CS 374 A

Induction/Recursion

Σ = alphabet = $\{0,1\}$
any finite set

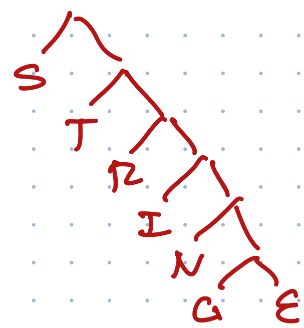
$\boxed{\begin{array}{llll} \text{String} = \text{either} & \varepsilon & & \text{empty} \\ \text{or} & a \cdot x & & \text{symbol } a \in \Sigma \\ & & & \text{string } x \in \Sigma^* \end{array}}$

Σ^* = all strings over Σ

All strings are finite, but Σ^* is infinite

sequence of ⟨foo⟩s is ε
or $a \cdot x$ where a is ⟨foo⟩
x is seq of ⟨foos⟩

STRING = S · TRING
= S · (T · RING)



$$\text{length } |w| = \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = a \cdot x \end{cases}$$

$$\text{concatenation } w \bullet z = \begin{cases} z & w = \varepsilon \\ a \cdot (x \bullet z) & w = ax \end{cases}$$

NOW • HERE ⟶ NOWHERE

Lemma: For all strings w and z:

$$|w \circ z| = |w| + |z|$$

Proof:

Let w and z be arbitrary strings

Assume for all strings x shorter than w that $|x \circ z| = |x| + |z|$

There are two cases:

- $w = \varepsilon$

$$\begin{aligned} |w \circ z| &= |\varepsilon \circ z| && w = \varepsilon \\ &= |z| && \text{def } \circ \\ &= 0 + |z| && \text{duh} \\ &= |\varepsilon| + |z| && \text{def } |\varepsilon| \\ &= |w| + |z| && \text{def } w \end{aligned}$$

- $w = a \cdot x$

$$\begin{aligned} |w \circ z| &= |ax \circ z| && \text{def } w \\ &= |a(x \circ z)| && \text{def } \circ \\ &= 1 + |x \circ z| && \text{def } |\,| \\ &= 1 + |x| + |z| && \text{IH!} \\ &= |ax| + |z| && \text{def } |\,| \\ &= |w| + |z| && \text{def } w \end{aligned}$$

Therefore, $|w \circ z| = |w| + |z|$.