5.2
Closure Properties of Regular Languages

Regular Languages

Regular languages have three different characterizations

- Inductive definition via base cases and closure under union, concatenation and Kleene star
- Languages accepted by DFAs
- Languages accepted by NFAs

Regular language closed under many operations:

- union, concatenation, Kleene star via inductive definition or NFAs
- complement, union, intersection via DFAs
- homomorphism, inverse homomorphism, reverse, ...

Different representations allow for flexibility in proofs

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Example: PREFIX

Let L be a language over Σ.

Definition

$\text{PREFIX}(L) = \{w \mid wx \in L, x \in \Sigma^*\}$

Theorem

If L is regular then PREFIX(L) is regular.

Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA that recognizes L

$X = \{q \in Q \mid s \text{ can reach } q \text{ in } M\}$ $Y = \{q \in Q \mid q \text{ can reach some state in } A\}$

$Z = X \cap Y$

Create new DFA $M' = (Q, \Sigma, \delta, s, Z)$

Claim: $L(M') = \text{PREFIX}(L)$.

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Exercise: SUFFIX

Let L be a language over Σ.

Definition

$\mathrm{SUFFIX}(L) = \{w \mid xw \in L, x \in \Sigma^*\}$

Prove the following:

Theorem

If L is regular then $PREFIX(L)$ is regular.

Exercise: SUFFIX

An alternative "proof" using a figure

THE END

...

(for now)