

Midterm 1 grades.

Extra office hours: Emily 2-4pm Wednesday &
Thursday
2336 Siebel

Jeff: Friday 9-11am &
1-3pm
3237 Siebel

Check Gradescope for homework scores.

Coursework:

HW 6 today.

GPS + HW 7 due next week.

Given strings $A[1..m]$ & $B[1..n]$ over the same alphabet.

Edit distance (Levenshtein, Udam, ~~Vintsyuk~~):
minimum number of insertions, deletions,
and substitutions to change A into B .

ALGORITHM
↓ ↓ ↓ ↓ ↓
AL TRUISTIC

6 edits $\Rightarrow$ ED is ≤ 6

Guess an edit and recurse.

too many subproblems? (all strings?)

A L G O R I T H M

A L T R U I S T I C

↑
del

↑
substitute

↑

insertion

Guess ~~any~~ column?

a

the rightmost column?

ALGOR	I		T	H	M
ALTRU	I	S	T	I	C

want edit distance
of ALGOR +
ALTRU

Edit(i, j): Edit distance of $A[1..i] + B[1..j]$.
Specification! 

Want to compute Edit(m, n) to get
ED of $A[1..m] + B[1..n]$.

Three cases for "next" column, Edit(i, j):

Insertion:

ALGOR	
ALTR	U

 costs $1 + \text{Edit}(i, j-1)$
ED between $A[1..i] + B[1..j-1]$

Deletion:

ALGO	R
ALTRU	

 costs $1 + \text{Edit}(i-1, j)$

"Substitution"

ALGO	R
ALTR	U

- or -

ALGO	R
ALT	R

costs $Edit(i-1, j-1)$
 $+ [A[i] \neq B[j]]$

$\uparrow$
 indicator

$+ (A[i] \neq B[j])$
 $+ (A[i] \neq B[j])_{as}$
 int

take min of those options.

If $i=0$, $Edit(i, j) = j$

$j=0$, $Edit(i, j) = i$

$(Edit(0, 0) = 0)$

$$\text{Edit}(i, j) = \begin{cases} i & \text{if } j = 0 \\ j & \text{if } i = 0 \end{cases}$$

Goal:

$$\text{Edit}(m, n)$$

$$\min \begin{cases} \text{Edit}(i, j-1) + 1 \\ \text{Edit}(i-1, j) + 1 \\ \text{Edit}(i-1, j-1) + [A[i] \neq B[j]] \end{cases}$$

otherwise

Subproblems: $0 \leq i \leq m$

$$0 \leq j \leq n$$

$$\text{Time: } O(i) \cdot O(mn) = O(mn)$$

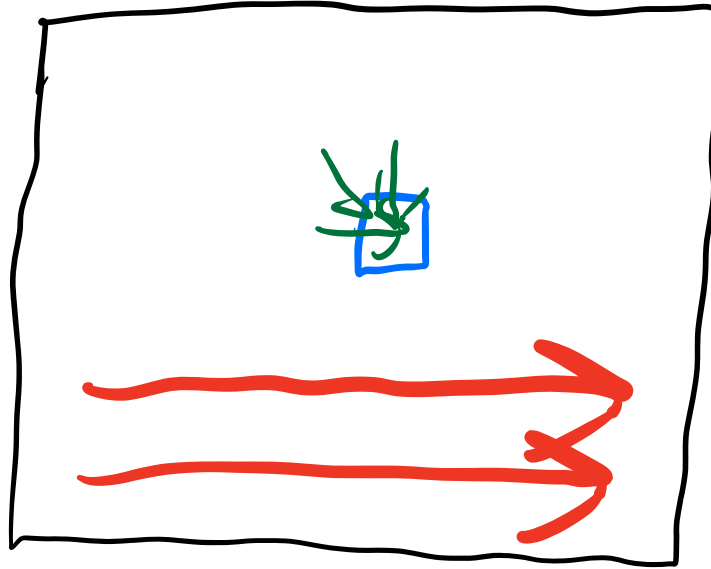
Data structure: $Edit[0..m, 0..n]$.

Dependencies:

Eval order: i

"row-major
order"

top-down, left-to-right j



EDITDISTANCE($A[1..m], B[1..n]$):

for $j \leftarrow 0$ to n

$Edit[0, j] \leftarrow j$

for $i \leftarrow 1$ to m

$Edit[i, 0] \leftarrow i$

 for $j \leftarrow 1$ to n

$ins \leftarrow Edit[i, j-1] + 1$

$del \leftarrow Edit[i-1, j] + 1$

 if $A[i] = B[j]$

$rep \leftarrow Edit[i-1, j-1]$

 else

$rep \leftarrow Edit[i-1, j-1] + 1$

$Edit[i, j] \leftarrow \min\{ins, del, rep\}$

return $Edit[m, n]$

	A L G O R I T H M									
	0	1	2	3	4	5	6	7	8	9
A	1	0	1	2	3	4	5	6	7	8
L	2	1	0	1	2	3	4	5	6	7
T	3	2	1	1	2	3	4	4	5	6
R	4	3	2	2	2	2	3	4	5	6
U	5	4	3	3	3	3	3	4	5	6
I	6	5	4	4	4	4	3	4	5	6
S	7	6	5	5	5	5	4	4	5	6
T	8	7	6	6	6	6	5	4	5	6
I	9	8	7	7	7	7	6	5	5	6
C	10	9	8	8	8	8	7	6	6	6

A L G O R I T H M
A L T R U I S T I C

A L G O R I T H M
A L T R U I S T I C

A L G O R I T H M
A L T R U I S T I C

Follow arrows

(how you got
best values)
backwards for
best edits.