16.3.2
DFS with pre-post numbering

DFS

DFS with Visit Times

Keep track of when nodes are visited.

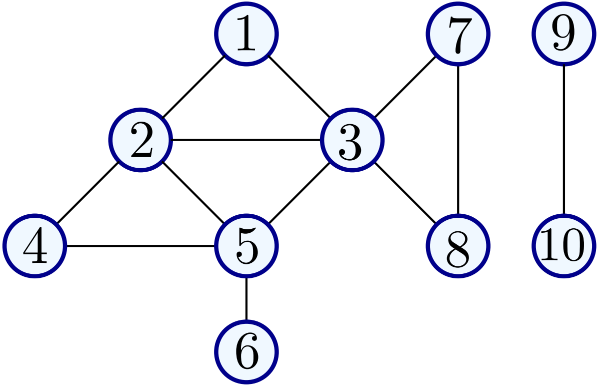
```
DFS(G)
    for all u ∈ V(G) do
        Mark u as unvisited
    T is set to ∅
    time = 0
    while ∃ unvisited u do
        DFS(u)
    Output T
```

```
DFS(u)
    Mark u as visited
    pre(u) = ++time
    for each uv in Out(u) do
        if v is not marked then
            add edge uv to T
            DFS(v)
    post(u) = ++time
```

Animation

$time = 0$

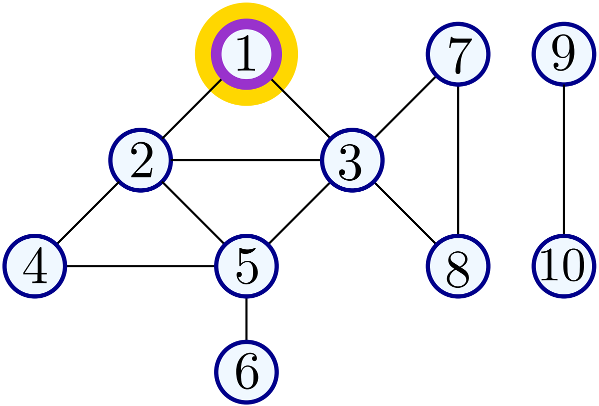
vertex	$[pre, post]$



Animation

$$\textit{time} = 1$$

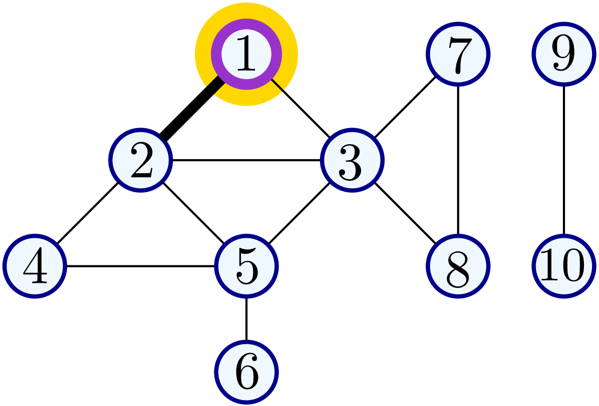
vertex	[*pre*, *post*]
1	[1,]



Animation

$time = 1$

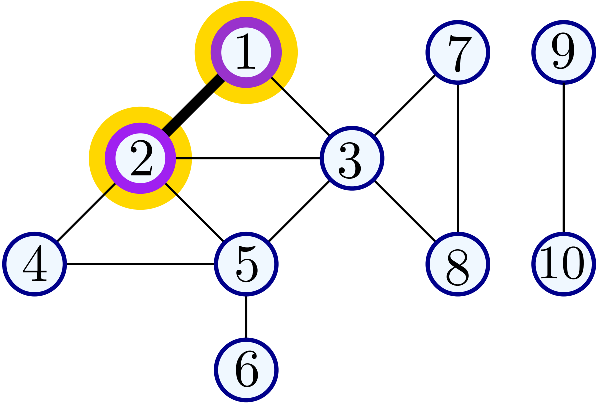
vertex	[*pre*, *post*]
1	[1,]



