15.4.1
Strong connected components

Connectivity and Strong Connected Components

Definition

Given a directed graph G, u is **strongly connected** to v if u can reach v and v can reach u. In other words $v \in \mathbf{rch}(u)$ and $u \in \mathbf{rch}(v)$.

Define relation C where uCv if u is (strongly) connected to v.

Proposition

C is an equivalence relation, that is reflexive, symmetric and transitive.

Equivalence classes of C: **strong connected components** of G.
They partition the vertices of G.
$\mathrm{SCC}(u)$: strongly connected component containing u.

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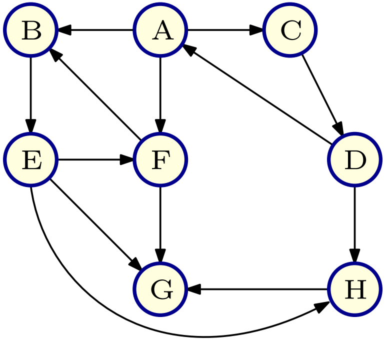
Strongly Connected Components: Example



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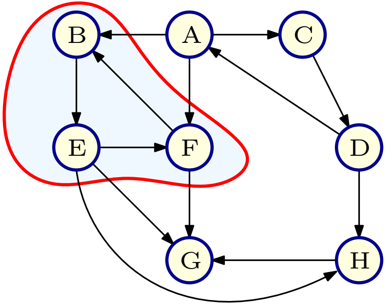

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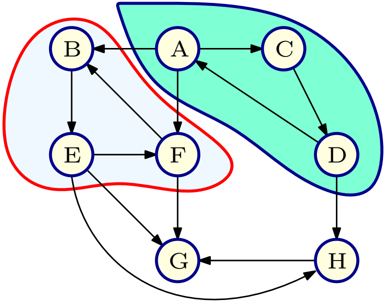

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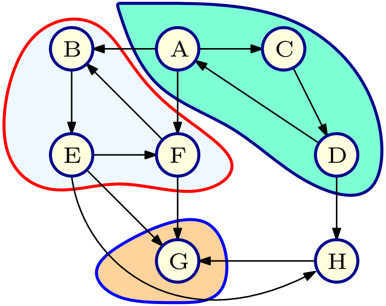

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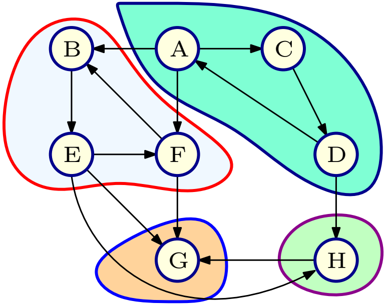

Directed Graph Connectivity Problems

1. Given G and nodes u and v, can u reach v?
2. Given G and u, compute $\mathsf{rch}(u)$.
3. Given G and u, compute all v that can reach u, that is all v such that $u \in \mathsf{rch}(v)$.
4. Find the strongly connected component containing node u, that is $\mathrm{SCC}(u)$.
5. Is G strongly connected (a single strong component)?
6. Compute <u>all</u> strongly connected components of G.

THE END

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(for now)