Homework: Citation list on separate page after each part. Even if 0 sources. "Nothing to cite / ∅"

↑ (a), (b), ...

LLM transcripts can be long-lived links.

o.w. use as many pages as necessary

No hard page limits.

Fix an alphabet Σ. Let x and y be strings,

L a language.

z is a distinguishing suffix of x and y if exactly one of xz and yz are in L.

$\Rightarrow$ x and y go to distinct states in a DFA.

A set F of strings is a fooling set for L if there is a distinguishing suffix for every pair in F.

$L: (0+1)^*(00+11)(0+1)^*$

$F = \{\varepsilon, 0, 1, 00\}$

(00 and 11 cannot be distinguished: $00z \in L$, $11z \in L$ for all z)

ε and 00 are distinguished by ε ($\varepsilon\varepsilon = \varepsilon \notin L$
$00\varepsilon = 00 \in L$)

0 and 00 — ε

ε and 1 — 1 ($1 \notin L$
$11 \in L$)

1 — 00 — ε

0 — 1 — 1

ε — 0 — 0

L needs ≥ 4 states

<u>Thm</u>: For any language L, max fooling set size
$=$ min # DFA states

So infinite fooling set $\Rightarrow$ no DFA.

Ex: $L = \{0^n 1^n \mid n \geq 0\}$ is not regular.

Proof: Consider infinite set $F = \{0^n \mid n \geq 0\} = 0^*$

Let x and y be distinct strings in F.

$x = 0^i$ and $y = 0^j$ for some $i \neq j$

Let $z = 1^i$

$xz = 0^i 1^i \in L$

$yz = 0^j 1^i \notin L$, because $i \neq j$

So F is a fooling set for L.

F is also infinite, so L is not regular.

Ex: Set of palindromes $L = \{w \mid w = w^R\}$ is not regular.

<u>Proof:</u> Consider the infinite set $F = \{0^n 1 \mid n \geq 0\}$.

Let x and y be distinct strings in F.

So $x = 0^i 1$ and $y = 0^j 1$ with $i \neq j$.

Let $z = 0^i$.

$xz = 0^i 1 0^i \in L$.

$yz = 0^j 1 0^i \notin L$.

F is a fooling set for L.

F is infinite, so L is not regular.

Ex: L: All strings <u>except</u> $\{0^n 1^n \mid n \geq 0\}$ is not regular.

Proof: Suppose for contradiction that L is regular.

Then $\overline{L} = \Sigma^* \setminus L$ is regular,

We just proved $\overline{L}$ is <u>not</u> regular, so contradiction.

So L is <u>not</u> regular.

Ex: L: Binary strings w such that $\#(w, 0) = \#(w, 1)$.

Suppose for contradiction L is regular.

$0^* 1^*$ is regular, so $L \cap 0^* 1^* = \{0^n 1^n \mid n \geq 0\}$

is regular,

Contradiction!

So L is not regular.

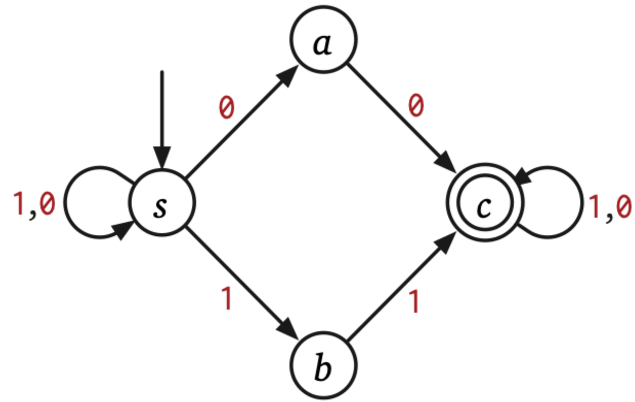
Go in correct direction!

Anti-example:

$L = \{0^n 1^n \mid n \geq 0\}$ & $\overline{L}$ are not regular.