Animation

$$time = 2$$

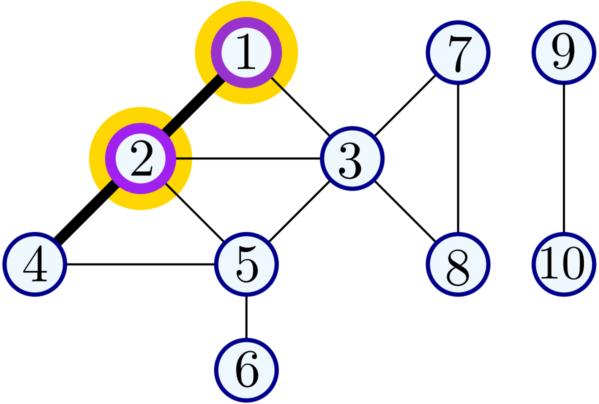
vertex	[*pre*, *post*]
1	[1,]
2	[2,]



Animation

$$time = 2$$

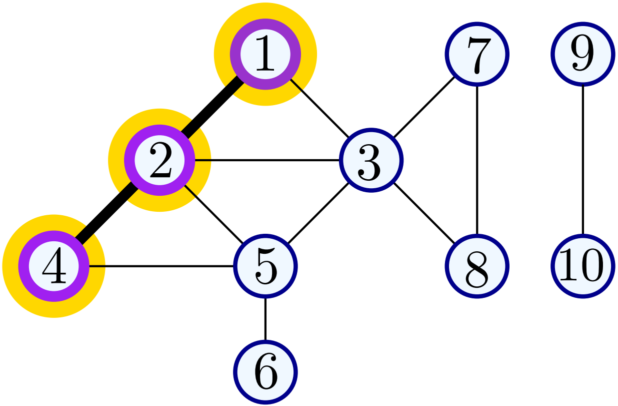
vertex	[*pre*, *post*]
1	[1,]
2	[2,]



Animation

$$time = 3$$

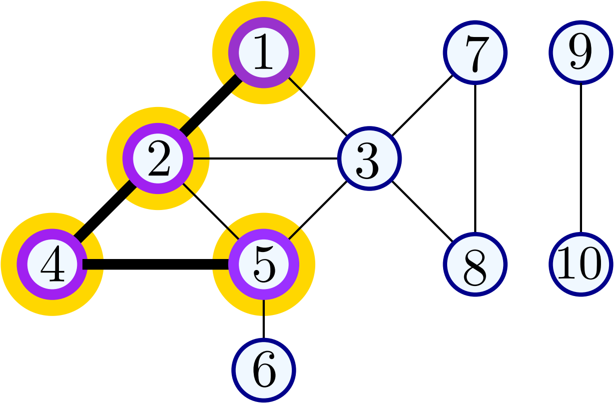
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]



Animation

$$time = 4$$

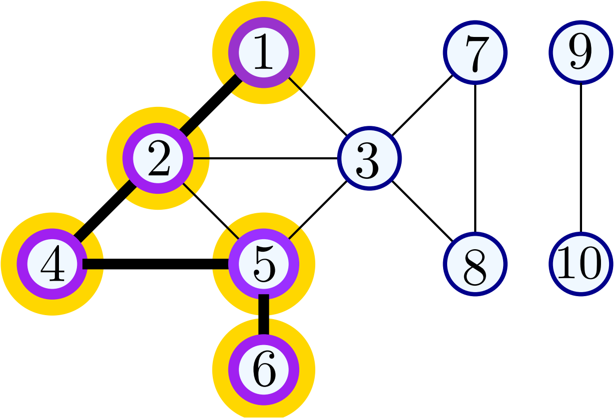
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]



Animation

$$time = 5$$

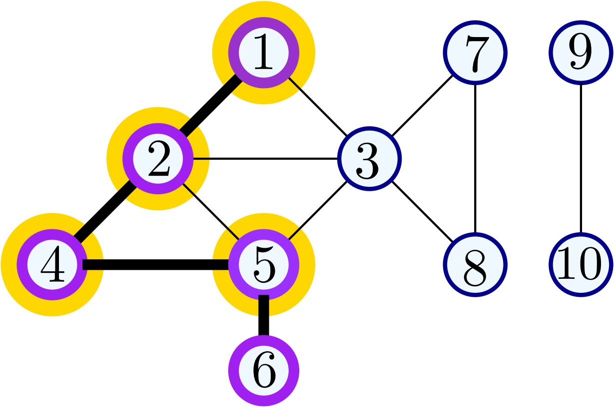
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5,]



