GPS 10 is due Mon, 17th.

HW 10 is due Tue, 18th.

HW 11 is due Tue, Dec 2nd.

GPS 11 due Mon, Dec 8th

HW 12 "due" Mon, the 8th (not graded)

<u>Reduce</u> a problem A <u>to</u> a problem B: call algorithms for B as <u>a black-box</u>.

Given undirected graph $G=(V,E)$ as input.

Maximum Independent Set:

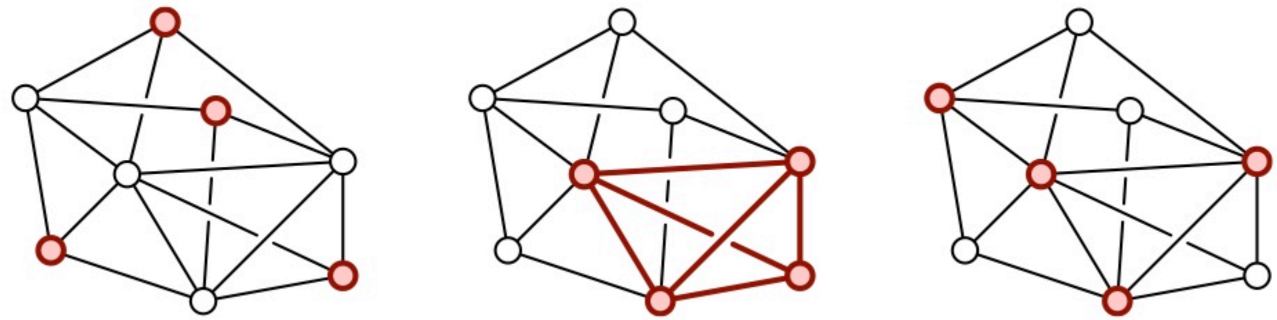
- independent set $A \subseteq V$: no two members of A share an edge
- want # vertices in max size independent set

Maximum Clique:

- clique: complete subgraph
- want # vertices in max size clique

Minimum Vertex Cover:

- vertex cover: subset $C \subseteq V$ s.t. every edge in G incident to <u>at least one</u> member of C
- want # vertices in smallest vertex cover



independent set — clique — vertex cover

Want to solve Max Ind Set. Given graph $G=(V,E)$:

← compliment

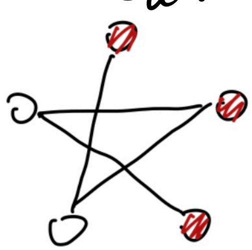
Build graph $\overline{G}=(V,\overline{E})$

$V=V$;

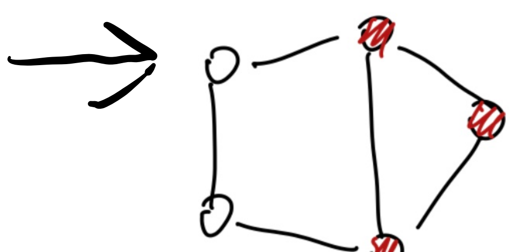
$\overline{E}=\{uv \mid uv \notin E\}$

$A \subseteq V$ is ind. in G iff A makes a clique in $\overline{G}$.

