13.1.2
Automatic/implicit memoization

Automatic Memoization

Can we convert recursive algorithm into an efficient algorithm without explicitly doing an iterative algorithm?

```
Fib(n):
    if (n = 0)
        return 0
    if (n = 1)
        return 1
    if (Fib(n) was previously computed)
        return stored value of Fib(n)
    else
        return Fib(n − 1) + Fib(n − 2)
```

How do we keep track of previously computed values?

Two methods: explicitly and implicitly (via data structure)

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How do we keep track of previously computed values?
Two methods: explicitly and implicitly (via data structure)

Automatic implicit memoization

Initialize a (dynamic) dictionary data structure D to empty

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Fib(n):
        if (n = 0)
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            return 1
        if (n is already in D)
            return value stored with n in D
        val ⇐ Fib(n − 1) + Fib(n − 2)
        Store (n, val) in D
        return val
```

Use hash-table or a map to remember which values were already computed.

Explicit memoization (not automatic)

1. Initialize table/array M of size n: $M[i] = -1$ for $i = 0, \ldots, n$.
2. Resulting code:

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    M[n] ⇐ Fib(n − 1) + Fib(n − 2)
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3. Need to know upfront the number of subproblems to allocate memory.

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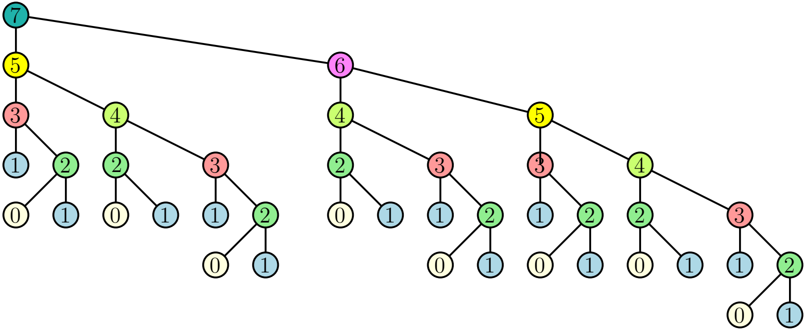
Recursion tree for the memoized Fib...



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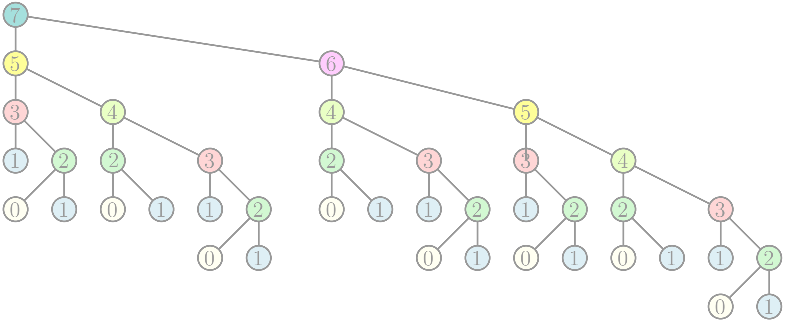

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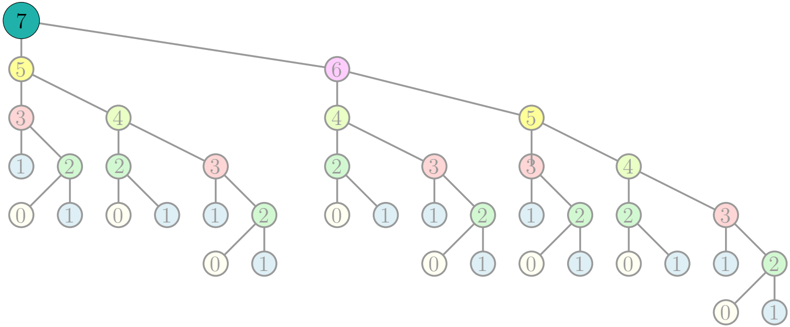

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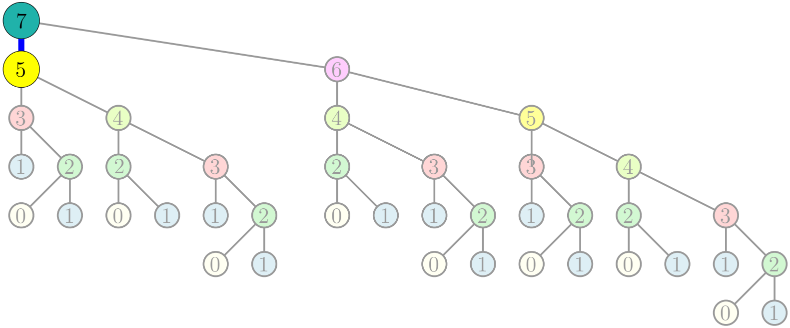

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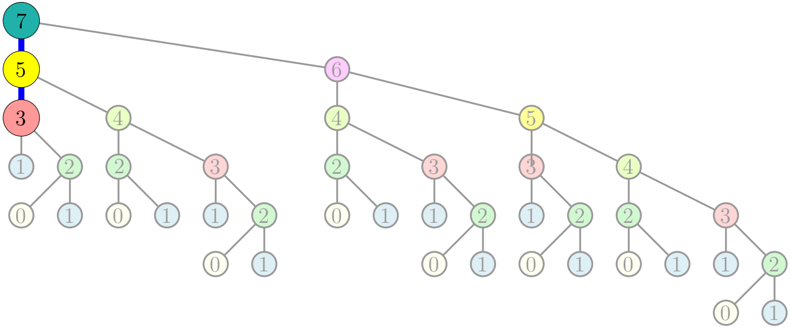

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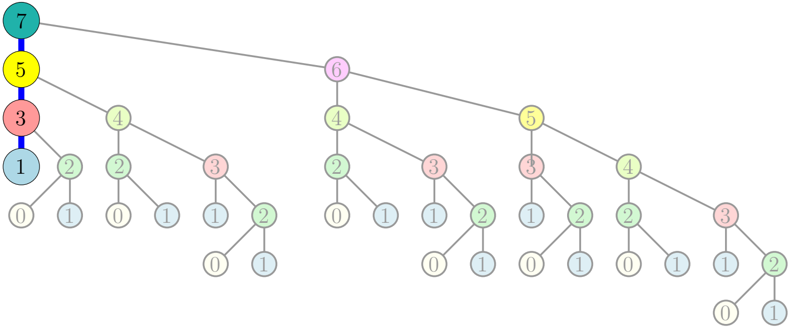

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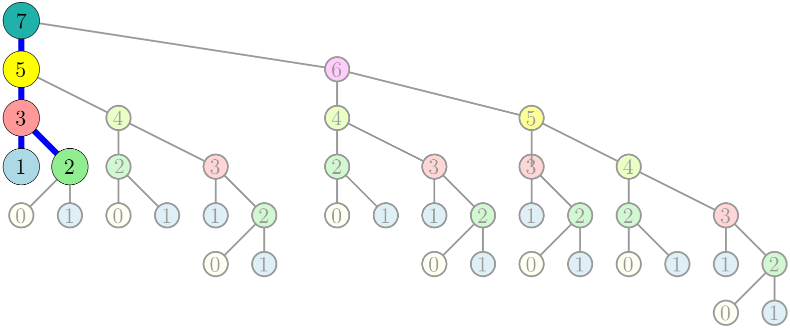

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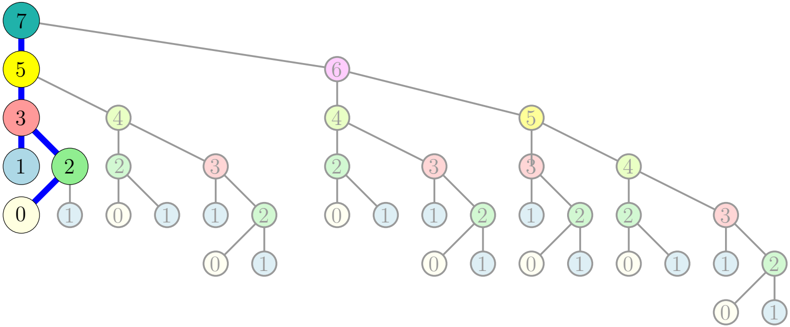

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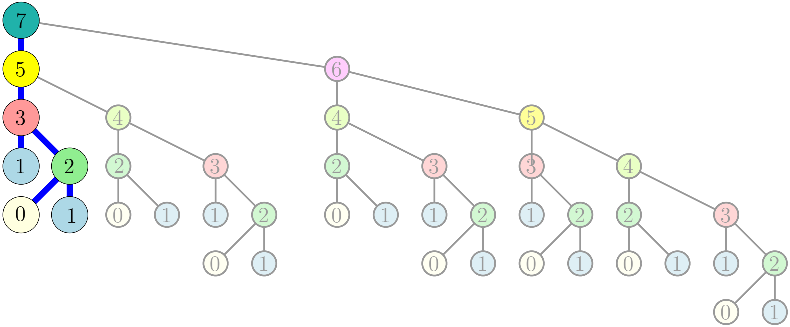

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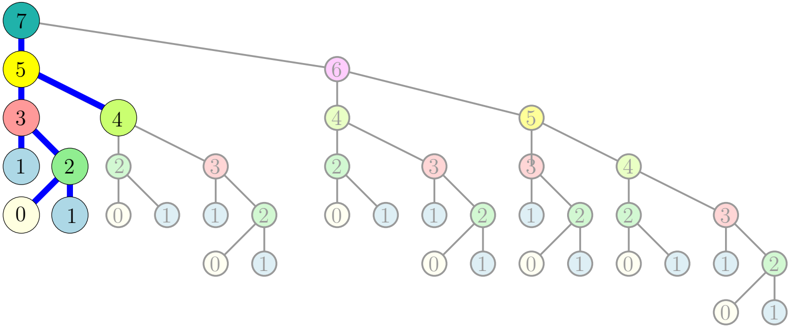

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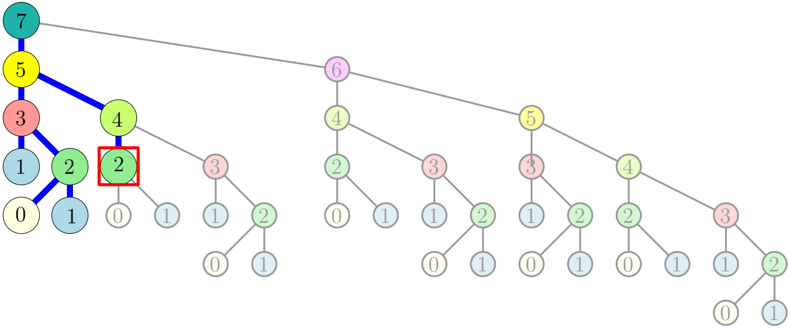

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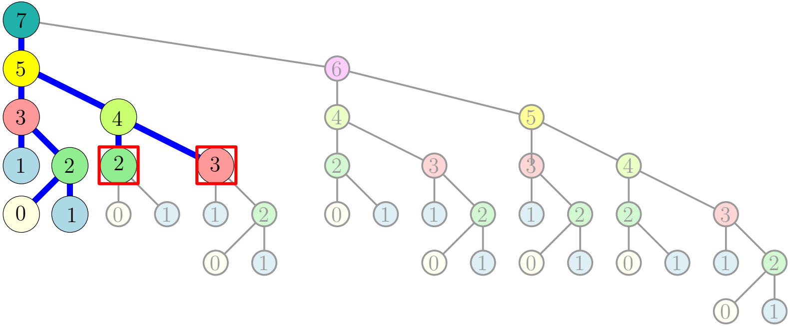

