24.3
NP-Completeness of Graph Coloring

24.3.1
The coloring problem

Graph Coloring

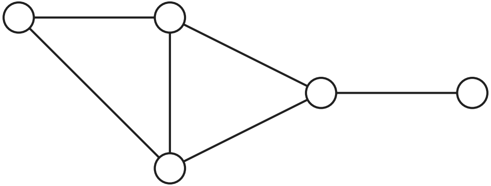
Problem: Graph Coloring

Instance: $G = (V, E)$: Undirected graph, integer k.
Question: Can the vertices of the graph be colored using k colors so that vertices connected by an edge do not get the same color?

Graph 3-Coloring

Problem: 3 Coloring

Instance: $G = (V, E)$: Undirected graph.
Question: Can the vertices of the graph be colored using 3 colors so that vertices connected by an edge do not get the same color?



Graph 3-Coloring

Problem: 3 Coloring

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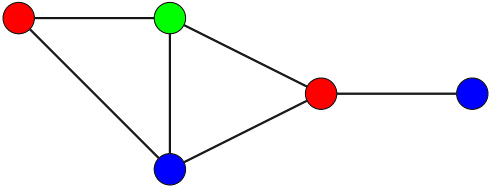
Question: Can the vertices of the graph be colored using **3** colors so that vertices connected by an edge do not get the same color?



Graph Coloring

1. Observation: If G is colored with k colors then each color class (nodes of same color) form an independent set in G.
2. G can be partitioned into k independent sets $\iff$ G is k-colorable.
3. Graph 2-Coloring can be decided in polynomial time.
4. G is 2-colorable $\iff$ G is bipartite.
5. There is a linear time algorithm to check if G is bipartite using **BFS** (we saw this earlier).

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THE END

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(for now)