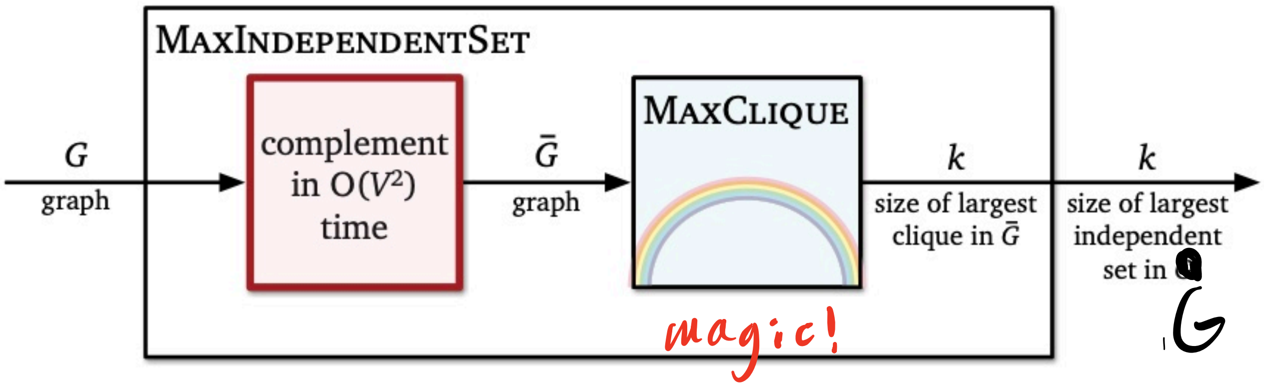
Call an algorithm for max clique



Max Ind Set



← Clique

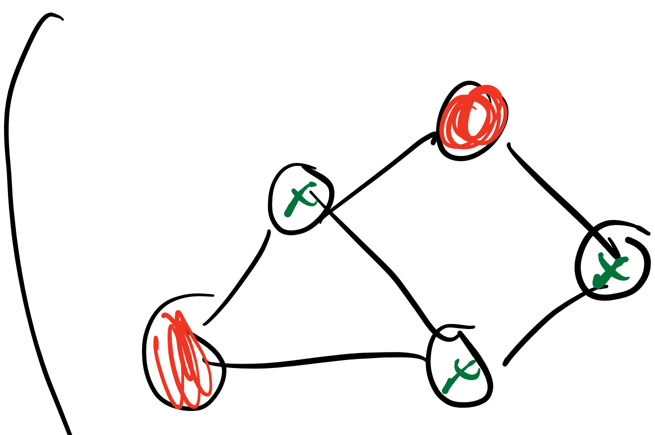


$T_{indset}(G) \leq O(V^2) + T_{clique}(\bar{G})$

$\Rightarrow$ if max clique has a poly time alg, then so does max ind set.

Say we want to solve Max Ind Set another way.

Given $G=(V,E)$. Want a reduction to min vertex cover.



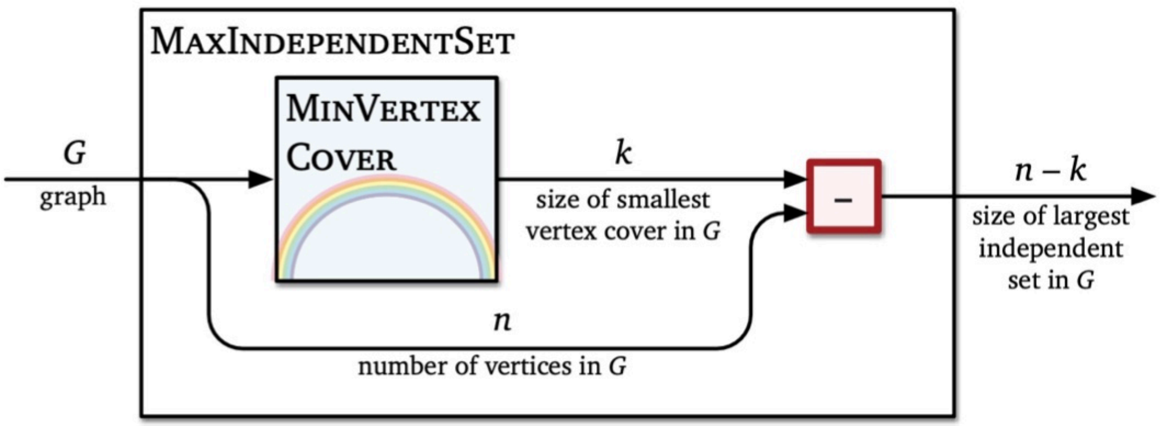
$A \subseteq V$ is an ind. set iff $V \setminus A$ is a vertex cover.

$\Rightarrow$ want to return compliment

of a min vertex cover.

$n \leftarrow |V|$

return $n -$ MinVertexCover(G) ← magic algorithm/ black-box



$$T_{\text{max ind set}}(G) \le O(1) + T_{\text{min vertex cover}}(G)$$

These reductions reverse "easily" so a poly time algo for any of them $\Rightarrow$ the same for all of them.