Animation

$time = 6$

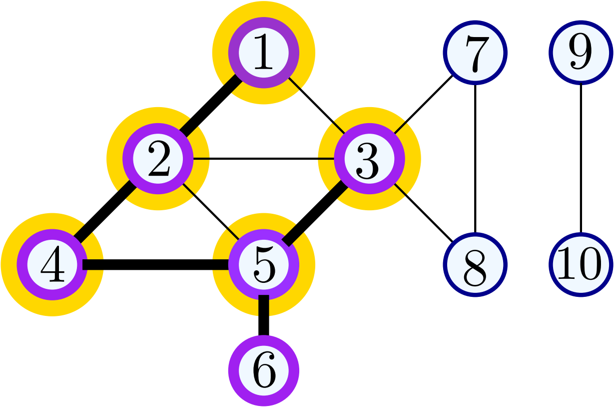
vertex	$[pre, post]$
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]



Animation

$$\textit{time} = 7$$

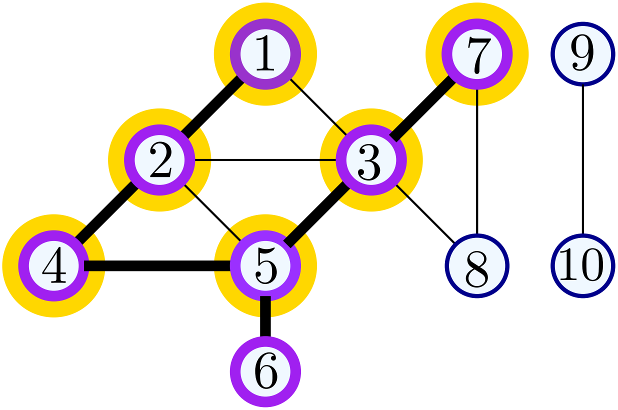
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]
3	[7,]



Animation

$$time = 8$$

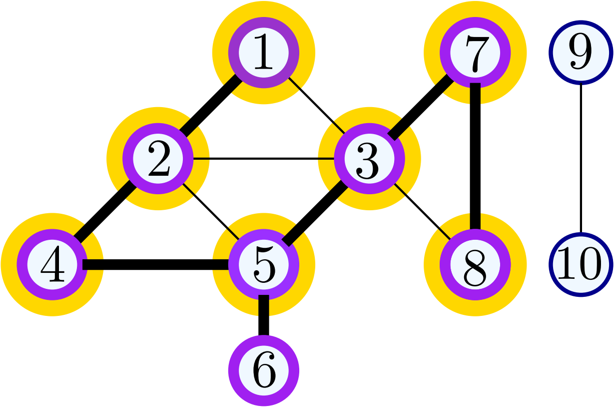
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]
3	[7,]
7	[8,]



Animation

$time = 9$

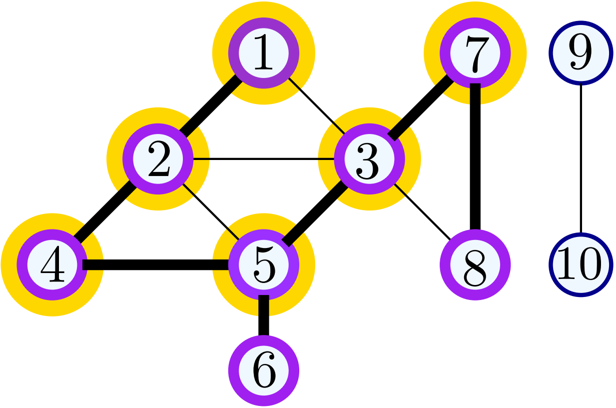
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]
3	[7,]
7	[8,]
8	[9,]