$L \cup \overline{L} = \Sigma^*$ is regular. :-)

Nondeterministic finite-state automata (NFAs):



subsets of Q

↓

powerset of Q

↓

Formally, an NFA has components:

<u>input alphabet</u> Σ

finite set of <u>states</u> Q

a <u>transition function</u> $\delta : Q \times \Sigma \to 2^Q$

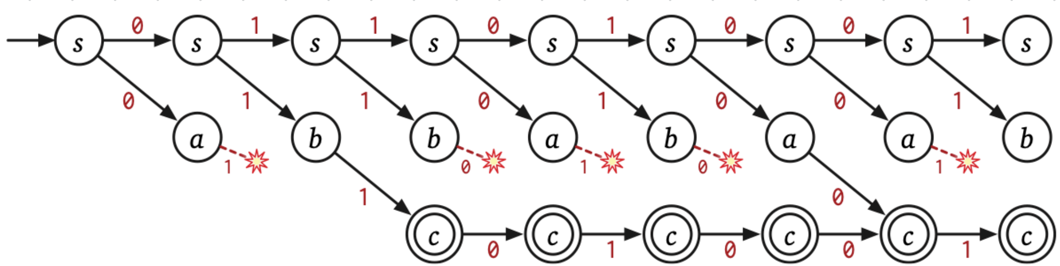
a <u>start state</u> $s \in Q$

set of <u>accepting states</u> $A \subseteq Q$

extended transition function $\delta^*: Q \times \Sigma^* \to 2^Q$

$$\delta^*(q,w) := \begin{cases} \{q\} & \text{if } w = \varepsilon \\ \displaystyle\bigcup_{r \in \delta(q,a)} \delta^*(r,x) & \text{if } w = ax \end{cases}$$

NFA accepts w iff $\delta^*(s,w) \cap A \neq \emptyset$



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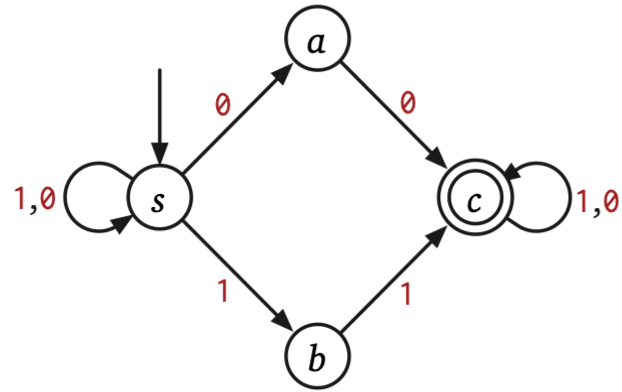
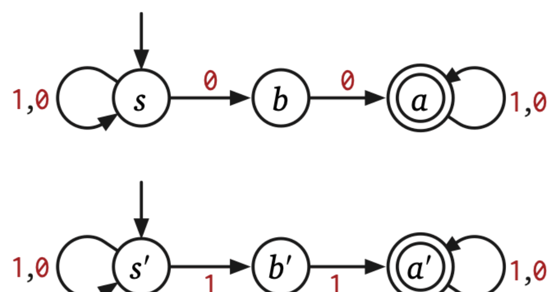
is <u>accepted!</u>



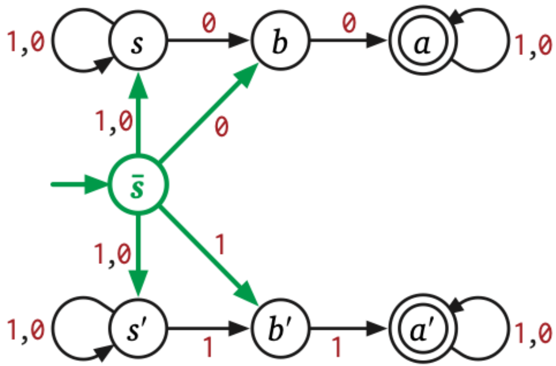
Extension:

Multiple start states:



Uses a set $S \subseteq Q$ of start states.

Accept w iff $\left[\bigcup_{s \in S} \delta^*(s,w)\right] \cap A \neq \emptyset$



1,0
s
0
b
0
a
1,0
1,0
0
s̄
1
1,0
1,0
s′
1
b′
1
a′
1,0