Given a undirected / directed graph $G=(V,E)$, a Hamiltonian cycle includes <u>every vertex</u> exactly once.

UndirHamCycle: Given <u>undirected</u> G, is there a Ham. cycle?

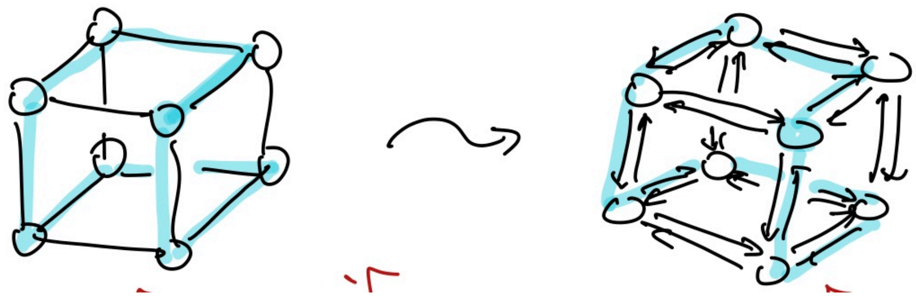
DirHamCycle: Given <u>directed</u> G, is there a Ham. cycle?

Given instance of UndirHamCycle (undirected $G=(V,E)$), want to reduce to DirHamCycle.

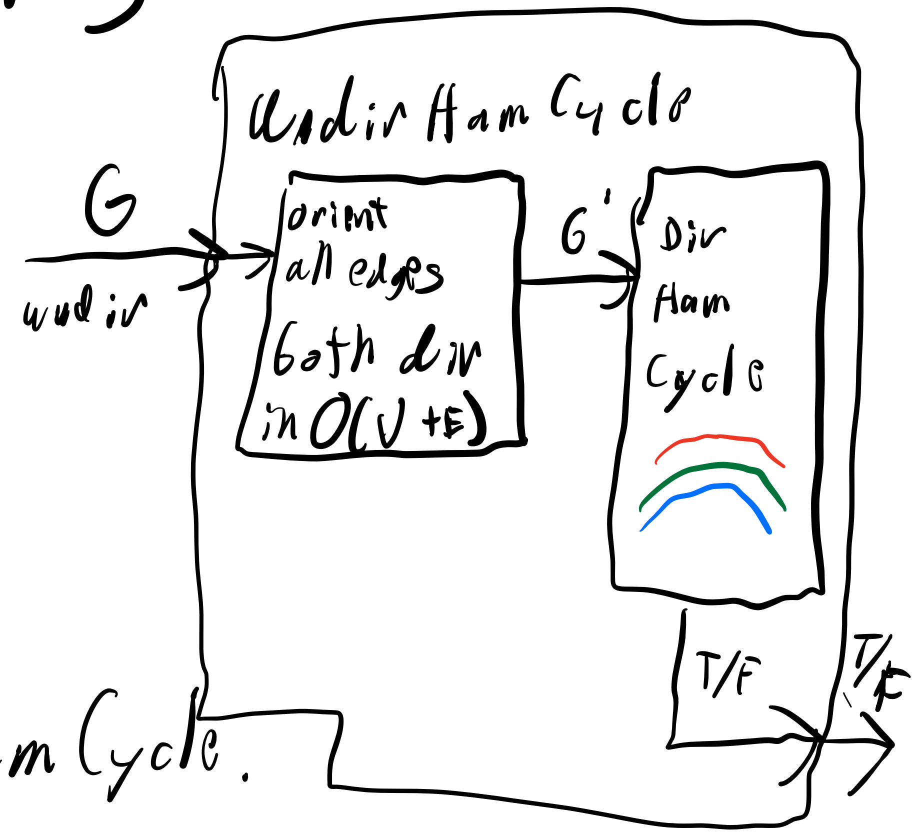
- Build a <u>directed</u> graph $G'=(V',E')$

$V' = V$

$E' = \{u \to v \mid uv \in E\}$



G G'



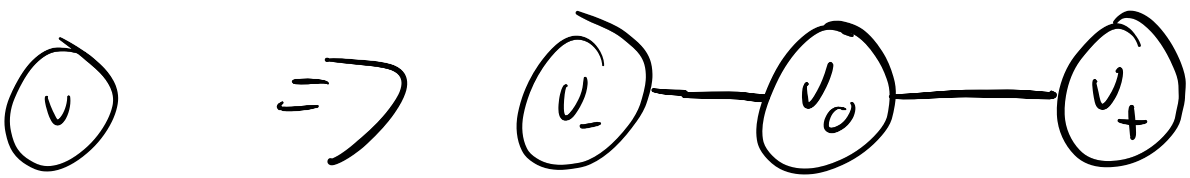
Return result of DirHamCycle.