Animation

$\textit{time} = 10$

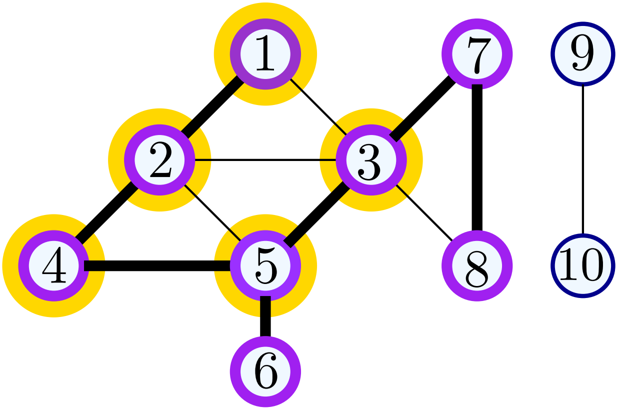
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]
3	[7,]
7	[8,]
8	[9, 10]



Animation

$$time = 11$$

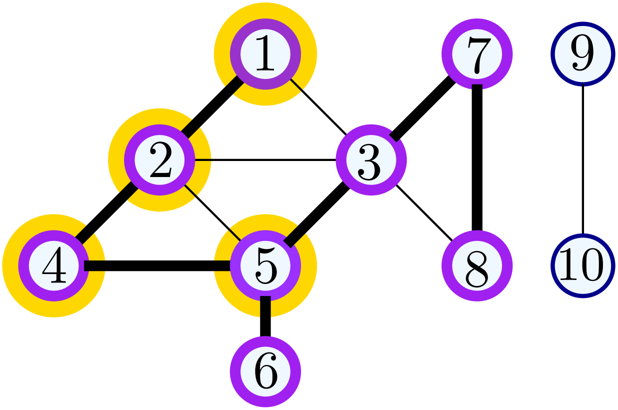
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]
3	[7,]
7	[8, 11]
8	[9, 10]



Animation

$$time = 12$$

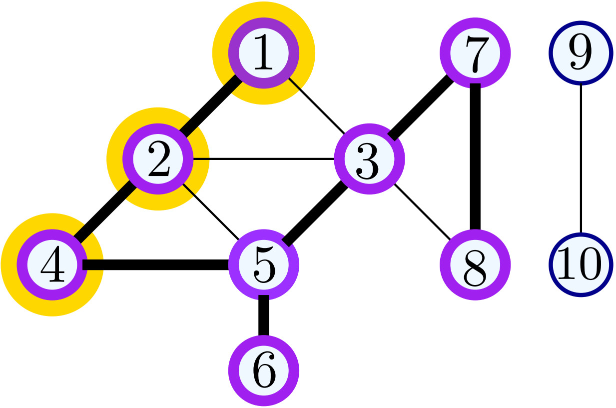
vertex	$[pre, post]$
1	[1,]
2	[2,]
4	[3,]
5	[4,]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]



Animation

$time = 13$

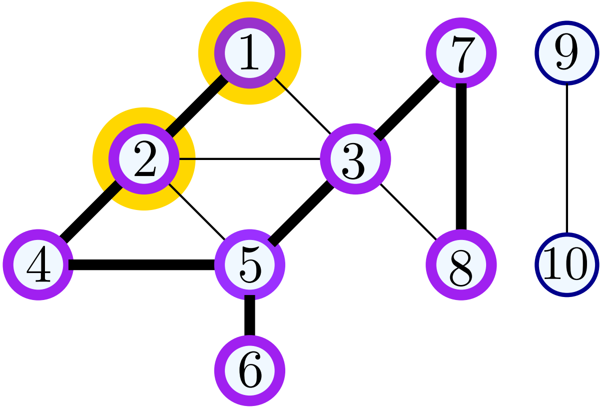
vertex	$[pre, post]$
1	[1,]
2	[2,]
4	[3,]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]



