7.4.2
Parse trees and ambiguity

Parse Trees or Derivation Trees

A tree to represent the derivation $S \rightsquigarrow^* w$.

- Rooted tree with root labeled S
- Non-terminals at each internal node of tree
- Terminals at leaves
- Children of internal node indicate how non-terminal was expanded using a production rule

A picture is worth a thousand words

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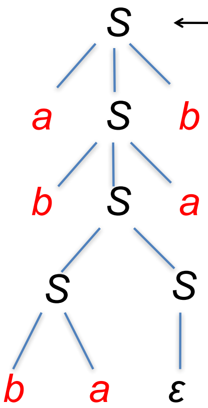
Example



A derivation tree for *abbaab*

(also called "parse tree")

$S \rightarrow aSb \mid bSa \mid SS \mid ab \mid ba \mid \varepsilon$

A corresponding derivation of *abbaab*

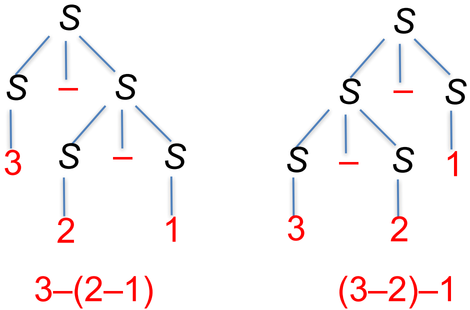
$S \rightarrow aSb \rightarrow abSab \rightarrow abSSab \rightarrow abbaSab \rightarrow abbaab$

Ambiguity in CFLs

Definition

A CFG G is ambiguous if there is a string $w \in L(G)$ with two different parse trees. If there is no such string then G is unambiguous.

Example: $S \rightarrow S - S \mid 1 \mid 2 \mid 3$

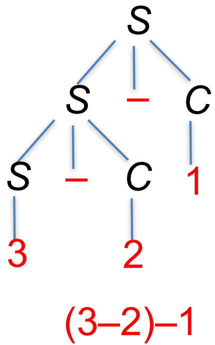


Ambiguity in CFLs

- Original grammar: $S \rightarrow S - S \mid 1 \mid 2 \mid 3$
- Unambiguous grammar:

  $S \rightarrow S - C \mid 1 \mid 2 \mid 3$

  $C \rightarrow 1 \mid 2 \mid 3$



The grammar forces a parse corresponding to left-to-right evaluation.

Inherently ambiguous languages

Definition

A CFL L is inherently ambiguous if there is no unambiguous CFG G such that $L = L(G)$.

- There exist inherently ambiguous CFLs.
  Example: $L = \{a^n b^m c^k \mid n = m \text{ or } m = k\}$
- Given a grammar G it is undecidable to check whether $L(G)$ is inherently ambiguous. No algorithm!

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THE END

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(for now)