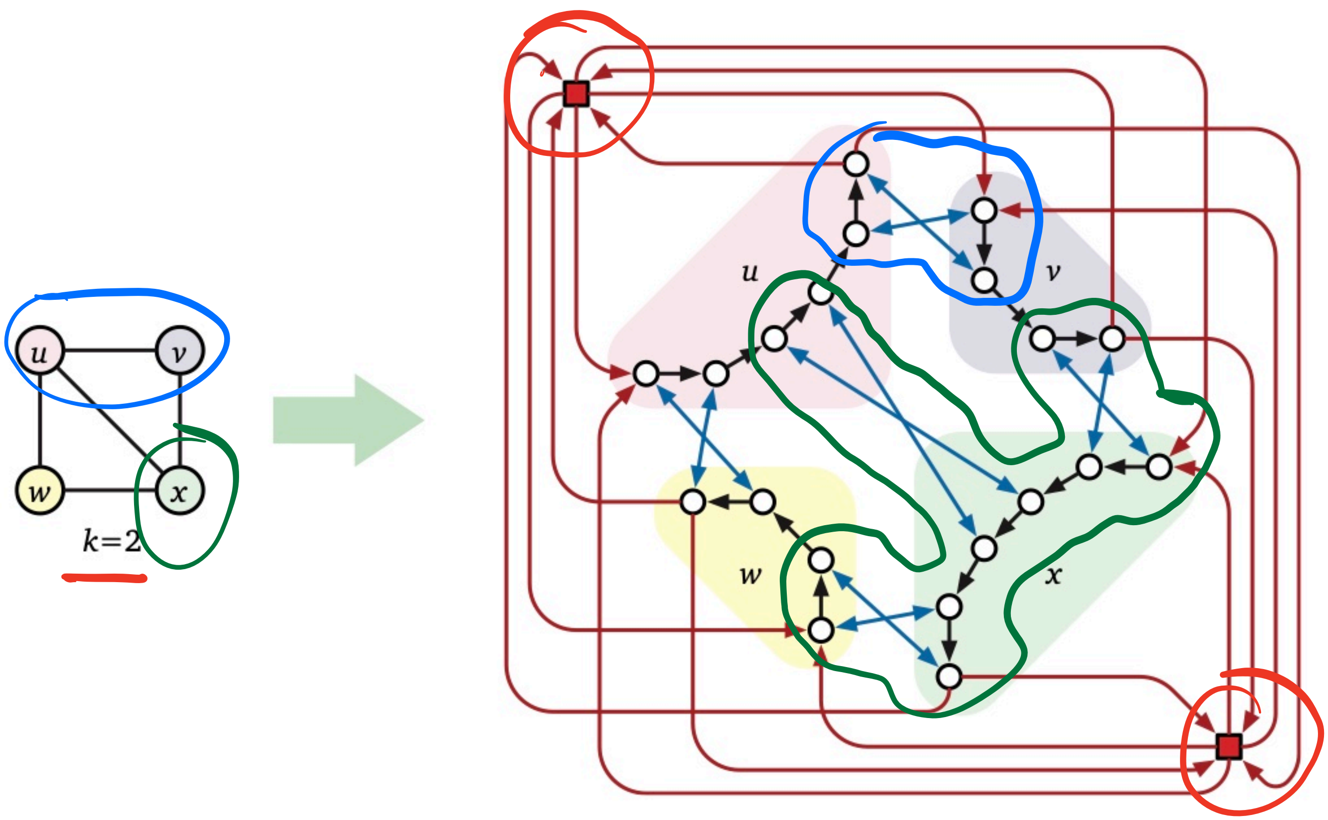
yes use u

don't use u

Add k cover vertices

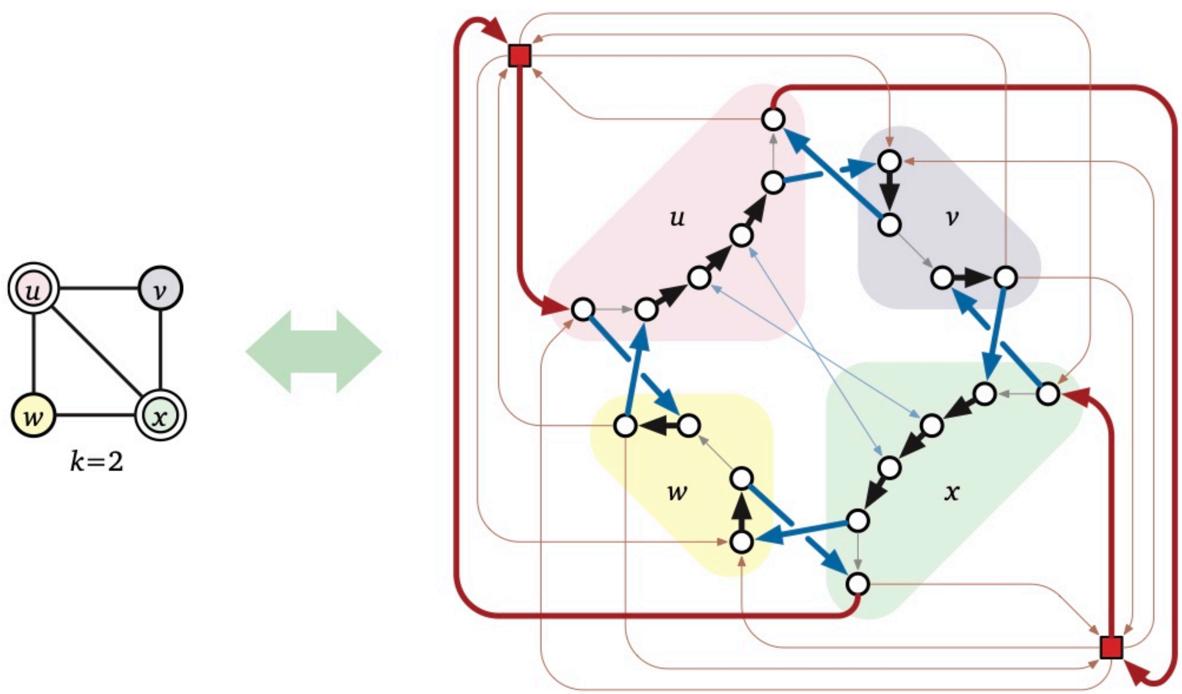
$x_0, x_1, \ldots, x_{k-1}$.

Each x_i has edges to all (u, v_1, in) + from all (u, v_d, out).



Need to prove G has a vertex cover of size k iff H has a Ham cycle.

$\Rightarrow$ Suppose $C = \{u_0, u_1, \ldots, u_{k-1}\}$ be a vertex cover of G.



For i from 0 to $k-1$, walk from x_i to the chain for u_i to cover vertex $x_{i+1 \bmod k}$.

For each $u_i v \in E$, if $v \in C$, then we do the detour through $(v, u_i, -)$. o.w. go straight from (u_i, v, in) to (u_i, v, out).

$\Leftarrow$ Suppose H has a Ham. cycle C.

We go from each x_i to a vertex chain for som u.

Include u in a vertex cover.

C touch each edge uv's gadget at

least once, so we use either u or v in cover.

See 3Color in book for another example!