Animation

$$time = 14$$

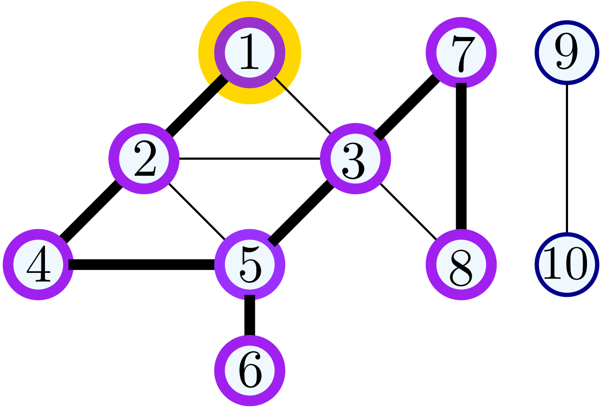
vertex	[*pre*, *post*]
1	[1,]
2	[2,]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]



Animation

$$time = 15$$

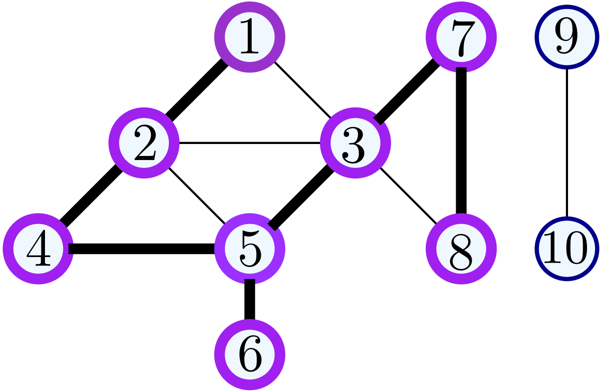
vertex	[*pre*, *post*]
1	[1,]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]



Animation

$time = 16$

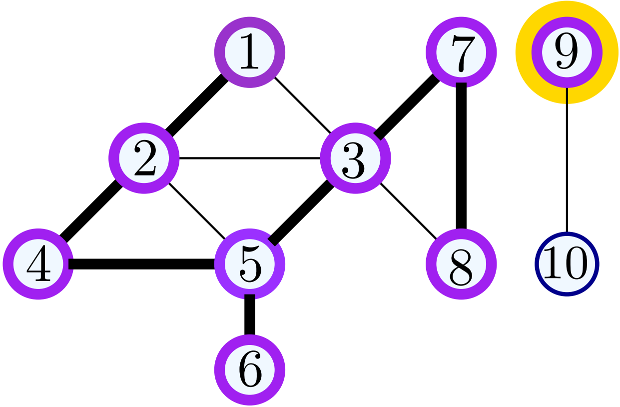
vertex	[*pre*, *post*]
1	[1, 16]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]



Animation

$time = 17$

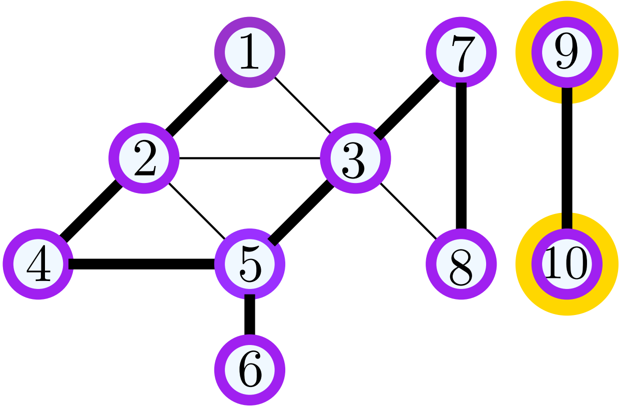
vertex	[*pre*, *post*]
1	[1, 16]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]
9	[17,]



Animation

$time = 18$

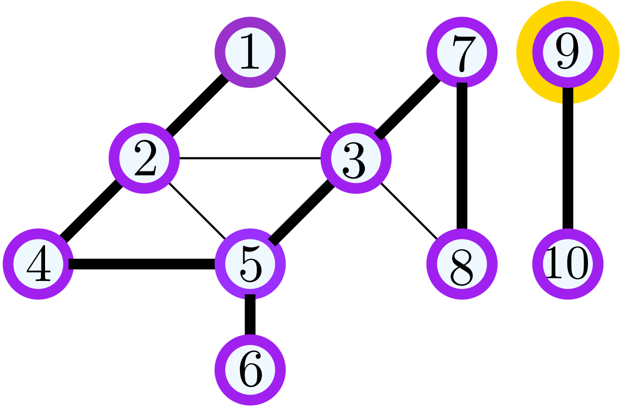
vertex	[*pre*, *post*]
1	[1, 16]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]
9	[17,]
10	[18,]