Want to solve DirHamCycle. Given directed $G=(V,E)$.

Build an <u>undirected</u> $G'=(V',E')$

$V' = \{v_-, v_0, v_+ \mid v \in V\}$ $|V'| = 3|V|$

$|E'| = |E| + 2|V|$



$E' = \{u_+ v_- \mid u \to v \in E\}$

magic.

Return result of UndirHamCycle algorithm.

So fast UndirHamCycle $\Rightarrow$ fast DirHamCycle.
(poly time)

Claim 1: If G has a Ham. cycle, then G' has a Ham. cycle.

Proof: Let $C = u \to v \to w \to \dots \to u$

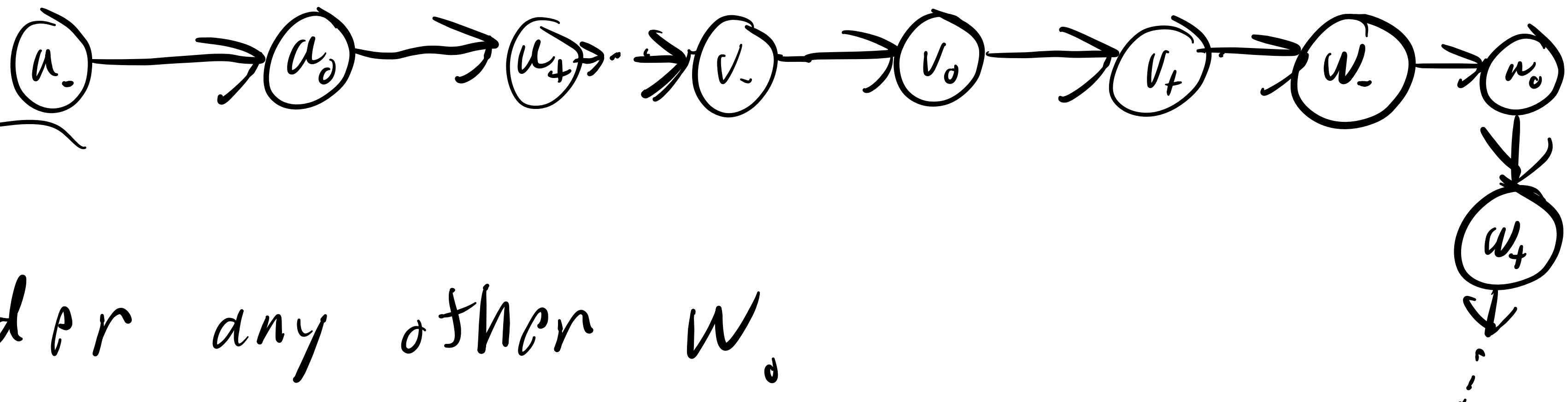
Let $C' = u_- \to u_0 \to u_+ \to v_- \to v_0 \to v_+ \to \dots \to u_-$

Claim 2: If G' has a Ham. cycle, then G has a Ham. cycle.

Proof: Let C' be a Ham. cycle in G'.

Pick any u_0. u_0 is adjacent to u_- & u_+

G' is undirected, so we may assume C' goes $u_- \to u_0 \to u_+$ by reversing it if necessary.



Consider any other w_0

By induction on # vertices from u_- to w_0, we visit w's three vertice along an edge $v_+ w_-$

$\Rightarrow$ it goes $w_- \to w_0 \to w_+$

$$u \to \cdots \to v \to w \to \cdots$$

These $+$ to $-$ hops work with dirs in G. Also we use every vertex in G' once, so take the x_0 vertices in order to get a Ham. cycle in G.