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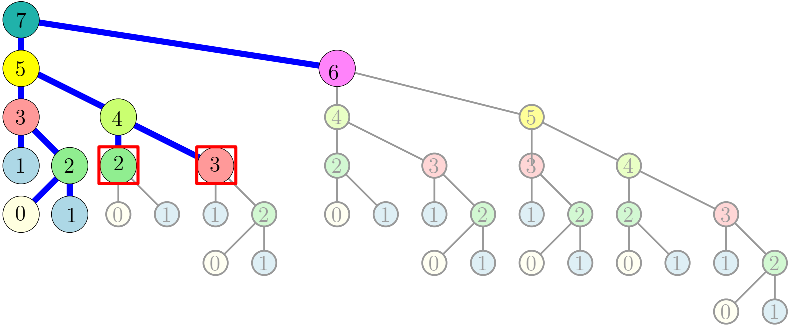

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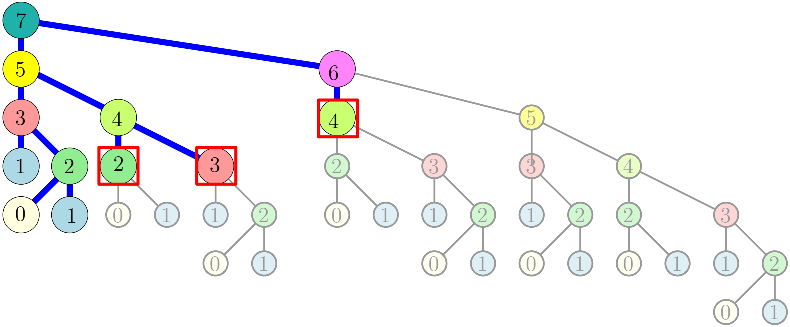

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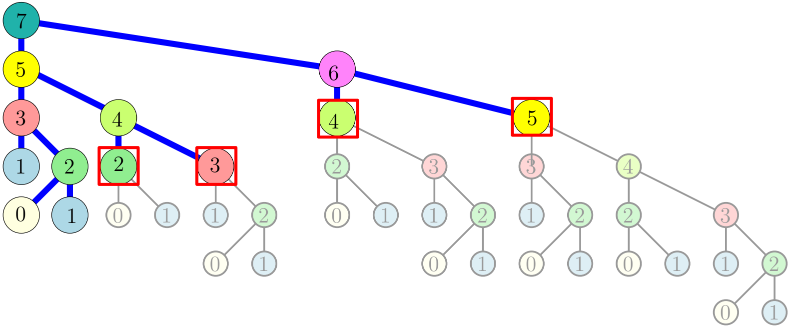

Automatic Memoization

1. Recursive version:

   $f(x_1, x_2, \ldots, x_d)$:
   CODE

2. Recursive version with memoization:

   $g(x_1, x_2, \ldots, x_d)$:
   if f already computed for $(x_1, x_2, \ldots, x_d)$ **then**
   return value already computed
   NEW_CODE

3. NEW_CODE:
   1. Replaces any "**return** α" with
   2. Remember "$f(x_1, \ldots, x_d) = \alpha$"; **return** α.

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Explicit vs Implicit Memoization

1. Explicit memoization (on the way to iterative algorithm) preferred:
   1. analyze problem ahead of time
   2. Allows for efficient memory allocation and access.
2. Implicit (automatic) memoization:
   1. problem structure or algorithm is not well understood.
   2. Need to pay overhead of data-structure.
   3. Functional languages (e.g., LISP) automatically do memoization, usually via hashing based dictionaries.

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Explicit/implicit memoization for Fibonacci