Animation

$$time = 19$$

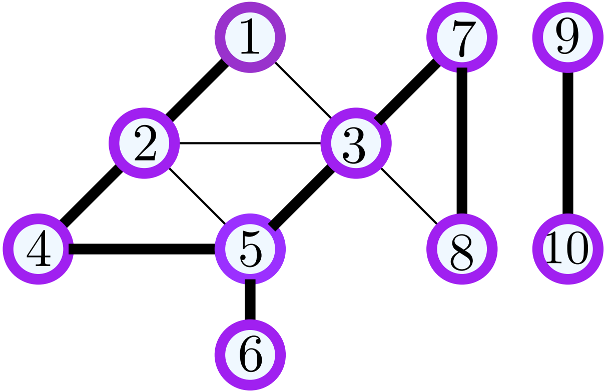
vertex	[*pre*, *post*]
1	[1, 16]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]
9	[17,]
10	[18, 19]



Animation

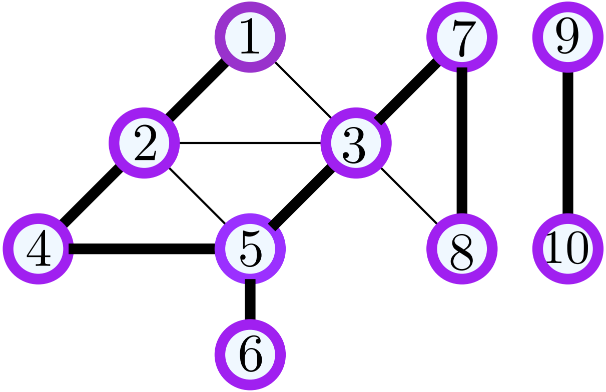
$$time = 20$$

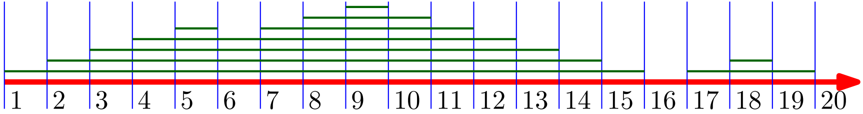
vertex	[*pre*, *post*]
1	[1, 16]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]
9	[17, 20]
10	[18, 19]



Animation

vertex	[*pre*, *post*]
1	[1, 16]
2	[2, 15]
4	[3, 14]
5	[4, 13]
6	[5, 6]
3	[7, 12]
7	[8, 11]
8	[9, 10]
9	[17, 20]
10	[18, 19]





pre and post numbers

Node u is **active** in time interval $[\mathrm{pre}(u), \mathrm{post}(u)]$

Proposition

For any two nodes u and v, the two intervals $[\mathrm{pre}(u), \mathrm{post}(u)]$ and $[\mathrm{pre}(v), \mathrm{post}(v)]$ are disjoint or one is contained in the other.

Proof.

- Assume without loss of generality that $\mathrm{pre}(u) < \mathrm{pre}(v)$. Then v visited after u.
- If **DFS**(v) invoked before **DFS**(u) finished, $\mathrm{post}(v) < \mathrm{post}(u)$.
- If **DFS**(v) invoked after **DFS**(u) finished, $\mathrm{pre}(v) > \mathrm{post}(u)$. □

pre and post numbers useful in several applications of **DFS**

$\mathbf{pre}$ and $\mathbf{post}$ numbers

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pre and post numbers useful in several applications of DFS

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pre and post numbers useful in several applications of **DFS**

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pre and post numbers useful in several applications of **DFS**

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Proposition

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$\mathbf{pre}$ and $\mathbf{post}$ numbers useful in several applications of **DFS**

THE END

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(for now)