```
Init:   M[i] = −1, i = 0,...,n.

Fib(k):
    if (k = 0)
        return 0
    if (k = 1)
        return 1
    if (M[k] ≠ −1)
        return M[n]
    M[k] ⇐ Fib(k − 1) + Fib(k − 2)
    return M[k]
```

Explicit memoization

```
Init:   Init dictionary D

Fib(n):
    if (n = 0)
        return 0
    if (n = 1)
        return 1
    if (n is already in D)
        return value stored with n in D
        val ⇐ Fib(n − 1) + Fib(n − 2)
    Store (n, val) in D
    return val
```

Implicit memoization

How many distinct calls?

```
binom(t, b)    // computes (t choose b)
    if t = 0 then return 0
    if b = t or b = 0 then return 1
    return binom(t − 1, b − 1) + binom(t − 1, b).
```

How many distinct calls does **binom**$(n, \lfloor n/2 \rfloor)$ makes during its recursive execution?

- $\Theta(1)$.
- $\Theta(n)$.
- $\Theta(n \log n)$.
- $\Theta(n^2)$.
- $\Theta\left(\binom{n}{\lfloor n/2 \rfloor}\right)$.

That is, if the algorithm calls recursively **binom**$(17, 5)$ about 5000 times during the computation, we count this is a single distinct call.

Running time of memoized binom?

```
D: Initially an empty dictionary.
binomM(t, b)    // computes (t choose b)
    if b = t then return 1
    if b = 0 then return 0
    if D[t, b] is defined then return D[t, b]
    D[t, b] ⇐ binomM(t − 1, b − 1) + binomM(t − 1, b).
    return D[t, b]
```

Assuming that every arithmetic operation takes $O(1)$ time, What is the running time of **binomM**$(n, \lfloor n/2 \rfloor)$?

- $\Theta(1)$.
- $\Theta(n)$.
- $\Theta(n^2)$.
- $\Theta(n^3)$.
- $\Theta\left(\binom{n}{\lfloor n/2 \rfloor}\right)$.

THE END

...

(for now)